WCT Signal Simulation Algorithms

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October 16, 2020

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1 Introduction

This note explains the Wire-Cell Toolkit (WCT) signal simulation algorithm.

2 Inputs

The inputs to the algorithm are listed briefly with additional details in the following subsections:

- *drifted depos* describe a distribution of ionization electrons over a *response plane* and over time.
- *wire geometry* describes the physical location and logical channel connection of the sense wires
- *field response* describes how a drifting charge induces current on a sense wire
- *electronics response* describes the output voltage response of the readout electronics to input current
- detector response describes the convolution of field and electronics responses

2.1 Depos

A single *depo* ("deposition") represents a localized group of ionization electrons produced over a small segment of a charged particle trajectory (aka a "Geant4 step"). It is modeled as a 7-tuple:

$$(x, y, z, t, N_e, \sigma_\parallel, \sigma_\perp) \tag{1}$$

The first four parameters given the space-time location of the center of the group, N_e gives the number of electrons and the last two parameters give an extent of the group in two dimensions:

- the $\sigma_\parallel$ width measured in the *longitudinal* dimension that is along the nominal drift direction (typically $-x$)
- the $\sigma_\perp$ width is measured in the *transverse* dimension that is parallel to the wire plane, perpendicular to the nominal drift direction

Typically depo information is input to WCT at the point of ionization and so the widths are zero. Optionally the energy loss (dE) and step size (dx) may be given and WCT will calculate the number of electrons N_e.

The depos must be *drifted* to a *response plane* which is parallel to and near the wire planes. WCT provides this operation taking into account effects such as diffusion (broadening of the group widths), absorption (reduction of N_e) and stochastic fluctuations.

Once drifted, all depos have common x values lying on the response plane which is typically $\approx$ 10cm from the wire planes. They will have a variety of values for the remaining elements of the 7-tuple and must be ordered by their time value t.

2.2 Wires

The sense wires of the detector are individually described by their two endpoints and an identifier for the channel to which their current is directed. Furthermore, sets of these wire 7-tuples are formed by associating them with their wire plane. The wire centers may be used to order the wires within a plane and assigning each wire a wire-in-plane index number. The algorithm assumes wires in a plane have uniform pitch and direction.

2.3 Field response

Each plane has an associated *field response* which gives the electric current induced in a particular wire-of-interest (WOI) by the drifting of an electron in the applied electric field. In general, to provide a field response requires solving many Laplace equations. One equation must be solved to provide the electrostatic *drift field* for a particular arrangement of conductors. Then, one Laplace equation must be solved for each WOI.

Current WCT simulation employs approximations to reduce the computational complexity of providing field response. The fields are calculated in a two-dimensional space (ie, longitudinal vs transverse, assuming all wires are parallel and infinite). A translational symmetry along the wire pitch direction is taken. And, finally, there must exist a *sampling scale* such that within a sample the variation of the field response may be neglected.

Given a drift field and a starting point of an electron the *drift path* of that electron is determined. Only a subset of possible drift paths are considered and which are said to originate on an *impact position*. These positions form a regular, one-dimensional lattice along the *response plane* (itself a

line in the 2D model). Because of the translational symmetry assumption, the set of considered paths are further limited to be near to a single WOI.

Given this description we may represent a field response for a WOI in wire plane p as a 2-tensor: $F^{\alpha}_{p,s}$ with indices α marking the plane, p marking the drift path and s marking the step in the path. Details of how this discretization is performed will be omitted here but this tensor has a shape of 210×2000. The 210 p indices cover ten bins across the region around the WOI and extend over the ten nearest wires on either side. The drift path sampling s covers a fine sampling frequency which is a multiple of the final ADC sampling.

2.4 Electronics response

The *electronics response* is in the form of a parameterized function giving the output response voltage of an electronics channel to an induced unit of current. It is internally sampled at the same fine time scale as the field response s index. A nominal electronics response is specified for all channels and per-channel variation may also be specified for application as a convolutional filter by the simulation. The nominal electronics response, sampled along fine drift time samples, is written as E_s and the per-channel variation filter may be written as $f^{w}_{w,t}$ or $f^{c}_{c,t}$ depending of if it is applied on a per-wire or per-channel basis.

3 Internals

This section describes the internal algorithm steps.

3.1 Detector response

As an optimization, the discrete field response for plane α and the sampled, nominal electronics response function may be immediately convolved to produce a smaller *detector response* tensor.

$$R^{\alpha}_{p,s} = \sum_{s'} F^{\alpha}_{p,s-s'} E_{s'} = (F^{\alpha} \otimes E)_{p,s}$$

We evaluate this 1-D convolution so that the resulting s indices cover the larger drift path step time sampling which matches that of the ADC, (2 MHz / 0.5 μs). Thus, this tensor is sized typically 210×200.

This tensor gives the partial contribution to a voltage input to one ADC sample from a wire in plane α due to an electron drifting through sample time s on a path that began at impact position p relative to the wire of interest.

3.2 Ionization electron distribution

From the *drifted depo* information we conceptually construct a three-dimensional *signal tensor* for a given wire plane α that describes the distribution of ionization electrons transversely at the *response plane* and in time: $S^{\alpha}_{p,s}$. As above, the p index runs along impact positions and the s index runs along the drift path steps.

This signal tensor is populated from *drifted depo* information in the following manner. Each depo of finite width is assumed to represent a 2D Gaussian distribution (in transverse *vs* longitudinal space). This distribution is sampled along bins of p and s and truncated at some fixed number of σ in both dimensions (enlarged where needed to cover integral number of bins). The sampling is

fluctuated preserving normalization. The result is called a "patch" as it covers some rectangular patch of the *pvss* space.

The ensemble of patches from all drifted depos have a variety of size and shape and many will be mutually overlapping. The final signal tensor is constructed as the added composition of all patches. Fully expanded this signal tensor can be rather large. For the largest plane of MicroBooNE it is about 10,000 bins in the longitudinal direction and 30,000 bins transverse. ProtoDUNE and DUNE single phase readout time and wire count is smaller and would produce about 10,000 by 10,000 signal tensor. Below we discuss how to factor the signal tensor to manageable chunks.

3.3 Detector-signal convolution

The *voltage waveform* tensor gives partial waveforms due to current induced on each sense wire as if that wire was attached to an electronics channel in isolation. We write this tensor for plane α as

$$W^{\alpha}_{p_w,s} = \sum_{p',s'} R^{\alpha}_{p_w-p',s-s'} S^{\alpha}_{p',s'}$$

Here, p_w represents the impact position bin (specifically its lower edge) at which the center of wire w exists. The index p' effectively runs over values such that $p_w - p'$ spans the width in impact bins of the detector response. As before s' runs over the drift steps and spans a range which is the sum of the longitudinal width of both the signal tensor and the detector response tensor.

3.4 Convolution factoring

Note, that as $W^{\alpha}_{p_w,s}$ is built one must iterate over i and this requires recomputing the same summand multiple times. We may swap iteration and sum and define a *per-impact voltage waveform* tensor:

$$W^{\alpha,i}_{p_w,s} = \sum_{p',s'} R^{\alpha}_{p_w-p',s-s'} S^{\alpha}_{p',s'}$$

Where i index runs over the number of impact bins n_{imp} covering one wire region (eg 10) and p' runs over the set $\{(p \mod n_{imp}) == i\}$. That is, it skips every n_{imp} impact bin with an offset of i.

Due to this "skipping" each convolution is striped over the signal tensor and thus the full signal tensor need not be constructed at one time. Instead its size in the transverse dimension (impact positions) is reduced by a factor equal to the number of impact bins per wire (eg 10). This has two practical computational benefits. First, the redundant summand calculations are avoided and second the amount of memory required is reduced. This memory reduction is not a factor of (eg) 10 but of 5 as ultimately one must maintain an accumulator $W^{\alpha}_{p_w,s}$ tensor as one iterates over i.

3.5 Wire-channel aggregation

Some detectors, notably ProtoDUNE-SP and DUNE-SP have multiple wires feeding to one channel. The channel identifier in the wire geometry information is used to correlate wires on common channels and sum their partial waveforms.

$$M^{\alpha}_{c,s} = \sum_{w \in c,i} W^{\alpha,i}_{p_w,s}$$

That is the measured voltage in channel c over sample time s is found by simply summing over the relative impact bins i for all per-impact voltage waveforms elements associated to wires w

attached to the channel. Of course for detectors that have one wire per channel, the $w \in c$ selection is trivial.

4 Output processing

The channel-level signal tensor, $M^{\alpha}_{c,s}$, may be summed with a same-sized voltage-level *noise tensor* generated from one of WCT's noise simulation models. As it is voltage-level it also has the nominal electronics response applied. Per-channel response variations may then be applied as a set of 1D convolutional filters over time index s. Finally, WCT provides a simple ADC model to digitize the single-precision voltage values into limited (eg 12 bit) integers.