Muon Anomalous Magnetic Moment

Outline

1) Present Exp (82) vs Theory
\[ \Delta Q_{\mu} = z(59)_{EP} (35)_{LBL} (12)_{EW} (63)_{Exp} \times 10^{-11} \]

2) New Physics Implications (SUSY...)

3) Standard Model Uncertainties
   (The \( \tau \) \( \rightarrow 3 \pi \) Problem)

4) Future Theory + Exp. (E89 at BNL)

5) Conclusions
\[ a_\mu = \frac{g_\mu-2}{2} = \frac{\alpha}{2\pi} + \ldots \]

\[ E821 \rightarrow a_\mu^{\exp} = 116.592080(63) \times 10^{-11} \]

\[ \text{Standard Model} \rightarrow a_\mu^{SM} = 116.591812(59)_{VP}(35)_{LBL}(2)_{EW} \times 10^{-11} \]

\[ \Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} = 268(59)_{VP}(35)_{LBL}(2)_{EW}(63)_{exp} \times 10^{-11} \]

\[ \text{(69)} \]

\[ \text{(93)} \]

2.9 sigma Discrepancy!

Future goal > 5sigma
2) New Physics Implications:

SUSY Loops:

$\Delta Q_\mu = 268 \times 10^{-11}$

Natural Explanation:

$\overline{m}_{\text{SUSY}} = 70 \sqrt{\tan \beta \ GeV} \approx 100 - 500 \ GeV$

$\tan \beta = 2 \left( \overline{m}_{\text{SUSY}} \right)^2$ (Best Determination)}
ii) Muon Mass Generation via Loops (e.g., Dynamics, Extra Dim...,)

\[ m_\mu^0 = 0, \quad m_\mu = \mu \quad \Delta a_\mu = \mu \quad \propto \Lambda \text{ scale of new physics} \]

\[ \Delta a_\mu = z68 \times 10^{-11} \quad \Lambda \approx 2 \text{ TeV} \quad \text{(Scale Probed by LHC)} \]

Many Examples!

E821 Results > 350 Citations
3.) **Standard Model Uncertainties**

\[ a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EN} + a_\mu^{Hadronic} \]

i) \[ a_\mu^{QED} = \frac{\alpha}{2\pi} + 0.765857376 \left( \frac{\alpha}{\pi} \right)^2 + 24.05050898 \left( \frac{\alpha}{\pi} \right)^3 + 131.0 \left( \frac{\alpha}{\pi} \right)^4 + 663(80) \left( \frac{\alpha}{\pi} \right)^5 \ldots \]

\[ \alpha^{-1} = 137.03599877(40) \text{ from } a_\mu^{QED} \]

\[ a_\mu^{QED} = 116584718.9(0.9)(0.1) \times 10^{-11} \text{ Very Precise} \]

ii) \[ a_\mu^{EN} = \text{diagrams} + \text{diagrams} + \text{2 loops} \ldots \]

(tiny)
2 loop order: 

\[ A_\mu (2\text{loop}) = -40.7 \times 10^{-11} \] 

\[ A_\mu (3\text{loop}) \approx O(10^{-12}) \quad \text{Negligible} \] 

\[ A_\mu = 154(2) \times 10^{-11} \]

iii) Hadronic 

\[ A_\mu = \text{Hadrons} + \text{VP} + \text{LBL} \]

Vacuum Pol.

Light by Light
Use $e^+e^- \rightarrow \text{Hadrons}$ Cross Section + Dispersion Relation

$e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ Dominates $\sim 72\%$ ($\rho$ dominated)

Alternatives:

$\pi^+(\pi^+ \rightarrow \pi^-\pi^0) + \text{Isospin Corrections} \quad \not\!\pi^+ \not\!\pi^- \not\!\pi^0$

or Pure Lattice Calculation (T. Blum)

For LBL: Pion Pole + Short-Distance

or Lattice (Same Day)

\[ a_\mu^{LBL} = +120 (35) \times 10^{-11} \text{ Conservative Error} \]

Interesting History

Now Stable
Role of $e^+e^- \rightarrow \pi^+\pi^-$ in evaluation of the hadronic contribution to muon $(g-2)$

Hadronic contribution to the muon $(g-2)$ is calculated via dispersion integral:

$$a^\text{had}_\mu \ (l.o.) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m^2_\pi}^\infty ds \frac{K(s)}{s^2} R(s)$$

Contribution to the integral from different modes $e^+e^- \rightarrow \text{hadrons}$:

- $e^+e^- \rightarrow 2\pi$ gives dominant contribution both to the value and to the uncertainty of the hadronic contribution
At low $s$ the cross-section is measured independently for each final state.
**Hadronic Vacuum Polarization (L.O.) (lowest order)**

New $e^+e^-$ results \( \text{CMDZ, SMD, KLOE} \)

\( e^+e^- \rightarrow \pi^+\pi^- \quad e^+e^- \rightarrow \gamma\pi^+\pi^- \) (Radiative Return)

*All Consistent Now*

\[
q_\mu (\text{L.O.}) = 6917(59) \times 10^{-11}
\]

+ Higher Order \((-98 \times 10^{-11})\) + LBL \((+120(35) \times 10^{-11})\)

Hadronic

\[
q_\mu = 6939(59)_{\text{wp}}(35)_{\text{LBL}} \times 10^{-11}
\]

\[
q_\mu^{SN} = 116591812(59)(35)_{\text{wp}}(2)_{\text{SN}} \times 10^{-11}
\]

\[
\Delta q_\mu = 268(59)_{\text{wp}}(35)_{\text{LBL}}(2)_{\text{EN}}(63)_{\text{Exp}} \times 10^{-11}
\]
Pion formfactor (CMD-2)

Analysis finished. Not published yet.

Systematic error
0.7%  0.6% (95)/ 0.8% (98)  1.2-4.2%
Pion formfactor (SND)

Systematic error
3.2% 1.3%
Comparison of CMD2(95) and CMD2(98)

\[ \Delta(95-98) \approx 0.7\% \pm 0.5\% \]

Plotted is

\[
\frac{\Delta F}{F} = \frac{|F_\pi|^2 \text{ (exp)}}{|F_\pi|^2 \text{ (CMD-2 fit)}} - 1
\]
Comparison with KLOE
Implication to $a_\mu$ (very unofficial)

Davier, Marciano-2004:

$$\Delta a_\mu = (23.9 \pm 7.2_{\text{had,LO}} \pm 3.5_{\text{other}} \pm 5.8_{\text{exp}}) \times 10^{-10}$$

- $0.610 < \sqrt{s} < 0.960$ GeV

CMD-2 (95): $379.7 \pm 2.6 \pm 2.3$ (3.5)

CMD-2 (98): $374.3 \pm 1.8 \pm 3.0$ (3.5)

CMD-2 (comb): $377.05 \pm 2.2 \pm 1.5$ (2.7)

SND: $376.7 \pm 1.3 \pm 4.9$ (5.1)

KLOE: $375.6 \pm 0.8 \pm 4.9$ (5.0)

S. Eidelman, private communication
The $\tau \rightarrow \pi \pi \nu$ Problem

$\Gamma (\tau \rightarrow \pi \pi \nu) + Isospin \ (QED) \text{ Corrections seem to disagree with } e^+ e^- \rightarrow \pi^+ \pi^-$

Data From Aleph, Cleo, Opal

Alleviates $\sim \frac{1}{2}$ Discrepancy

But $\sigma (e^+ e^- \rightarrow \pi^+ \pi^-)$ data from CMD2, SND + KLOE also consistent

Main Difference $\sqrt{s} \gtrsim 16 \text{GeV Spectrum}$

Recent Belle: $\tau \rightarrow 4 \pi \pi \nu$ data statistically powerful!

Spectrum Shape Disagrees with Aleph!

$M_\rho = 773.9^{+0.1} _{-0.0} \text{ MeV vs } 775.5^{+0.7} _{-0.7} \text{ MeV Aleph}$

+ other differences $\Gamma$, $M_\rho$, $M_\omega$ BR($\tau \rightarrow \nu \pi^+ \pi^- \pi^0$) somewhat smaller (25.15%)

But claim to agree with Aleph on $Q_\mu^{\text{Hadronic}}$? Isospin Corr.!
Comparison with ALEPH ($\tau \rightarrow \pi \pi^{0} \nu$)
Recent Belle analysis of tau decays

Pion formfactor, calculated from the spectral function of $\tau^{-}\rightarrow\pi^{-}\pi^{0}v_{\tau}$ decay

EPS2005 Proceedings - hep-ex/0512071
Comparison with ALEPH and BELLE tau decay data
4) Future Theory + Exp (E969 at BNL)

\[ \Delta a_\mu = 268 (36)_{\text{Exp}} (27)_{\text{Exp}} \times 10^{-11} \]

BaBar, Belle, CLEO, Nooosibirsk

Tau Data? → (25)_{VP}

\[ \text{(30)}_{\text{Exp}} \] or \[ \text{(27)}_{\text{Exp}} \] if ave with E821

\[ \text{(E969)} \]

\[ \Delta a_\mu = 268 (36)_{\text{Exp}} (27)_{\text{Exp}} \times 10^{-11} \] or \[ 268 (32)_{\text{Exp}} (27)_{\text{Exp}} \times 10^{-11} \]

\[ \text{6.5sigma} \] or \[ \text{6.4 sigma} \]

Remains 5\sigma down to → 225 \times 10^{-11}

\[ 209 \times 10^{-11} \]

\[ (167 \times 10^{-11} \rightarrow 4\text{sigma}) \]
VEPP-2M collider: 0.36-1.4 GeV in c.m., $L \approx 10^{30} \text{1/cm}^2\text{s at 1 GeV}$

- Detectors CMD-2 and SND: 50 pb$^{-1}$ collected in 1993-2000
Future measurements at VEPP-2000

- Factor >10 in luminosity
- Up to 2 GeV c.m. energy
- CMD-3: major upgrade of CMD-2 (new drift chamber, LXe calorimeter)

- measure $2\pi$ mode to 0.2-0.3%
- measure $4\pi$ mode to 1-2%
- overall improvement in $R$ precision by factor 2-3
Conclusions: E969 Goal ±30x10^-11 Well Motivated
> 5σ Discovery Potential
(Even if T-data moves δψ down somewhat)

Complements LHC Discoveries of tanβ_{susy}

Exp. & Theory Unc. Well Matched
Over Next Few Years

E969 - Must Do Experiment