PROTON DECAY: THE MISSING PIECE OF GRAND UNIFICATION

(The Need for A NEXT Generation Detector)

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UDIG WORK SHOP, BNL (oct, 2008)

REFERENCES:

Babu, Pati, Wilczek, hep-ph/9812538

Babu, Pati, Tavartkiladze - TO APPEAR

JCP, Review Talk - hep-ph/0606089

I. Introduction

A) Improved Studies of

① Proton Decay

② ν oscillations

2 indispensable tools to probe nature at the shortest distance $\sim 10^{-30}$ cm.

In a broader sense, these two processes get intimately connected with each other within Grand Unification.

In fact both are predictions of a well-motivated class of Grand Unification symmetries.

I will therefore begin by first listing evidence which has built up over the years in favor of Grand Unification, in particular for a certain class of Grand Unification Symmetries.

B) Alt. Routes To Grand Unification

$$G(213) = SU(2)_L \times U(1)_Y \times SU(3)^C$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3}, \quad (u_{r,y,b})_R^{4/3}, \quad (d_{r,y,b})_R^{-2/3}, \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1}, \quad (e^-_R)^{-2}$$

5 disconn. multiplets // Arb. Y_W, $SU(2)$, $SU(3)^C$ Q. Nos.

↓

1972 $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R} \times (L \leftrightarrow R)$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \quad F^e_L = (2,1,4) \quad F^e_R = (1,2,4)$$

$$Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}$$

Advantages of G(224)

- All 16 in one L-R Conj. multiplet // L-R symm
- Explain Y_W & All Q. Nos. // Quantize Q_{em} //
- $Q_{e^-} = -Q_p$ // (ν_R // B-L) ← Need For seesaw & Leptogenesis

↓

SO(10): 16 (one coupling)

All Advantages of G(224) retained by SO(10), not SU(5)

SU(5): $\bar{5} + 10$ → Problem with ν Masses & Leptogenesis.

NO ν_R, NO B-L

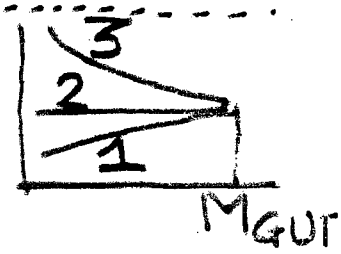
C) EVIDENCE FOR GRAND UNIFICATION

1) Family Structure - Quantum Nos.

2) Charge Quantization

3) $Q_{e^-} = -Q_p$

4) Meeting of The 3 gauge Couplings
SUSY Grand Unification
$M_{GUT} \simeq 2 \times 10^{16}$ GeV



5) $\sqrt{\Delta m^2(\nu)_{23}} \approx 1/20$ eV $\leftrightarrow SU(4)_f \rightarrow \nu_R$ // B-L
$m(\nu_\tau)_{Dirac}$ //
$M_{GUT} \rightarrow$ Maj ν_R
Seesaw

6) $m_b(GUT) = m_\tau$] $\rightarrow SU(4)^c$

7) $m(\nu_\tau)_{Dirac} = m_{top(GUT)}$]

8) $\theta(\nu_\mu - \nu_\tau) \approx \pi/4 \leftrightarrow V_{cb} \approx 0.04$

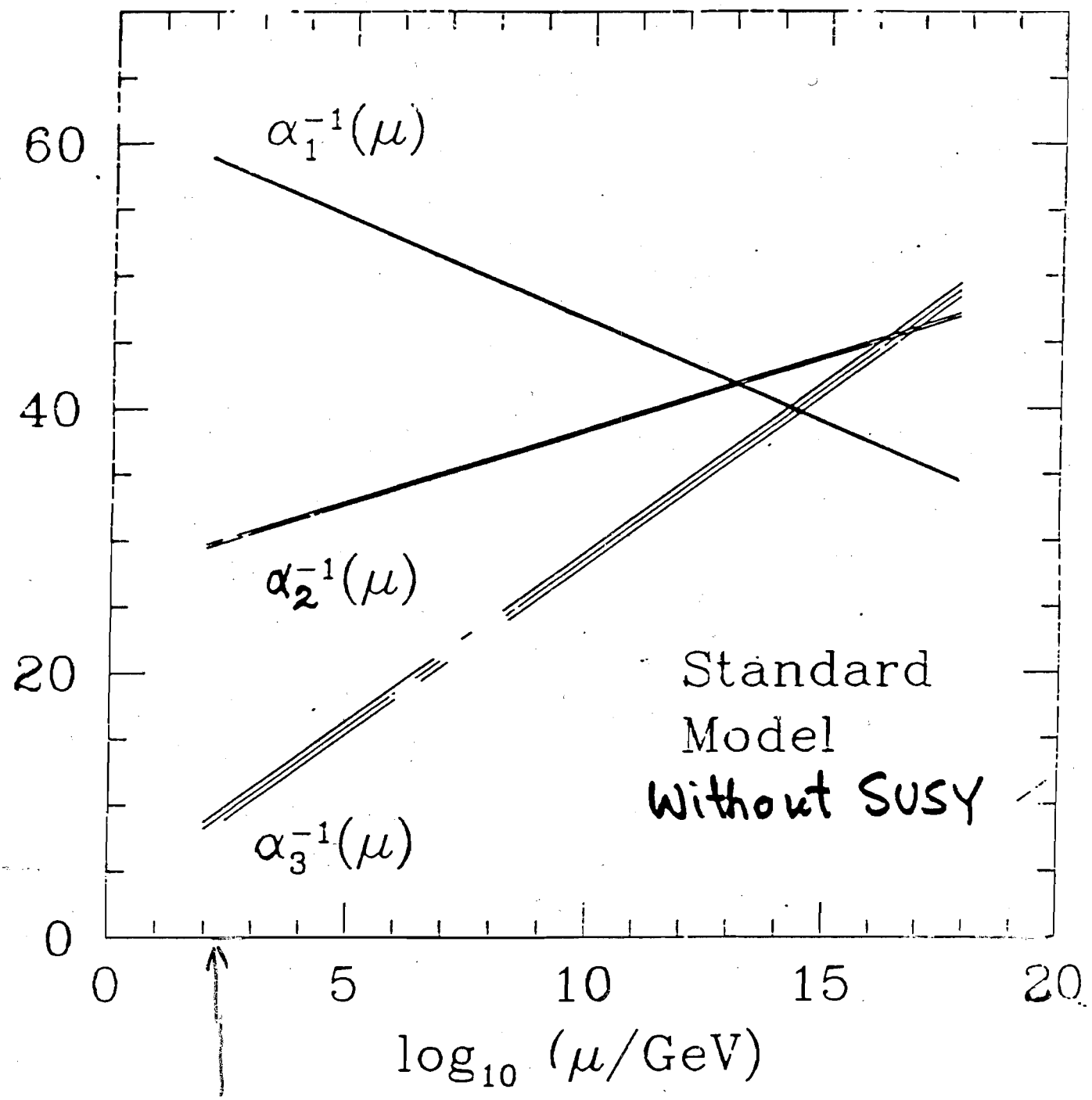
9) LEPTOGENESIS $\rightarrow Y_B \sim 10^{-10} \rightarrow \nu_R$/B-L

$SO(10)$ // $G(224)$

SUCCESS of All 9 Features Non-Trivial!

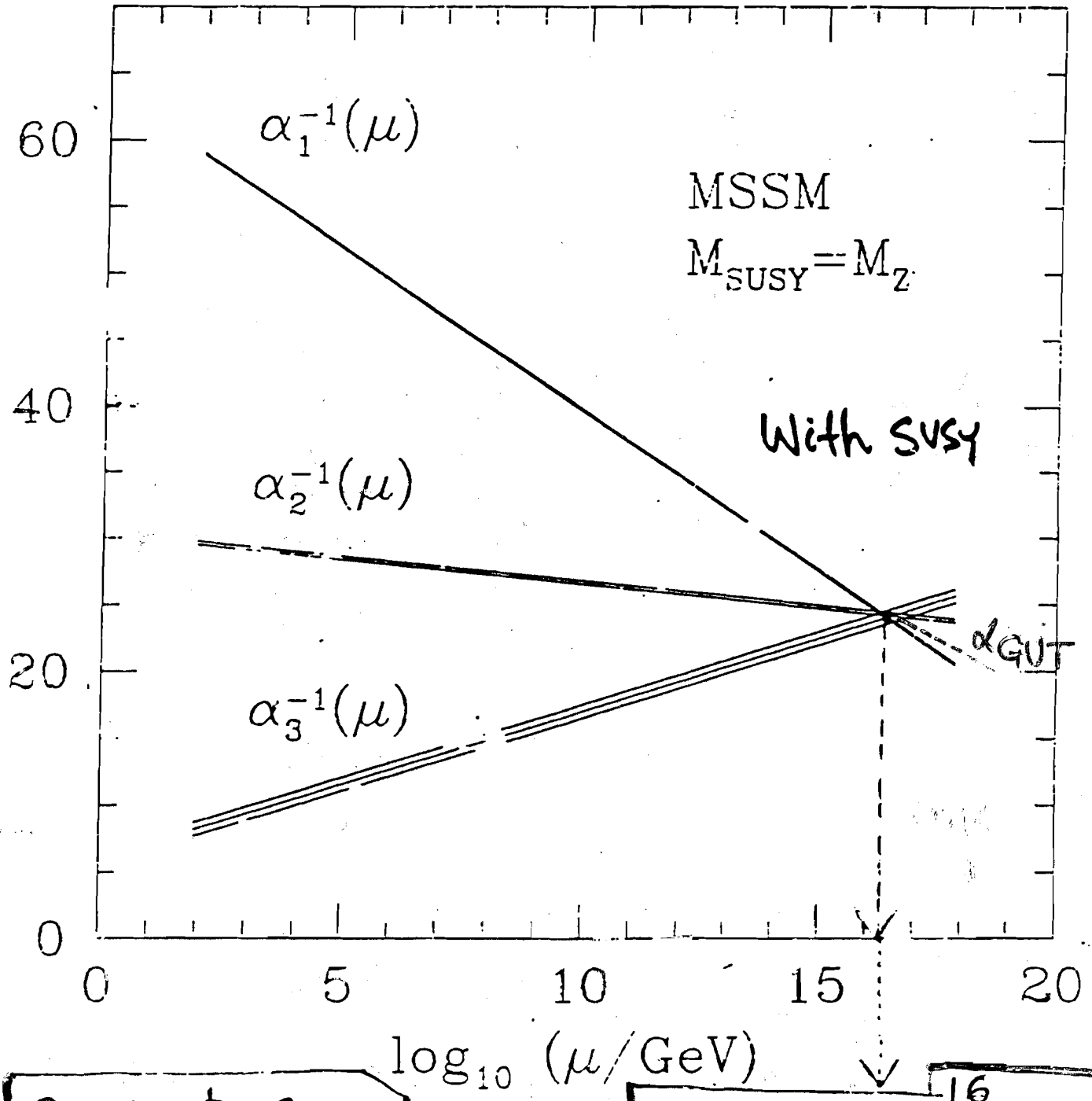
TOGETHER MAKE A STRONG CASE FOR SUSY GRAND UNIF // SIMULT. FOR AN EFF. SYMMETRY LIKE SO(10) OR MINIMALLY A STRING-DERIVED G(224) SYMMETRY

Coupling Unification Without SUSY



α_i's Measured at LEP
at $\sim$ 100 GeV

Gauge Coupling Unification With SUSY



Supports SUSY Unification

$M_{GUT} \approx 2 \times 10^{16}$ GeV

$\sin^2\theta_W(m_Z)_{th} = 0.2315 \pm .003$

$\sin^2\theta_W(m_Z)_{EXPT} = 0.23124 \pm .00017$

$\alpha_3(m_Z)^{-1}_{theory} \simeq 0.125 - 0.13$

$\alpha_3(m_Z)_{obs} = 0.118 \pm .003$

GUT THRESHOLD CORRECTION

c) Insight From SuperK Result: $\sqrt{\Delta m^2_{23}} \approx 1/20$ eV

SeeSaw (ignore mixing for a moment)

$$m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{Dirac})^2}{M(\nu_R^\tau)}$$

(a) $m(\nu^\tau_{Dirac}) \approx m_t(M_X) \approx 120\,GeV$ ← SU(4) - Color, SU(5), $[SU(3')]^3$

$m_b^\circ \approx m_\tau^\circ$

(b) Get $M(\nu_R^\tau)$ from SUSY Unif. Scale: $M_X \approx 2\times10^{16}$ GeV

$$f_{33}\,16_3\,16_3\,\langle\overline{16}_H\rangle\langle\overline{16}_H\rangle \Rightarrow M(\nu_R^\tau) \sim \frac{(2\times10^{16}\,GeV)^2}{10^{18}\,GeV}$$

($f_{33} \approx 1$), $M \to \sim 10^{18}$ GeV

$$\approx 4\times10^{14}\,GeV\,(1/2\text{-}2)$$

$$m(\nu_L^\tau) \sim \frac{(120\,GeV)^2}{4\times10^{14}\,GeV} \approx \left(\frac{1}{30}\,eV\right)\left(\frac{1}{2}\text{ to }2\right)$$

Also get $m(\nu_L^\mu) \sim \frac{m(\nu_L^\tau)}{10} \Rightarrow \sqrt{\Delta m^2(\nu_2\nu_3)_{Th}} \approx \left(\frac{1}{30}\,eV\right)\left(\frac{1}{2}\text{-}2\right)$

Thus SuperK result brings to light the existence of ν_R // reinforces the ideas of a) See Saw // b) SU(4) Color // & c) SUSY Unif

In Short just this single piece of information

$(\sqrt{\Delta m^2(\nu_\mu \nu_\tau)} \sim 1/20 \text{ eV})$

Disfavors

- $SU(5)$, $[SU(3)]^3$ or SM
- Intermed. Scale ($\lesssim 10^{13}$ GeV) Breaking of B-L
- Large Extra Dim (ν_R in Bulk)

Favors

- $SU(2)_L \times SU(2)_R \times SU(4)^C$ // SO(10) Route To Higher Unification With SUSY
- Single Step Breaking of G(224) // SO(10) TO SM. symmetry at $M_X \sim 2 \times 10^{16}$ GeV

Thus,

> ν's hold the Key to the Nature of The Unification Symmetry !

As does the need for leptogenesis.

Some General Predictions of SUSY Grand Unif Based on SO(10) // String G(224)

1) Coupling Unification

2) ν - Oscillations

3) CP & Flavor Violations
$K^0-\bar{K}^0$, $B-\bar{B}$, edm, $\mu\to e\gamma$, $\tau\to\mu\gamma$

4) Baryogenesis via Leptogenesis

&

5) Proton Decay

↓

Correlated with Fermion Masses & Mixings

↓

∴ First Discuss Fermion Masses & Mixings in G(224) // SO(10)

II) SO(10) BREAKING / FERMION MASSES / ν OSC

Babu, Pati, Wilczek

$SO(10) \xrightarrow{\langle 45\rangle,\ \langle 16_H\rangle = \langle \overline{16}_H\rangle} G(213) \xrightarrow{\langle 10_H\rangle} U(1)\times SU(3)^C$

LOWEST DIM. HIGGSES (NO 126, 120, 210 ..) → GENERICALLY TOO LARE THRESHOLD CORR.

YUKAWA:
$16_i\ 16_j\ 10_H$
$16_i\ 16_j\ 10_H \cdot 45_H/M$
$16_i\ 16_j\ 16_H\ 16_H/M$

HIERARCHICAL BY U(1) FLAVOR SYMM $(S/M)^{n_{ij}}$

⇓

"33" ≫ "23" ~ "32" ≫ "22" ≫ "12" ETC.

$16_i\ 16_j\ \overline{16}_H\ \overline{16}_H/M$ → MAJ. MASS (ν_R)

PATTERN DET. BY SO(10)//G(224) AND Flavor sym.

⇓

① A PREDICTIVE FRAMEWORK FOR FERMION MASSES // ν OSC } BPW

⇕

② CP & FLAVOR VIOLS ($B-\bar{B}$, ϵ'/ϵ, edm, $\mu\to e\gamma$) } Babu, Pati, Rastogi

③ LEPTOGENESIS : $\boxed{Y_B/\sin 2\phi \sim (10\text{-}100)\times 10^{-10}}$ JCP

⇓

④ A PREDICTIVE FRAMEWORK FOR PROTON DECAY

$d=6 \to p\to e^+\pi^0$

$d=5 \to p\to \bar{\nu} K^+$

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\}$ + "S"

1st consider only μ & τ Families (Flavor sym: $\mu \neq \tau$) (BPW)

$\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3 \langle 10_H \rangle \longrightarrow$ 3rd Family: $m_b^0 = m_\tau^0$

$\propto (1) \leftarrow \cdots + \tilde{h}_{23}\, 16_2\, 16_3 \langle 10_H \rangle \langle S/M \rangle \longrightarrow$ 2nd Family

$\propto (\sigma) \leftarrow$ ANTISYMMETRIC, $\propto B-L$

$\propto (\epsilon) \leftarrow + a_{23}\, 16_2\, 16_3 \dfrac{\langle 10_H \rangle \langle 45_H \rangle}{M} \rightarrow m_\mu^0 \neq m_s^0$

$\propto (\hat{\eta}) \leftarrow \cdots + g_{23}\, 16_2\, 16_3 \langle 16_H \rangle \langle \overline{16}_H \rangle^D_{EW}/M$

$\hookrightarrow$ CKM $\neq 1$

$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3)$ Flavor sym

$$U = \begin{pmatrix} c & t \\ 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \qquad D = \begin{pmatrix} s & b \\ 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N_{Dirac} = \begin{pmatrix} \nu_\mu & \nu_\tau \\ 0 & -3\epsilon+\sigma \\ +3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \qquad L = \begin{pmatrix} \mu & \tau \\ 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{pmatrix} m_D^0$$

Note $q-\ell$ correlation ($SU(4)$-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \quad ; \quad D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \quad L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix} \times m_D^0$$

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2 New param (ϵ', η'), but 5 new observables just in (q,ℓ) system $\Rightarrow$ 3 New predictions for (q,ℓ) // With $\epsilon' = 0 \rightarrow m_u \rightarrow 0$

ν Majorana Masses

Saw before.

$M_R \approx 10^{15}$ GeV

Expect $y \sim 1/10$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

Note same hier. pattern as in Dirac sector

$$f_{ij}\, 16_i\, 16_j \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle / M_{st}$$

$$\downarrow$$

$$f_{ij}\, \nu_{iR}^T\, \bar{C}\, \nu_{jR}\, \langle \overline{16}_H \rangle^2 / M_{st}$$

$$(M_R^\nu)_{ij} = f_{ij} \langle \overline{16}_H \rangle^2 / M_{st}$$

Summary on Fermion Masses & Mixings (Babu, Pati Wilczek)

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,GeV$	$\approx 4.2\,GeV$
$\sqrt{\Delta m^2_{23}(\nu)} \sim \left(\frac{1}{24}eV\right)\left(\frac{1}{2}-2\right)$	$\approx (1/15 - 1/25)$ eV ⊛
$V_{cb} \approx 0.043$	≈ 0.04
$\sin^2 2\theta^{osc}_{\nu_\mu\nu_\tau} \approx \boxed{0.92} \leftrightarrow \boxed{0.995}$	$\approx 0.92 \leftrightarrow 1$
$m_{\nu_2}/m_{\nu_3} \approx 1/10 - 1/5$	
$V_{us} \approx 0.21$	≈ 0.22
$\lvert V_{ub}\rvert \approx 0.0032$	$\approx 0.003 - 0.004$
$m_d(1GeV) \approx 8\,MeV$	$\approx 6-8\,MeV$

$m(\nu_2) \approx (2-10)\times 10^{-3}\,eV \leftrightarrow$ { SMA $\sim 3\times 10^{-3}$ eV ; **LMA** $\approx 7\times 10^{-3}$ eV }

$m(\nu_1) \sim (1\ \text{to few})\times 10^{-3}\,eV$

Consistent with the framework

$M(\nu_R^\tau, \nu_R^\mu, \boxed{\nu_R^e}) \approx (10^{15}, 2\times 10^{12}, (1/3-3)\times 10^{10})$ GeV

Just right for Lepto/Baryogenesis

NOTE: ν_R Masses Necessarily Hierarchical

III PROTON DECAY: HALLMARK OF GRAND UNIFICATION

① d = 6 Gauge Med. SUSY SU(5)/SO(10)

$M_X \approx M_Y \approx 10^{16}$ GeV (1±25%)



$\Rightarrow$ Amp $(p \to e^+ \pi^0) \propto g^2/M_X^2$

Chiral Lag Param $(D+F) \simeq 1.25$ // $A_R \simeq 3.4$

$\alpha_H = 0.01\ \text{GeV}^3$ $(1/\sqrt{2} - \sqrt{2})$

$\Gamma^{-1}(p \to e^+ \pi^0) \simeq 10^{35 \pm 1}$ yrs (Theory)

Can be as short as $\sim 10^{34}$ yrs.

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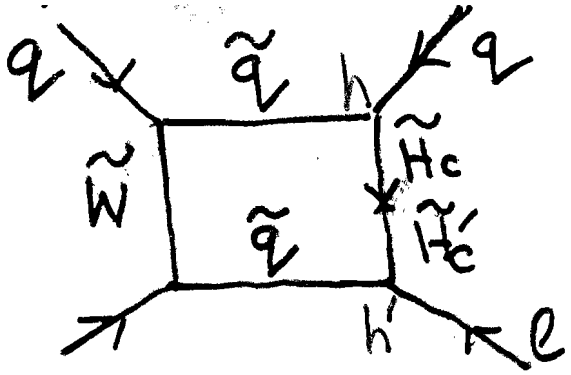
② SUSY SU(5) / SO(10) Standard d=5

Color Triplet Higgsino Med Sakai, Yanagida/Weinberg

Need D-T Splitting



$\tilde{H}_c \subset (10_H)_{SO(10)}$
$= (2,2,1) + (1,1,6)$

Amp $\propto (hh'/M)(m_{\tilde{W}}/m^2_{\tilde{q}})\alpha_2$

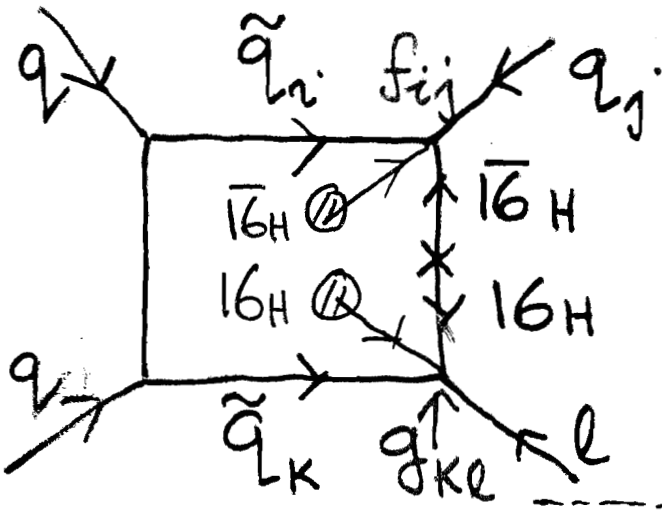
$M = M_{H_c}$ (SU(5))
$M = M_{eff}$ (SO(10))

$p \to \bar{\nu} K^+$ (dominant)
$\to \mu^+ K^0$ (suppressed) BR $\sim 10^{-5}$

$\Gamma^{-1}(p \to \bar{\nu} K^+) \sim 10^{30} - 10^{34}$ yrs.

③ New d=5 ν-Mass Related operators

Babu, Pati, Wilczek



$f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$\Downarrow$

Maj Masses of ν_R's

Generically $16_i\, \overline{16}_H$ in 45 & 1 of SO(10)

$\Longrightarrow g_{Kl}\, 16_K\, 16_l\, 16_H\, 16_H / M$

Indep of $\tan\beta$ $\quad\hookrightarrow V_{CKM} \neq \mathbb{1}$

$$\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34}\ \text{yrs}$$

SUSY SO(10) or G(224)

$$BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10 - 30)\%$$

Note these contributions from new d=5 would generically be present, even if "Standard" d=5 (with color triplets $\subset 10_H$) absent, as in SUSY G(224)!

The $\mu^+ K^0$ mode a signature of this mechanism.

Doublet-Triplet splitting By Fine Tuning

Consider SUSY SU(5)

Suppose $W \supset \lambda_1 \bar{5}_H' 24_H 5_H$

$+ \lambda_2 \bar{5}_H' 1_H 5_H$

$\langle 24_H \rangle = \text{diag}(x,x,x,y,y) \Rightarrow y = -\frac{3}{2}x$

$\text{Tr}\langle 24_H \rangle = 0$

$\langle 1_H \rangle = Z$ (Or Equivalently A Bare mass Term)

$5_H = 3^{*c} + 2_L$

$SU(2)_L$ Doublet Mass : WANT

$$\left(-\frac{3}{2}\lambda_1 x + \lambda_2 Z\right) \lesssim 10^{-13} (\lambda_1 x + \lambda_2 Z)$$

$\downarrow$

EXTREME FINE TUNING

UNNATURAL

c) NATURAL Doublet-Triplet splitting in SO(10) & M_{eff}

Dimopoulos, Wilczek // Babu, Barr
Barr, Raby // Babu, Pati, Wilczek

$$W_H = \lambda\, 10_H\, 45_H\, 10'_H + M_{10'}\, 10'^2_H + M_{16}\, 16_H\, \overline{16}_H$$
$$+ \lambda'\, \overline{16}_H\, \overline{16}_H\, 10_H$$

$$\langle 45_H \rangle = (a, a, a, 0, 0) \otimes i\tau_2 \quad ; \quad a \sim M_{GUT}$$

$$\langle 45_H \rangle \propto B-L. \rightarrow \langle (1,1,15) \rangle = \begin{pmatrix} a & a & a & -3a \end{pmatrix}.$$

$$(\bar{5}_{10_H} \quad \bar{5}_{10'_H} \quad \bar{5}_{16_H}) \begin{bmatrix} 0 & \lambda\langle 45_H\rangle & \lambda'\langle \overline{16}_H\rangle \\ -\lambda\langle 45_H\rangle & M_{10'} & 0 \\ 0 & 0 & M_{16} \end{bmatrix} \begin{pmatrix} 5_{10_H} \\ 5_{10'_H} \\ 5_{\overline{16}_H} \end{pmatrix}$$

$\Rightarrow$ One pair of Higgs doublets light, triplets Heavy

$$H_u = 10_u \; ; \quad H_d = \cos\gamma\; 10_d + \sin\gamma\; 16_d$$

(a) $\tan\gamma = \lambda' \langle \overline{16}_H \rangle / M_{16} \Rightarrow \boxed{\tan\beta = (\cos\gamma)(m_t/m_b)}$

(b) $$\boxed{A(d=5)_{std} \approx \left(h^2_{33}/M_{eff}\right)(2\times 10^{-5})}$$

$$\boxed{M_{eff} \equiv (\lambda a)^2 / M_{10'} \approx M^2_{GUT}/M_{10'}}$$

$\Rightarrow M_{eff} \neq M_{H_c}$; If $M_{10'} < M_{GUT} \Rightarrow \boxed{M_{eff} > M_{GUT}}$

D-W Pattern in G(224)-Language

$$G(224) = SU(2)_L \times SU(2)_R \times SU(4)^C$$

$$(45_H)_{SO(10)} = (3,1,1) + \boxed{(1,3,1)} + \boxed{(1,1,15^C)} + (2,2,6)$$

$$10_H = (2,2,1) + (1,1,6)$$

D-W Pattern $\rightarrow$ $\langle (1,1,15^C) \rangle \overset{\text{diag}}{=} (a,a,a,-3a)$

$SU(4)^C$ $\propto$ $(B-L)$

$$\langle (1,3,1) \rangle = 0$$

$$10_H . \langle 45_H \rangle 16_H$$

Do couple

$$= [(2,2,1) + (1,1,6)] [\langle (1,1,15) \rangle] [(2,2,1)' + (1,1,6)']$$

DO NOT COUPLE BECAUSE OF $SU(4)^C$

$$[6^C \times 6^C]_{SU(4)^C} = 1_S + 15_a + 20_s$$

$$\Rightarrow [6]_{SU(4)^C} = (3^C + \bar{3}^C) \text{ GET MASS}$$

$\Rightarrow$ COLOR TRIPLETS GET MASS

$SU(2)_L$ DOUBLETS MASSLESS

III FOUR ISSUES RELEVANT TO PROTON DECAY

NEED TO SPECIFY ALL FOUR RELEVANT TO PROTON DECAY

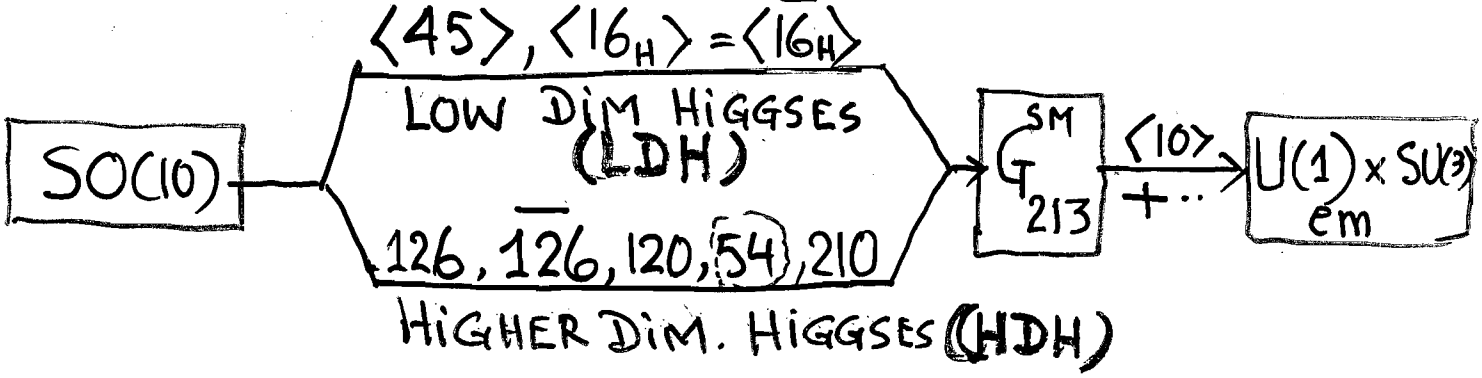
① HIGGSES FOR SO(10)-BREAKING

② ENSURE (NATURAL) D-T SPLITTING WITH STABILITY // NO PGB // NO FLAT DIR

③ PATTERN OF YUKAWA COUPLINGS ↔ FERMION MASSES // MIXINGS // ν-OSC IN A PREDICTIVE SYSTEM

④ INCLUDE ALL EW & GUT-SCALE THRESHOLD EFFECTS (CONSISTENT WITH) ↔ COUPLING UNIF. & PROTON DECAY

① → TWO DIFFERENT HIGGS SYSTEMS



ALL 3 ISSUES (2, 3 & 4) NOW ADDRESSED IN LDH // NOT IN HDH (2&4)

IV STABILIZING D-T SPLITTING // PROVIDING UPPER LIMITS ON $\bar{\Gamma}^{-1}(p\to\bar{\nu}K^+) \leftrightarrow \bar{\Gamma}^{-1}(p\to e^+\pi^0)$

Babu, Pati, Tavartkiladze (To Appear)

COMBINE BPW
SINGLE $45_H, 16_H, \overline{16}_H, 10, 10'$

$\oplus$

BARR, RABY
$16', \overline{16}'$

(S, Z) $\overline{16}_H \cdot 45_H \cdot 16'_H$

$U(1)_A \times Z_2 \longrightarrow$ DESIRED SUPERPOTENTIAL

$\langle 45_H \rangle = (a,a,a,0,0)\otimes Z_2 \sim M_{GUT}$

DIMOPOULOS, WILCZEK

→ • D-T SPLITTING ABSOLUTELY STABLE

→ NO PGB / NO FLAT DIR.

→ INCLUDE ALL TRHESHOLD CORR. (EW+GUT)

⇓ BONUS

CORELATE $M_{eff} \longleftrightarrow M_X$

$\Gamma^{-1}(p\to\bar{\nu}K^+) \longleftrightarrow \Gamma^{-1}(p\to e^+\pi^0)$

⇒ CONSTRAINED UPPER LIMITS ON BOTH FULLY ACCESSIBLE

D-T SPLITTING & SUPERPOTENTIAL

Babu, JCP, Tavartkiladze // (Barr, Raby Modified)

$\{45_H, 16_H, \overline{16}_H, 10_H, 10'_H\}, \{16'_H, \overline{16}'_H\}, (S, Z)$
$A, C, \bar{C}, H, H' \quad C', \bar{C}'$ SINGLETS

$$W(A) = M_A \, tr A^2 + \frac{\lambda_A}{M^*}(tr A^2)^2 + \frac{\lambda'_A}{M^*} tr A^4$$

Has soln For $\langle A \rangle$ in (B-L) dir

$$\langle A \rangle = i\sigma_2 \otimes Diag(a,a,a,0,0) \to a^2 = \frac{M_A M^*}{2(6\lambda_A + \lambda'_A)}$$

$$W(A,C,C') = C\left[\left(\frac{a_1}{M^*}\right)(Z\,A) + \left(\frac{b_1}{M^*}\right)(C\bar{C}) + (c_1\,S)\right]C'$$
$$+ C'\left[a_2(\;) + b_2(\;) + c_2(\;)\right]\bar{C}$$

$$W(H,H') = \lambda_1(HAH') + \lambda_{H'}\left(\frac{S^K H'^2}{M^{*K-1}}\right) + \lambda_2(H\bar{C}\bar{C})$$
$$+ \lambda_3 AH'CC'/M^*$$

$U(1)_A$	A	H	H′	C	$\bar{C}$	C′	$\bar{C}'$	Z	S	
	0	1	−1	$\frac{K+1}{2}$	$-\frac{1}{2}$	$-\left(\frac{K+2}{2}\right)$	$-\left(\frac{2K+7}{2}\right)$	$\frac{K+3}{2}$	$\frac{K+3}{2}$	Preferred
Z_2	−	+	−	+	+	+	+	−	+	K=2

⇒ NO OPERATOR ALLOWED IN ANY ORDER WHICH COULD UPSET DW VEV PATTERN!

In Limit of SUSY All $F_i = D_\alpha = 0$

→ In general a discrete set of deg. Vacua,

→ Assume SUSY BR provides the desired Vacuum

$$SO(10) \xrightarrow{\begin{cases} \langle A \rangle \propto B-L \\ \langle C \rangle \sim \text{"}\bar{\nu}_C\text{"} \end{cases}} SU(2)_L \times U(1)_Y \times SU(3)^C$$

ALLOWED CHOICE

ANOMALOUS $U(1)_A$ (PRESUMED OF STRING ORIGIN)

$\Rightarrow$ F-I TERM: $\boxed{\xi = \dfrac{g_A^2 \, \mathrm{Tr} Q_A \, \hat{M}_{Pl}^2}{192\pi^2}}$ (a)

$$D_A = \xi + q_i |\phi_i|^2 = 0$$

$$\Rightarrow \boxed{C^2 + Z^2 + S^2 = \frac{K}{2}\xi} \quad (b)$$

$$\bar{F}_{C'} \rightarrow \left(\frac{-3a_1}{M^*}\right) a \cdot Z + \left(\frac{b_1}{M^*}\right) C^2 + c_1 S = 0 \quad (c)$$

$$F_{C'} \rightarrow \left(\frac{-3a_2}{M^*}\right) a \cdot Z + \left(\frac{b_2}{M^*}\right) C^2 + c_2 S = 0 \quad (d)$$

(b), (c), (d) Fix Non-Zero VEV's OF

$$\langle C \rangle = \langle \bar{C} \rangle \sim \langle Z \rangle \sim M_{GUT}$$

$$\langle S \rangle \sim M_{GUT}^2 / M^* \; ; \; \langle C' \rangle = \langle \bar{C}' \rangle = 0 \quad (e)$$

RG STUDY : SUSY GUT

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(\Lambda) + \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \Delta_W^i + \boxed{\Delta_{GUT}^i}$$

$\Delta_W^i \rightarrow$ 2 loop RUNNING FROM $(M_Z \rightarrow \Lambda)$ including SUSY THRESHOLDS

$\Delta_{GUT}^i \rightarrow$ GUT-SCALE THRESHOLDS

$\Lambda \rightarrow$ UNIF. SCALE

MINIMALITY OF THE HIGGS SYSTEM & SYMMETRY OF THE MODEL PROVIDES REMARKABLE CANCELLATIONS IN GUT-SCALE THRESHOLD CORR

$\Leftarrow$

$\rightarrow$ CONTROLLED BY VERY FEW PARAM

$\Downarrow$

INCREASES PREDICTIVITY

$\Downarrow$

IN PARTICULAR AN INTRIGUING CORRELATION BETWEEN $\boxed{M_{eff} \leftrightarrow M_X}$

REMARKABLY ENOUGH

NET CONTRIB of HEAVY DOUBLETS $(D_i, \bar{D}_i)$ & HEAVY TRIPLETS TO DIFFERENTIAL RUNNING of $(\alpha_i^{-1} - \alpha_j^{-1})_{i \neq j}$ Depend on RATIO

SINGLE PARAM $M_{eff} \cos\gamma$

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{eff} \cos\gamma}$$

$$(M_{eff})^{-1} = (M_T^{-1})_{11} = M_H / \lambda_1^2 a^2$$

$$\cos\gamma = (m_b/m_t) \tan\beta \approx \tan\beta/59$$

SAME M_{eff} CONTROLS $A(p \to \bar{\nu} K^+)_{d=5}$

$$A(d=5) \propto (h_{33}^2 / M_{eff})$$

$$M_{eff} = \lambda_1^2 a^2 / M_{10'} \approx M_{GUT}^2 / M_{10'} \gg M_{GUT}$$

NATURALLY EXPECT

$$M_{eff} \approx (10^{17} - (4 \text{ to } 5) \times 10^{19}) \text{ GeV}$$

CORRELATION EQN

$$M_{eff} \simeq (10^{19}\,\text{GeV})\left(\frac{10^{16}\,\text{GeV}}{M_X}\right)^3\left(\frac{3}{\tan\beta}\right)\left(\frac{1/100}{r}\right) \times \left\{\frac{\exp\left[2\pi\left(\Delta_W^{(2)} - \Delta_W^{(3)} - \delta\alpha_3^{-1}\right)\right]}{2.07\times 10^{-2}}\right\}$$

→ FAIRLY INSENSITIVE TO SUSY SPECTRUM

$r \equiv m_\Sigma / M_X \quad (\Sigma = \underset{\sim}{8}^c \subset 45_H)$

$\delta\alpha_3^{-1}$ = Dev. of α_3^{-1} FROM CENTRAL VALUE

$= (\cdot 147, 0, -\cdot 1422) \rightarrow (\alpha_3(m_Z) = \cdot 1156, \cdot 1176, \cdot 1196)$

$$\Rightarrow \boxed{M_{eff} \propto 1/M_X^3}$$

$$\boxed{A(p\to e^+\pi^0) \propto g^2/M_X^2} \; ; \; \boxed{A(p\to \bar{\nu} K^+) \propto \frac{1}{M_{eff}}}$$

$\Rightarrow$ If RAISE M_X TOO MUCH TO SUPPRESS $d=6$ $(p\to e^+\pi^0)$, M_{eff} WILL BE TOO LOW → EXCLUDED BY (EMPIRICAL) LOWER LIMIT ON $\Gamma^{-1}(p\to \bar{\nu} K^+)$.

LIKEWISE IF RAISE M_{eff} TOO MUCH TO SUPPRESS $d=5$ $(p \to \bar{\nu} K^+)$, M_X WILL BE TOO LOW → EXCLUDED BY EMPIRICAL LOWER LIMIT ON $\tau^{-1}(p \to e^+ \pi^0)$

⇒ M_{eff} & M_X EACH RESTRICTED IN A CERTAIN RANGE

⇒ CONSTRAINED UPPER LIMITS ON BOTH $\tau^{-1}(p \to e^+ \pi^0)$ AND $\tau^{-1}(p \to \bar{\nu} K^+)$

SEE FIGURES

NOTE (a) $M_{eff} = \dfrac{\lambda_1^2 \langle 45_H \rangle^2}{M_{10'}} \approx \dfrac{M_{GUT}^2}{M_{10'}} \gg M_{GUT}$

⇒ $M_{eff} \approx 10^{17} - 5 \times 10^{19}$ GeV

(b) $r \equiv \dfrac{m_\Sigma}{M_X} \approx 4 \lambda_A' M_{GUT}/M_*$

⇒ CONSERVATIVELY $\dfrac{1}{10} \lesssim r \lesssim \dfrac{1}{200}$

From $M_\Sigma = \frac{2\lambda_A'}{6\lambda_A + \lambda_A'} M_A$

$$a^2 = \frac{M_X^2}{g^2} = \frac{M_A M^*}{2(6\lambda_A + \lambda_A')}$$

With $M_X \approx (M_{GUT}/2)(1/2 - 1)$

Get $r \approx 4\lambda_A'(1/2-1)^2 (M_{GUT}/M^*)$ $(\approx 1/100)$

$\lambda_A' \approx O(1) \approx 1/4 - 2$ (say)

$$\boxed{r \equiv \frac{M_\Sigma}{M_X} \approx \left(\frac{1}{10} - \frac{1}{200}\right)}$$

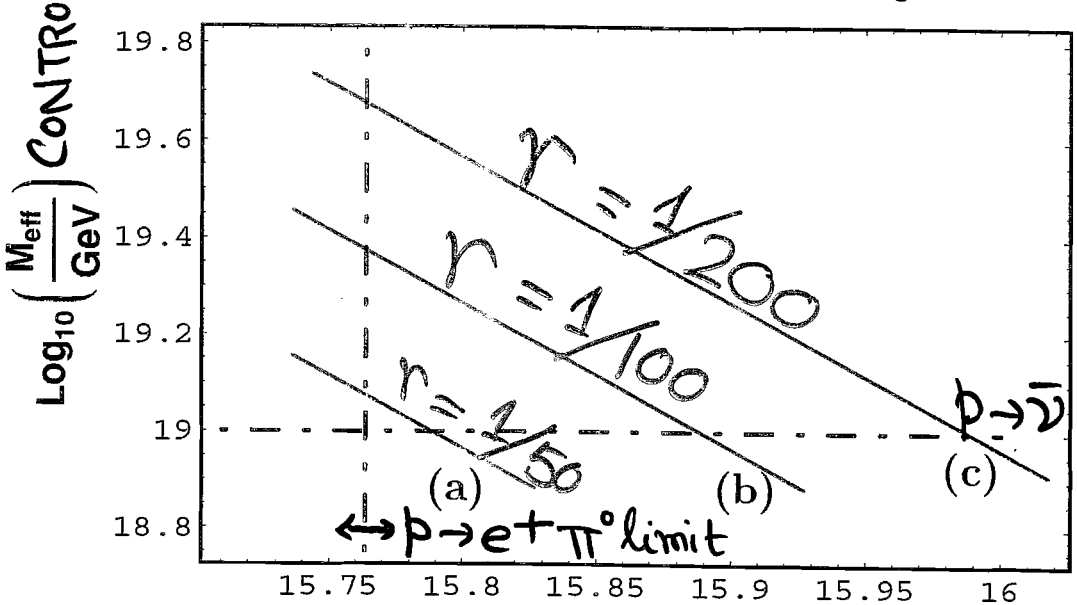
Correlation Eqn

$$M_X r^{1/3} = (0.745, 1.014, 1.366) \times 10^{15}\ \text{GeV} \times \left(\frac{0.643 \times 3}{\eta_\gamma \tan\beta}\right)^{1/3} \left(\frac{4.5 \times 10^{19}\ \text{GeV}}{M_{eff}}\right)^{1/3}$$

For Case 2 $\rightarrow m_{\tilde{q}} = 1.2$ TeV, $m_{\tilde{w}} = 130$ GeV

Similar for other cases.

CONSERVATIVE CASE: HEAVY SQUARK, LIGHTEST WINO



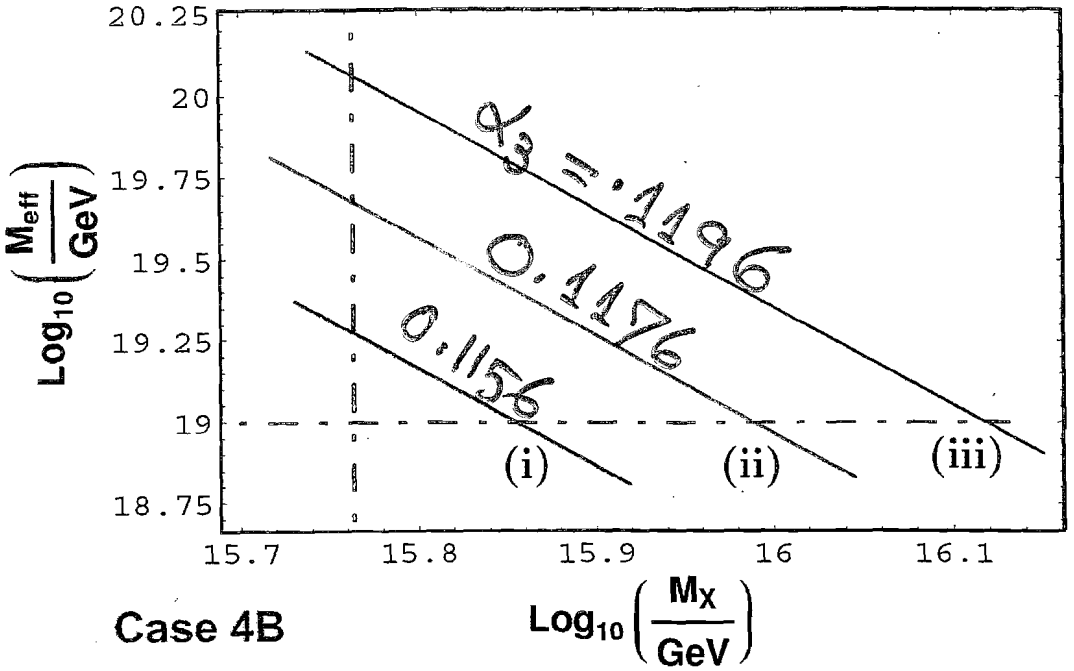
Case 4B

$(a, b, c) \to r = (1/50, 1/100, 1/200)$

$\tan\beta = 3, \quad \alpha_3(M_Z) = 0.1176$

$m_{\tilde{q}} = 1.5\ \mathrm{TeV}, \quad m_{\tilde{W}} = 125\ \mathrm{GeV}$

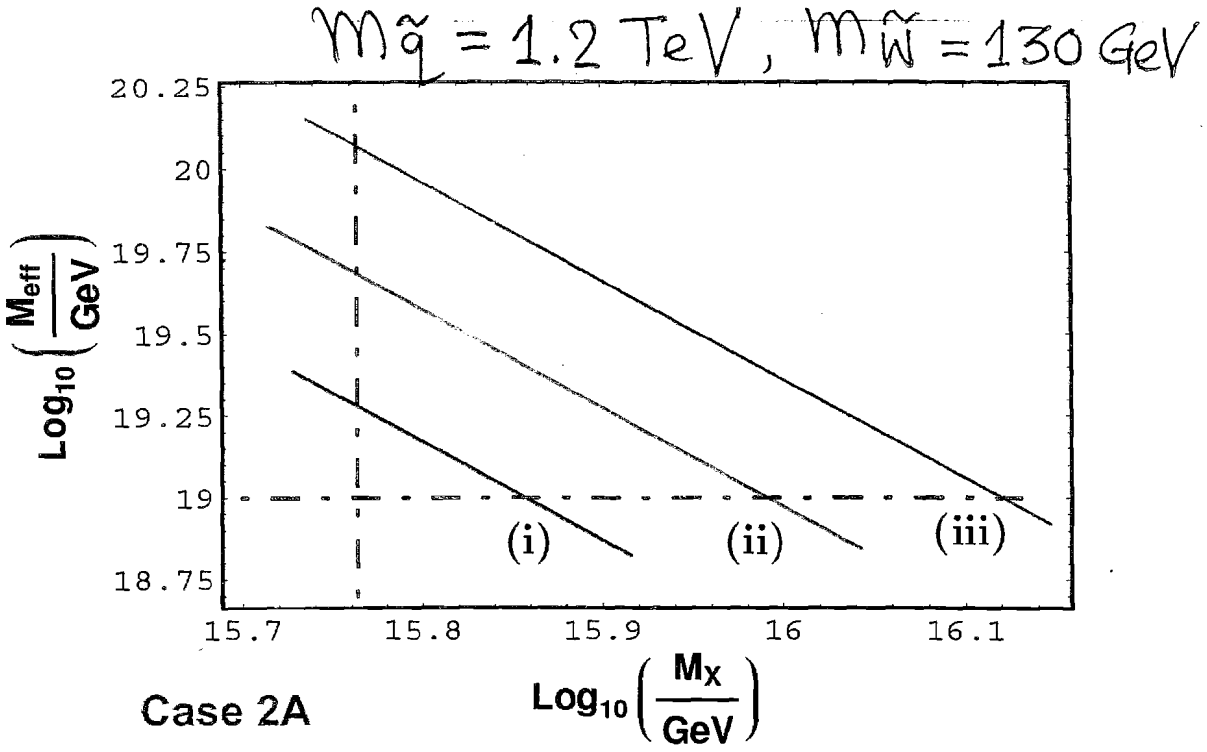
CONSERVATIVE CASE: HEAVY SQUARK LIGHTEST WINO



$$(\text{i, ii, iii}) \rightarrow \alpha_3(M_Z) = (0.1156,\ 0.1176,\ 0.1196)$$

$$\tan\beta = 3, \quad r = 1/200$$

$m_{\tilde{q}} = 1.5$ TeV, $m_{\tilde{W}} = 125$ GeV



$$(\text{i, ii, iii}) \rightarrow \alpha_3(M_Z) = (0.1156,\ 0.1176,\ 0.1196)$$

$$\tan\beta = 3, \quad r = 1/200$$

Getting UPPER LIMIT ON $\bar{\Gamma}^{-1}(p \to e^+ \pi^0)$

$$\bar{\Gamma}^{-1}(p \to \bar{\nu}_e K^+) \simeq (2.8 \times 10^{33}\ \text{yrs}) \left(\frac{0.01\ \text{GeV}^3}{\beta_H}\right)^2 \left(\frac{5.76}{A_S}\right)^2 \left(\frac{0.6}{\lambda_t(M_G)}\right)^4 \times \left(\frac{1.34}{R_L}\right)^2 \left(\frac{M_{eff}}{2.5 \times 10^{19}\ \text{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.2\ \text{TeV}}\right)^4 \left(\frac{130\ \text{GeV}}{m_{\tilde{W}}}\right)^2$$

Expt $\bar{\Gamma}^{-1}(p \to \bar{\nu} K^+) \gtrsim 2.3 \times 10^{33}$ yrs (SuperK)

$\Downarrow$ $m_{\tilde{q}} \leq 1.5$ TeV, $m_{\tilde{W}} \gtrsim 125$ GeV

Conservatively $(M_{eff})_{Min} \gtrsim 1.39 \times 10^{19}$ GeV

$\Downarrow$ Correlation Eqn
$r \gtrsim 1/200$, $\tan\beta \gtrsim 3$

$$(M_X)_{Max} \lesssim (6.41, 8.73, 11.75) \times 10^{15}\ \text{GeV}$$

$$\bar{\Gamma}^{-1}(p \to e^+ \pi^0)_{Max} \lesssim 3.9 \times 10^{34}\ \text{yrs}$$ (Central $\beta_H; \alpha_H, A_R, \bar{\alpha}(M_X)$)

$\Downarrow$ Allow $\pm$ 25% uncert M_X

$$\bar{\Gamma}^{-1}(p \to e^+ \pi^0)_{Max} \lesssim 2 \times 10^{35}\ \text{yrs}$$

CONSERVATIVE UPPER LIMIT

FERMION MASS & THRESHOLD CORREL SO(10) MODEL
FMTH SO(10) MODEL

$d = 6$, $p \to e^+ \pi^0$

$$\Gamma^{-1}(p \to e^+ \pi^0) \approx (10^{34}\ \text{yrs}) \times \left(\frac{0.01}{\alpha_H}\right)^2 \times \left(\frac{3.02}{A_R}\right)^2 \left[\frac{6.25}{\phi(p)}\right]^2 \left[\frac{1/19}{\alpha_X}\right]^2 \left[\frac{M_X}{6.21 \times 10^{15}\ \text{GeV}}\right]^4$$

$(1 \pm 0.25)^2$

$(1.56 \text{ to } 0.8)$ $(1 \pm 0.1)^2$ $\left(\frac{1}{2.5} \text{ to } 1.675\right)^4$

$(M_X) = 1.175 \times 10^{15}$ GeV LARGEST

$$\Gamma^{-1}(p \to e^+ \pi^0) \simeq (10^{32} \leftrightarrow 1.5 \times 10^{35})\ \text{Yrs}$$

$$\Downarrow$$

$$\boxed{\Gamma^{-1}(p \to e^+ \pi^0) \lesssim 2 \times 10^{35}\ \text{Yrs}}$$

FERMION MASS & THRESHOLD CORELATED SO(10) MODEL

FMTH SO(10) MODEL

$$\phi(p) = 5 + \frac{2}{(1+p^2)} + \frac{1}{(1+p^2)^2}$$

↖ SU(5) limit ($p \to \infty$)

$$= 5 \text{ to } 8 \quad (\text{for } p \to \infty \text{ to } 0)$$

$d=5$, $p \to \bar{\nu} K^+$

$$\Gamma^{-1}(p \to \bar{\nu} K^+) \simeq (2.8 \times 10^{33}\ \text{yrs}) \left(\frac{0.01\ \text{GeV}^3}{\beta_H}\right)^2 \xrightarrow{(1 \pm .25)^2}$$

$$\times \left(\frac{0.6}{\lambda_t(M_G)}\right)^4 \left(\frac{5.76}{A_S}\right)\left(\frac{1.34}{R_L}\right)^2$$

$$\times \left(\frac{M_{eff}}{2.5 \times 10^{19}\ \text{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.2\ \text{TeV}}\right)^4 \left(\frac{130\ \text{GeV}}{m_{\tilde{W}}}\right)^2$$

$M_{eff} \approx (10^{18} - 5 \times 10^{19})\ \text{GeV}$, $m_{\tilde{q}} \approx (600 - 1500)\ \text{GeV}$

$m_{\tilde{W}} \approx (125 - 500)\ \text{GeV}$,

VERY CONSERVATIVELY

$$\Gamma^{-1}(p \to \bar{\nu} K^+)_{Max} \lesssim 4 \times 10^{34}\ \text{yrs.}$$

Fermion Mass & Threshold Correl

SO(10) - Model

FMTH SO(10)

SUMMARY ON PROTON LIFETIMES

d = 5

SUSY Min. SU(5) $\quad \Gamma^{-1}(p \to \bar{\nu} K^+) < 10^{32}$ yrs Excluded

SUSY SO(10)$_{\text{GENERAL}}$ $\quad \Gamma^{-1}(p \to \bar{\nu} K^+) \sim (10^{30} - 10^{34})$ yrs

(D - T)

SUSY SO(10)
FMTH COREL

$\Gamma^{-1}(p(\bar{\nu} K^+)) \lesssim 2 \times 10^{33}$ yrs (Med)

$\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{Heavy}} \lesssim 4 \times 10^{34}$ yrs

d = 6

SUSY Min SU(5) $\quad \Gamma^{-1}(p \to e^+ \pi^0) \approx 10^{35.3 \pm 1.5}$ yrs

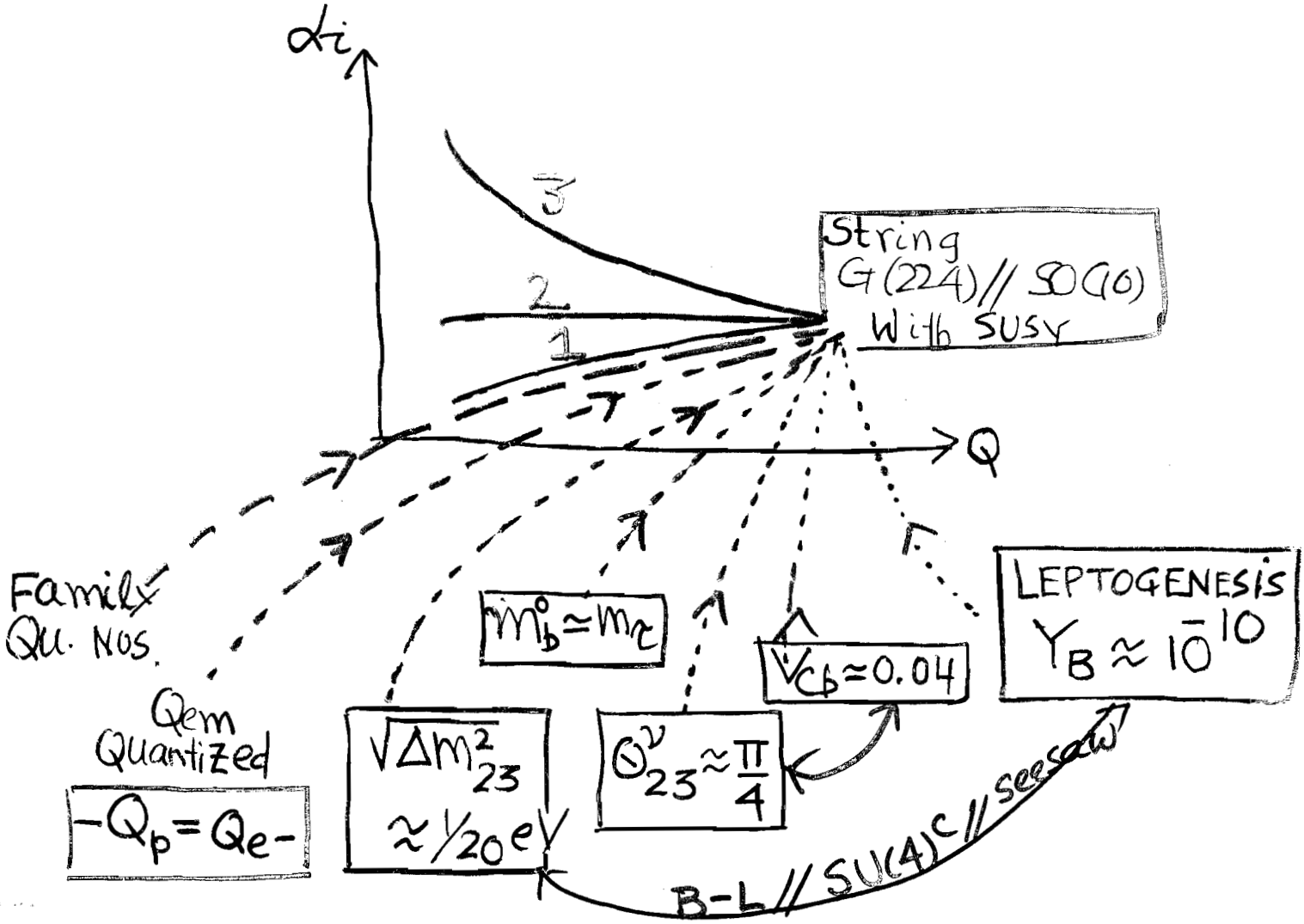
SUSY SO(10)$_{\text{FM COREL}}$ $\Gamma^{-1}(p \to e^+ \pi^0) \approx 10^{35 \pm 1}$ yrs

SUSY SO(10)
FMTH COREL

$\Gamma^{-1}(p \to e^+ \pi^0) \lesssim 2 \times 10^{35}$ yrs

PROTON DECAY OUGHT TO SHOW WITH IMPROVEMENT BY FACTOR OF 5 - 10.

Rendevouz In Particle Physics



ALL THESE FEATURES & MORE HANG TOGETHER NEATLY IN A SINGLE UNIFIED FRAMEWORK → HARD TO IMAGINE THIS CAN BE MERE COINCIDENCE

⇓

TWO Missing PIECES

- PROTON DECAY → Need NEXT GEN. DETECTOR
- SUPERSYMMETRY → LHC

PROTON DECAY THE HALLMARK OF GRAND UNIF.

Provides a Unique Window To View Physics at truly short dist $\lesssim 10^{-30}$ cm

$p \to \bar{\nu} K^+$ If seen $\Rightarrow$ { q-ℓ, q-$\bar{q}$, q-$\bar{\ell}$ unif // ; SUSY UNIF // ; $M_X \sim 10^{16}$ GeV }

$p \to \mu^+ K^0$ If seen $\Rightarrow$ { ν MASS Rel OP IMPORTANT }

If $e^+\pi^0$ not seen Say upto 10^{36} yrs, but $\bar{\nu} K^+$ seen $\Rightarrow$ { EFF. G(224) & ν MASS Rel $d=5$ IMPORTANT }

THUS PROTON DECAY IF SEEN WILL BRING A WEALTH OF KNOWLEDGE THAT CAN NOT BE GAINED BY ANY OTER MEANS

$\Downarrow$

DISCOVERY POTENTIAL HIGH

$\Downarrow$

MEGATON SIZE DETECTOR (or EQUIVALENT) SENSITIVE TO
- PROTON DECAY,
- ν Oscillations (LONG BASELINE)

AND
- SUPERNOVA ν'S.