$$\mathrm{Cov}[\hat{\theta}(\mathbf{x}), \mathrm{s}(\mathbf{x}\,|\,\theta)] = \mathrm{E}[\hat{\theta}(\mathbf{x})\mathrm{s}(\mathbf{x}\,|\,\theta)]$$