Neutrino Physics. The Basics

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BROOKHAVEN NATIONAL LABORATORY

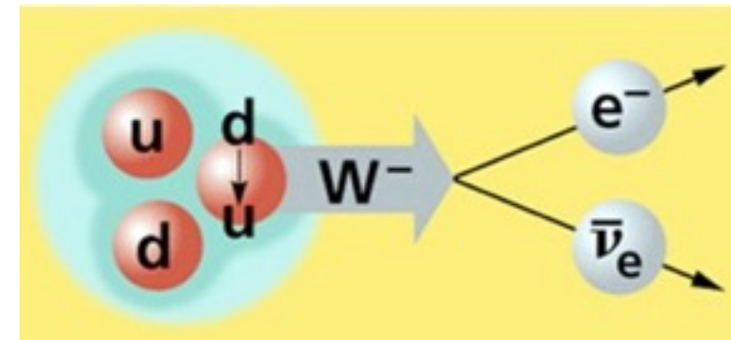
Outline

- **Brief review of neutrino particle properties**
- **Historical material**
- **Neutrinos in the Standard Model of Particle Physics**
- **Neutrino interaction rules**
- **Neutrino cross sections and interaction characteristics**
- **Summary**

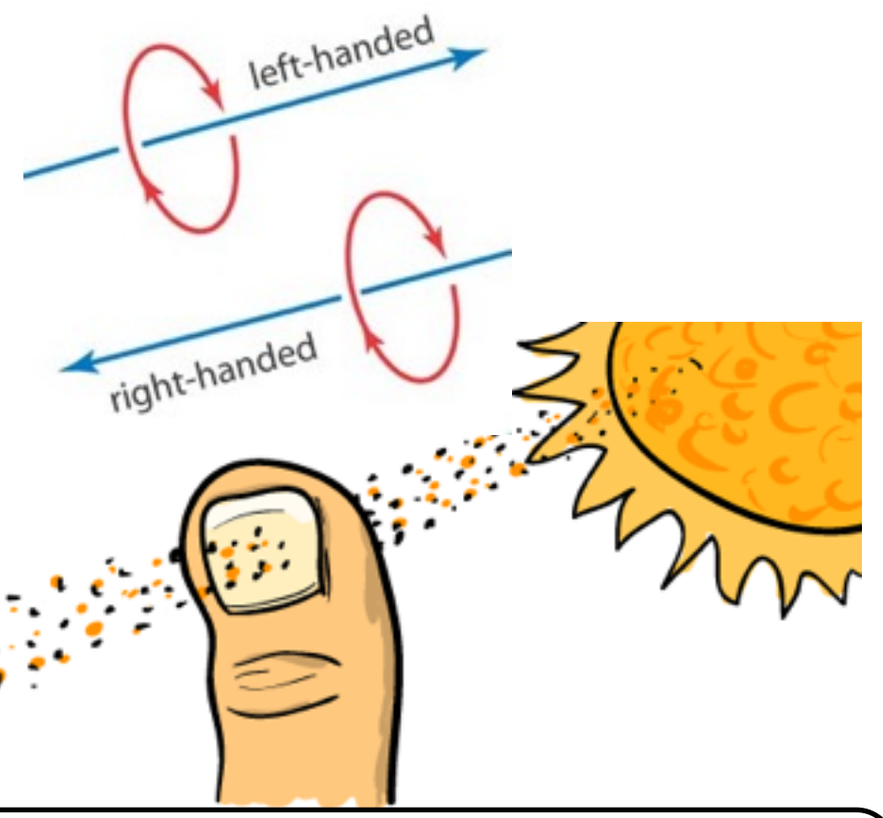
This is a fast survey of various types of neutrino interactions. The intention is to provide the tools and basics to allow simple calculations. This is very much from an experimental perspective.

What are neutrinos ?

$$n \rightarrow p\ e^{-}\bar{\nu}_e$$



- A particle with no electric charge. Predicted in 1930 by Pauli, and detected in 1957 by Reines and Cowan.
- It is emitted in radioactive decay. And has no other types of interactions.
- It has 1/2 unit of spin, and therefore is classified as a Fermion (or particle of matter.)
- Neutrino is extremely light.
- Neutrinos are produced in flavors !
- Neutrino is left handed ! Or has no mirror image !
- Neutrinos are as numerous as photons in the Universe.
- Important component of dark matter. May be responsible for matter/antimatter asym.



From the Sun:
10^{11} neutrinos/cm^2/sec

Some history

- By 1930 many confusing results concerning nuclear structure
  - Quantum Mechanics established. BUT
  - Spin and statistics of odd (A-Z) nuclei is wrong if made of electrons and protons. e.g. N14 (Z=7) has spin of 1. (a neutron cannot be a proton+electron since it would then have spin of 1 or 0, but neutron was not known then)
  - Nuclear spin is integer for all even A and half-integer for odd A.
  - Why is the beta spectrum from the nucleus continuous and not quantized ?

Beta decay spectrum ?

- Nucleus spits out only single electrons.
- There are no other mechanisms for energy to be lost.
- There are no unaccounted gamma rays.

The Average Energy of Disintegration of Radium E.

By C. D. Ellis, Ph.D., Lecturer in the University of Cambridge, and W. A. Wooster, B.A., Charles Abercrombie Smith Student of Peterhouse, Cambridge.

(Communicated by Sir Ernest Rutherford, O.M., P.R.S.—Received August 3, 1927.)

Bismuth 210 (beta 5 day) -> Po 210 (alpha 140 days) -> Pb 206

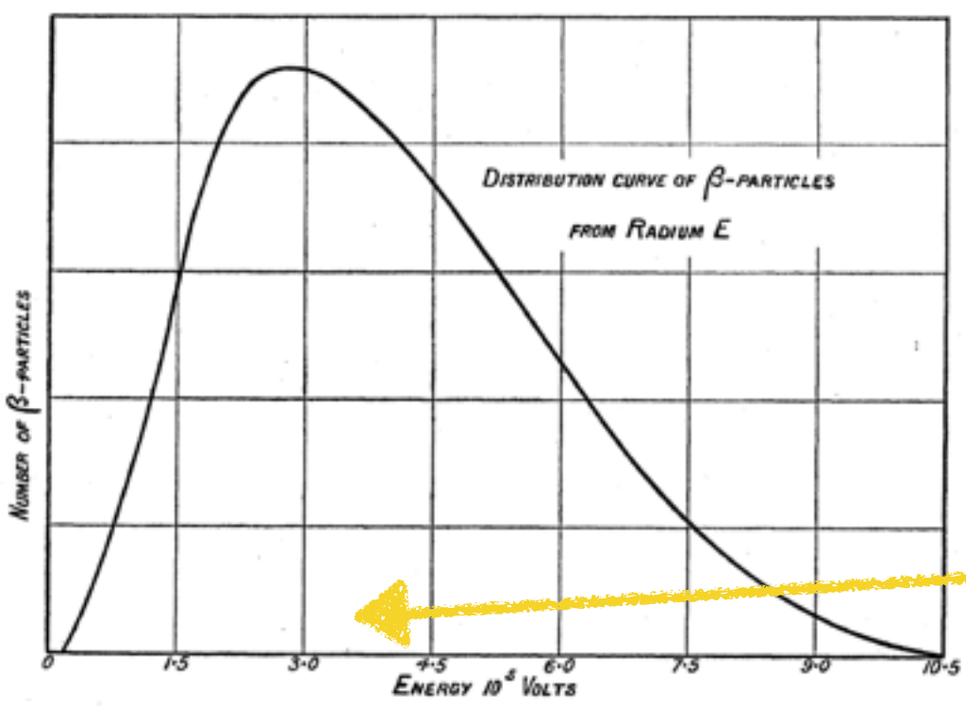


Table I.—Heating Effect of Radium E deduced from Curves of fig. 3.

True age.	Total heating.	Portion due to Po.	Portion due to Radium E.	x.	Disintegration energy of Radium E in volts.
days.	mm.				
2·25	22·0	3·68	18·3	15·4	339000
3·20	20·8	4·91	15·9	15·5	337000
5·20	19·0	6·99	12·0	15·5	337000
7·20	17·8	8·64	9·2	15·6	335000
11·20	16·1	10·53	5·6	14·5	360000
17·20	14·2	11·83	2·4	14·7	355000
29·20	12·85	12·18	0·67	15·1	346000

Confirm that only the mean beta energy leads to heating. Then were is rest going ?

Po210 Alpha energy was well known at that time to be 5 MeV. After it is subtracted in a fit, x = ratio of heat from Bi210 to Po210. This gives the average energy of the Bi210 Beta well below the maximum.

The experimental results and the understanding of the quantum mechanics could only mean either

1) No event by event energy conservation

or

2) Another penetrating particle.

Pauli proposes a particle

The letter in which Pauli proposed the neutrino, translated from the German of reference 5, reads as follows:

Zürich, 4 December 1930
Gloriastr.

Physical Institute of the
Federal Institute of Technology (ETH)
Zürich

Dear radioactive ladies and gentlemen,

As the bearer of these lines, to whom I ask you to listen graciously, will explain more exactly, considering the "false" statistics of N-14 and Li-6 nuclei, as well as the continuous β-spectrum, I have hit upon a desperate remedy to save the "exchange theorem" * of statistics and the energy theorem. Namely [there is] the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin $1/2$ and obey the exclusion principle, and additionally differ from light quanta in that they do not travel with the velocity of light: The mass of the neutron must be of the same order of magnitude as the electron mass and, in any case, not larger than 0.01 proton mass.—The continuous β-spectrum would then become understandable by the assumption that in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.

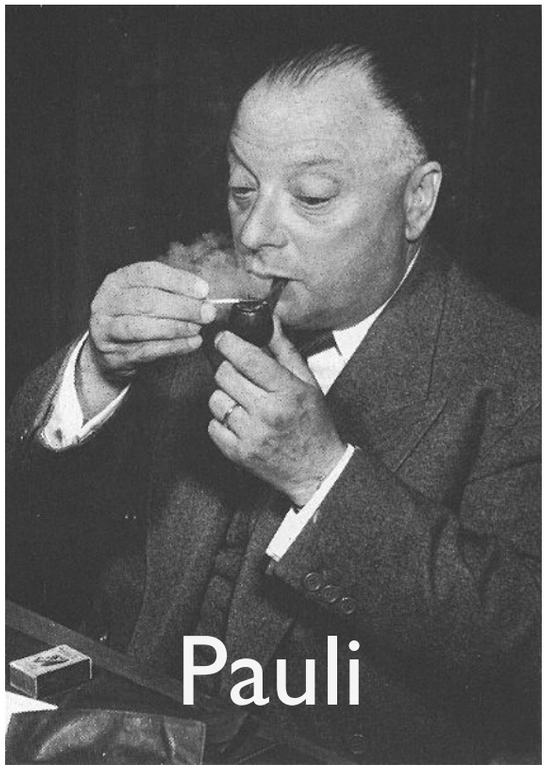
Now the next question is what forces act upon the neutrons. The most likely model for the neutron seems to me to be, on wave mechanical grounds (more details are known by the bearer of these lines), that the neutron at rest is a magnetic dipole of a certain moment μ. Experiment probably requires that the ionizing effect of such a neutron should not be larger than that of a γ ray, and thus μ should probably not be larger than $e.10^{-13}$ cm.

But I don't feel secure enough to publish anything about this idea, so I first turn confidently to you, dear radioactives, with the question as to the situation concerning experimental proof of such a neutron, if it has something like about 10 times the penetrating capacity of a γ ray.

I admit that my remedy may appear to have a small *a priori* probability because neutrons, if they exist, would probably have long ago been seen. However, only those who wager can win, and the seriousness of the situation of the continuous β-spectrum can be made clear by the saying of my honored predecessor in office, Mr. Debye, who told me a short while ago in Brussels, "One does best not to think about that at all, like the new taxes." Thus one should earnestly discuss every way of salvation.—So, dear radioactives, put it to the test and set it right.—Unfortunately I cannot personally appear in Tübingen, since I am indispensable here on account of a ball taking place in Zürich in the night from 6 to 7 of December.—With many greetings to you, also to Mr. Back, your devoted servant,

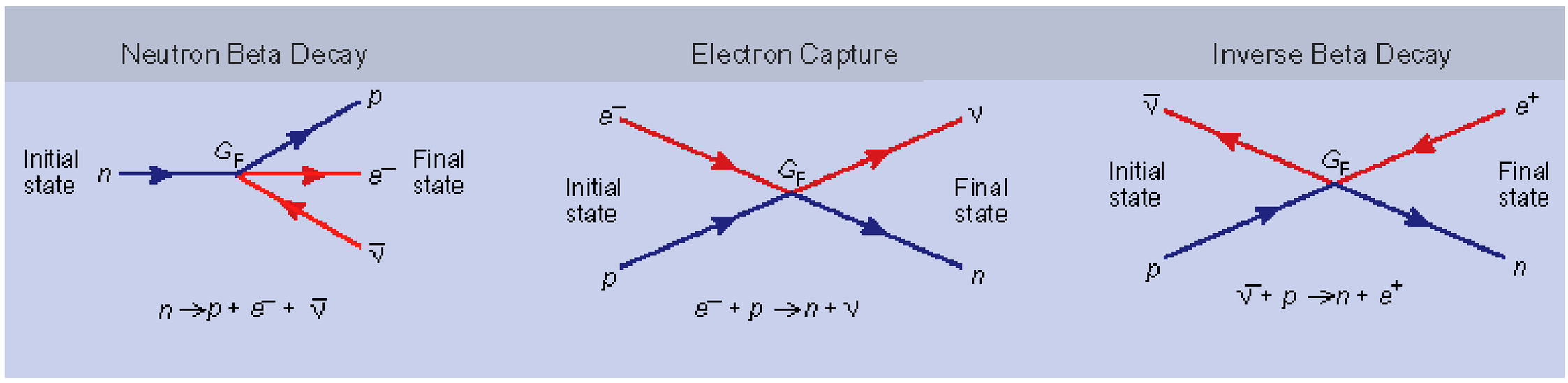
W. Pauli

* In the 1957 lecture, Pauli explains, "This reads: exclusion principle (Fermi statistics) and half-integer spin for an odd number of particles; Bose statistics and integer spin for an even number of particles."



Soon after Pauli's observation, an almost complete theory of beta decay was developed by Fermi (1933). This theory with important modifications allows calculation of decay and interaction rates.

We will discuss the interactions first before discussing the modern understanding of the physics of neutrinos.



Particles of matter

Lepton Number = +1

Quark Number = +1/3

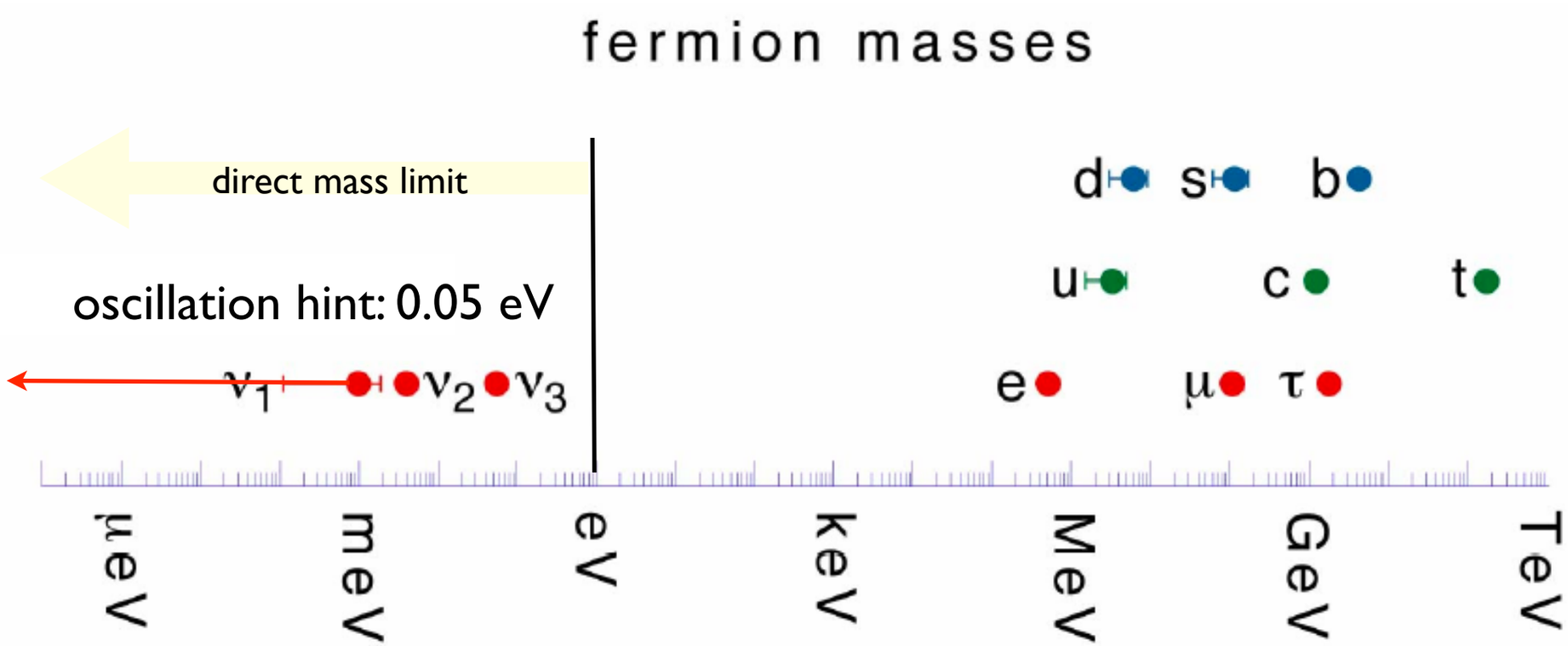
FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Generation	Leptons spin =1/2			Quarks spin =1/2		
	Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
First	ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0	u up	0.002	2/3
	e electron	0.000511	–1	d down	0.005	–1/3
Second	ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0	c charm	1.3	2/3
	μ muon	0.106	–1	s strange	0.1	–1/3
Third	ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0	t top	173	2/3
	τ tau	1.777	–1	b bottom	4.2	–1/3

- **Stable matter is formed from the first generation only.**
- **Fermion are created and destroyed in particle/antiparticle pairs only.**
- **Baryon and Lepton number is strictly conserved at observable energies.**
- **There is no definitive understanding of**
  - **Generations, Charge quantization,**
  - **Quarks versus leptons**
  - **The left/right asymmetry**

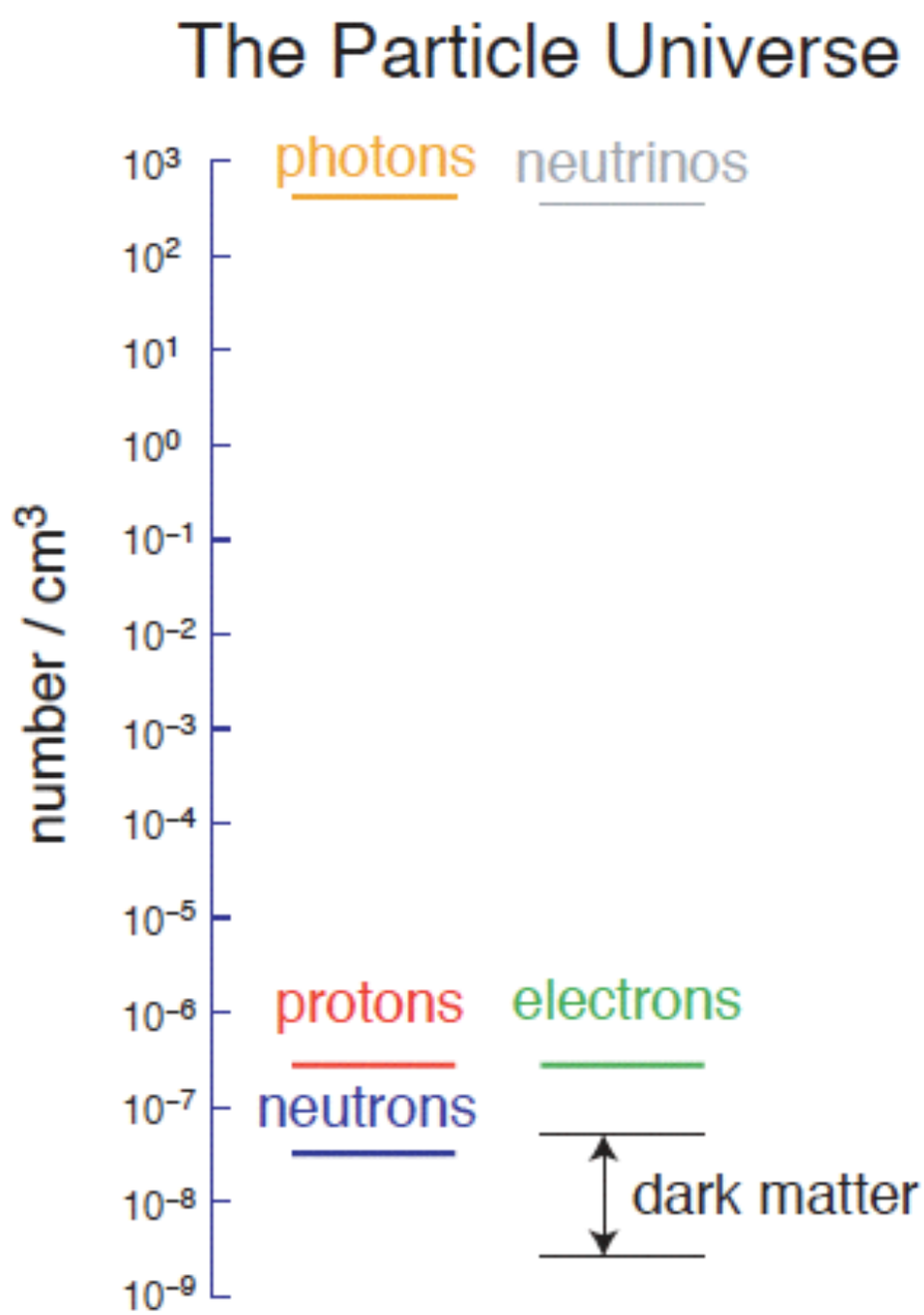
What else do we know and how do we know it ?

- **Neutrinos are definitely massive with extremely small mass - from oscillation experiments.**
- **Neutrino is the most abundant particle of matter with probably only 3 active types - cosmology and precision EW.**
- **Neutrino flavor states are not described by unique mass eigenstates, but are superpositions.**



Neutrinos in Cosmology

- The most abundant particle is the photon: ~400/cc
- The most abundant matter particle is the neutrino at 56/cc of each type.
- CNB (1.95K) is a relic of the big bang similar to the CMB (2.725K). Neutrinos decoupled at 2 sec while photons decoupled at ~400,000 yrs (3000 K, z~1000)
- For the period before photons decoupled, neutrinos played a crucial role in the development of cosmic structures and initial burning of stars.
- Neutrinos carry away 100 X energy in photons from core-collapse supernova.
- What is the typical wavelength of a CNB neutrino? ($K_B = 8.6\ 10^{-11}$ MeV/K)



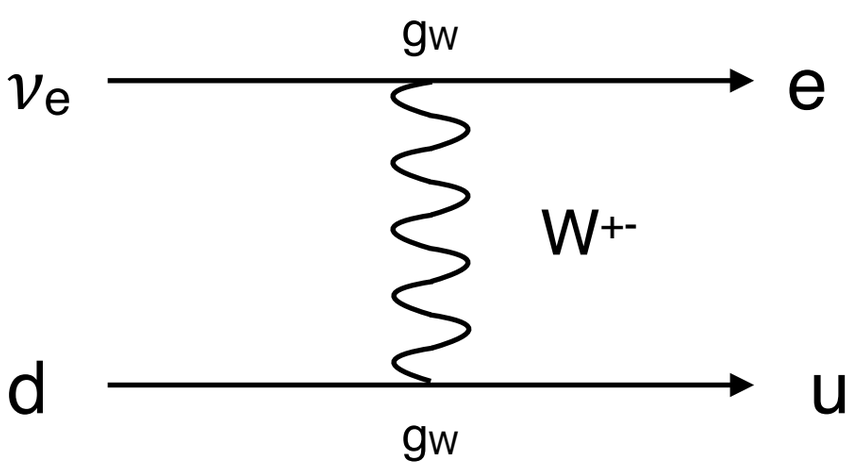
Interactions of quarks and leptons

	Gravity	Electromagnetic	Weak	Strong
Boson	Graviton	Photon	W, Z	Gluon
Spin/parity	2^+	1^-	$1^+, 1^-$	1^-
Mass (GeV)	0	0	80.2,91.2	0
Source	mass	electric charge	weak charge	color charge
Range (m)	∞	∞	10^{-18}	$<10^{-15}$
Strength	$GM^2/4\pi\hbar c$	$\alpha = e^2/4\pi\hbar c$	$G_F M^2 c^4/(\hbar c)^3$	α_s
	5×10^{-40}	1/137	1.17×10^{-5}	<1

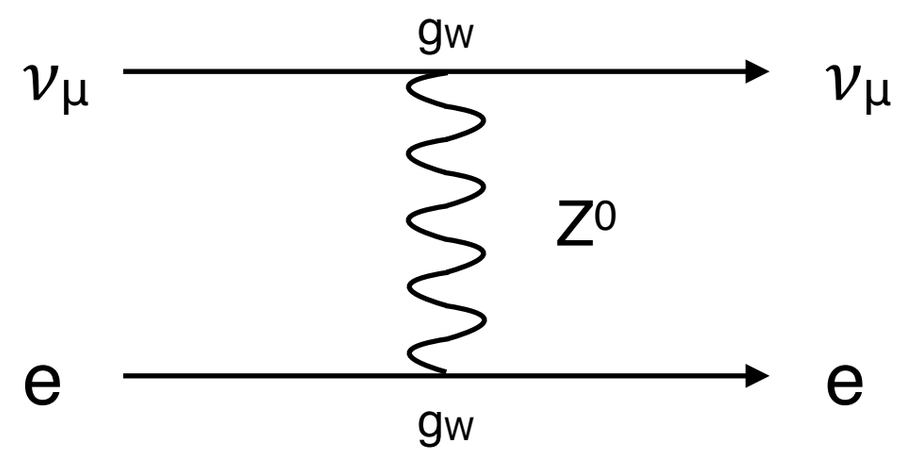
$M = 1$ GeV/c^2, $\hbar = 1$, $c = 1$, Calculate the relative strengths.

There is as yet no quantum theory of gravity, $M_{pl}=(\hbar c/G)^{1/2}$

The weak interaction



Charged current because the charge of the lepton and target changes



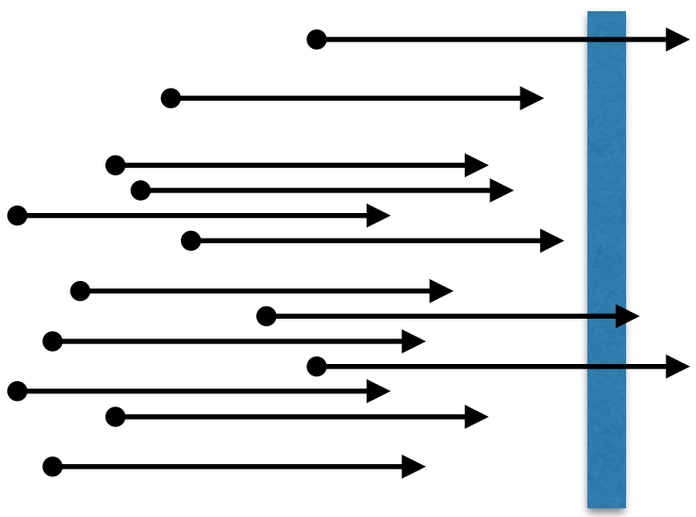
Neutral current because the charge of the lepton and target stays the same

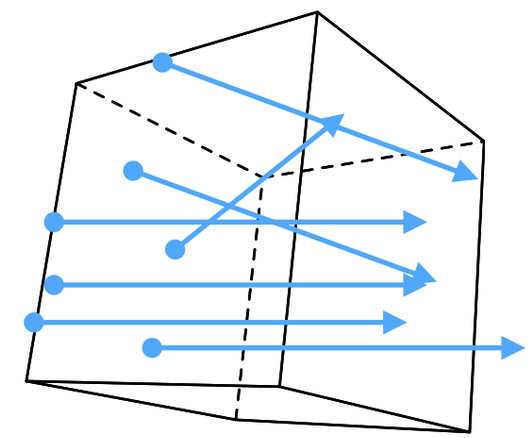
$$G_F = \lim_{q^2 \to 0} \frac{g_w^2}{(-q^2 + M_W^2)}$$

$$g_w \sim e \Rightarrow G_F \approx \frac{4\pi\alpha}{M_W^2}$$

- **q^2 is the square of 4-momentum transferred. It is always negative (Prove this).**
- **The neutrino has only weak interactions.**
- **Because of the large mass of W, Z, very short ranged and weak. Neutrinos cannot make bound states.**
- **In ordinary neutrino interactions q^2 can be ignored and the strength of the interactions (or cross section) is proportional to $G_F^2 = 5 \times 10^{-38} cm^2/GeV^2$**
- **Electroweak theory relates G_F, electric charge, the masses of the W, Z bosons.**

Particle Flux





Density of particles is N_a per cm^3

Velocity is V_a relative to the target

Then flux per unit area per unit time = $N_a \times V_a$

Take unit volume ΔV

Let T = the length of track within volume V per unit time Δt

Then Flux = $T/\Delta V/\Delta t$

As particles penetrate material slab of thickness dx, there is a reduction in the flux

$$F(x) = F(0)e^{-\sigma\rho x}$$

$$\lambda = 1/(\sigma\rho)$$

Here λ is the mean free path and σ is the cross section and ρ is the density of targets.
But for neutrino interactions the mean free path is so long that only the flux reduction can be completely ignored and only leading term used.

How to calculate neutrino event rate ?

- Events = Flux (/cm^2/sec)*Cross-section(cm^2)*Targets
- Targets are the number of particle targets in a detector volume. Detector itself serves as the target for interactions.
- 1 ton of water ~ 6 x 10^{29} protons and neutrons and ~3x10^{29} electrons
- Practical experiments will have efficiency as a function of energy.
- Typical cross section is 10^{-38} cm^2 x Energy (GeV)
- Neutrinos from various sources have huge energy range: eV to 10^{15} eV.
- Cross sections for low energies can be extremely small.
- Calculate the mean free path of 1 GeV neutrinos through the earth.

Weak interactions of neutrinos

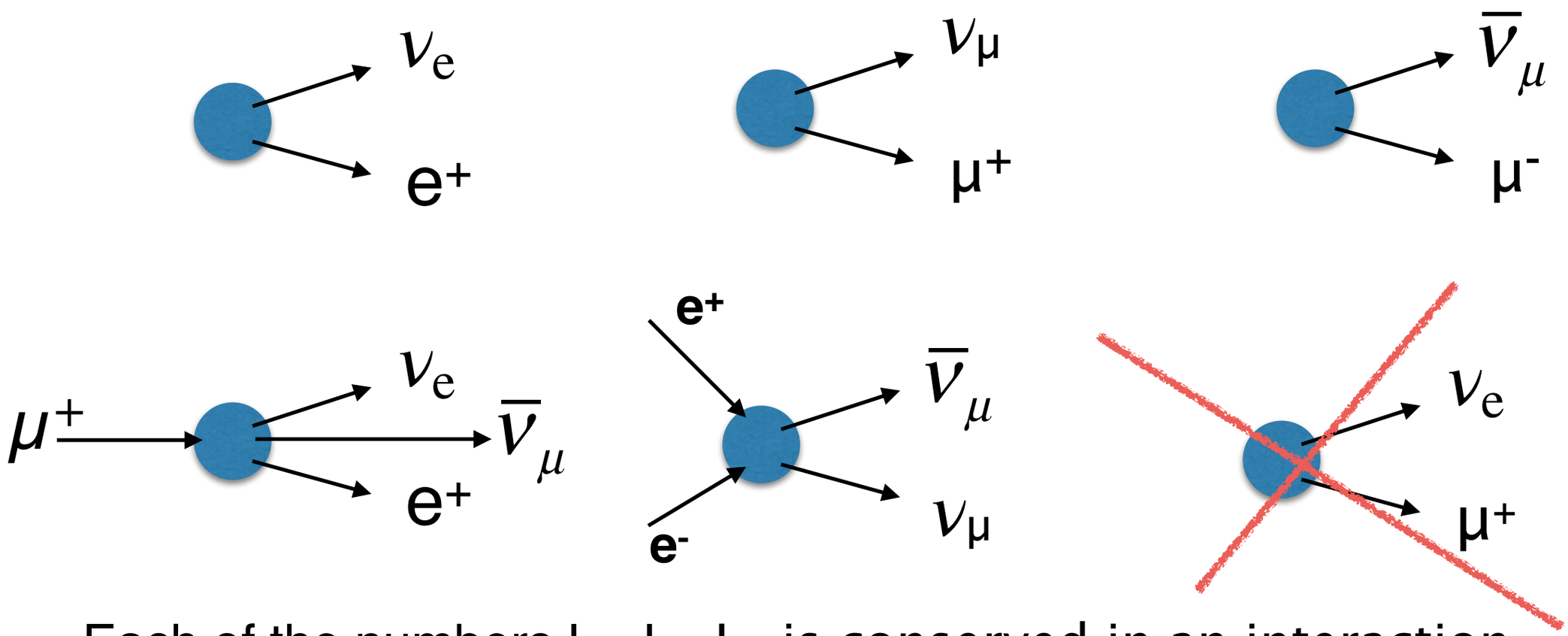
Particles of a given kind are all identical. All electrons are absolutely identical. There are no birth marks. Nevertheless, there are 3 kinds of electron type particles called flavors.

Particle	Symbol	Mass	Associated Neutrino
Electron	e	1	ν_e
Muon	μ	200	ν_μ
Tau	τ	3500	ν_τ
	Negative Electrical Charged		**Neutral**

All these have anti-particles with opposite charge. However, for neutral neutrinos the exact meaning of having anti-particles is not yet clear.

Neutrino creation

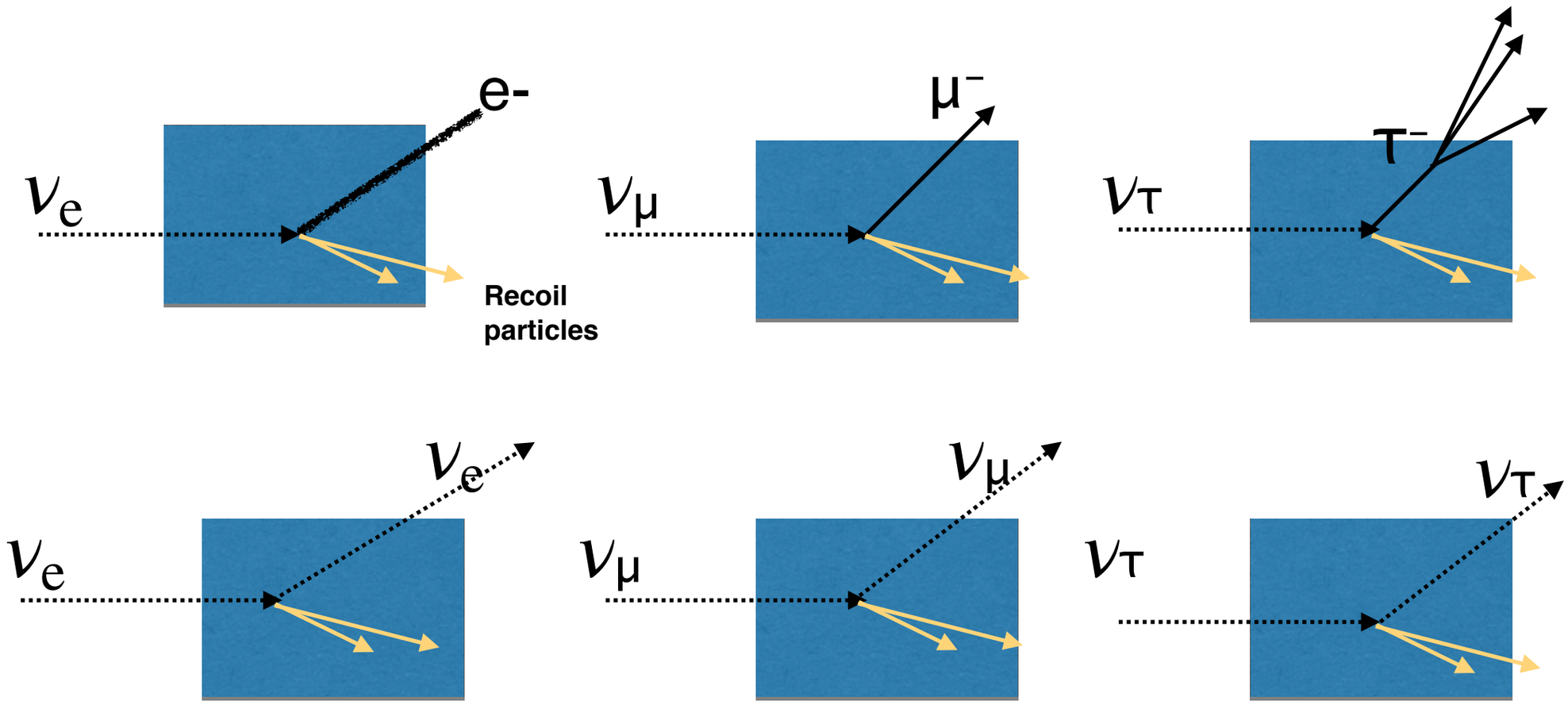
- At creation the lepton flavor is preserved. This is known from experiments to extreme precision.



Each of the numbers L_e, L_μ, L_τ is conserved in an interaction. The particles get (+1) and anti-particles get (−1)

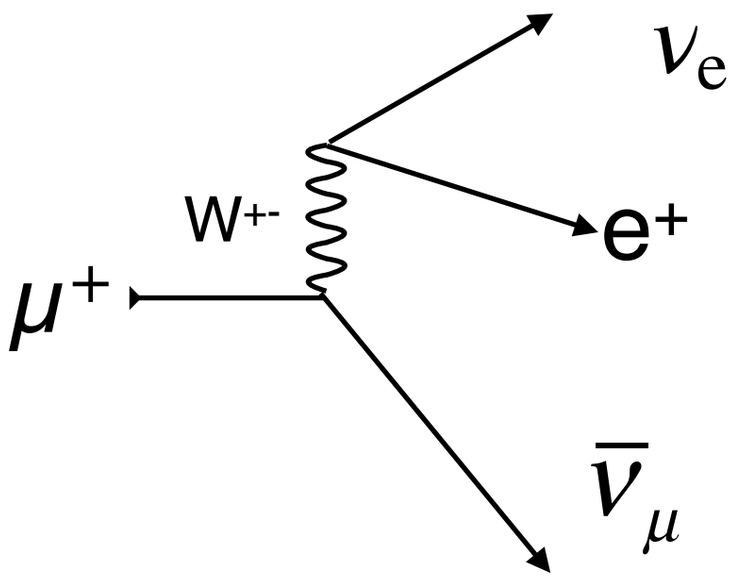
(The new understanding of neutrinos says that this is in fact violated in flight)

Neutrino Detection



- **The neutrino has no charge and so it is invisible as it enters a detector. Only very rarely it interacts and leaves charged particles that can be detected.**
- **Neutrino collision on atoms in detectors produces a charged lepton. (Charged Current)**
- **The electron, muon, tau have very different signatures in a detector.**
- **Neutrino can also collide and scatter away leaving observable energy.(Neutral Current)**

Muon decay



$$\Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \qquad \tau_\mu = \hbar / \Gamma = 2.197 \mu s$$

+ for anti-muons and - for muons

θ is measured between muon spin direction and electron direction.

$$x = E_e / E_{\max}$$

$$\frac{d^2N}{dx\,d(\cos\theta)} \sim x^2[(3-2x) \pm P_\mu \cos\theta(2x-1)]$$

P_μ is muon polarization.

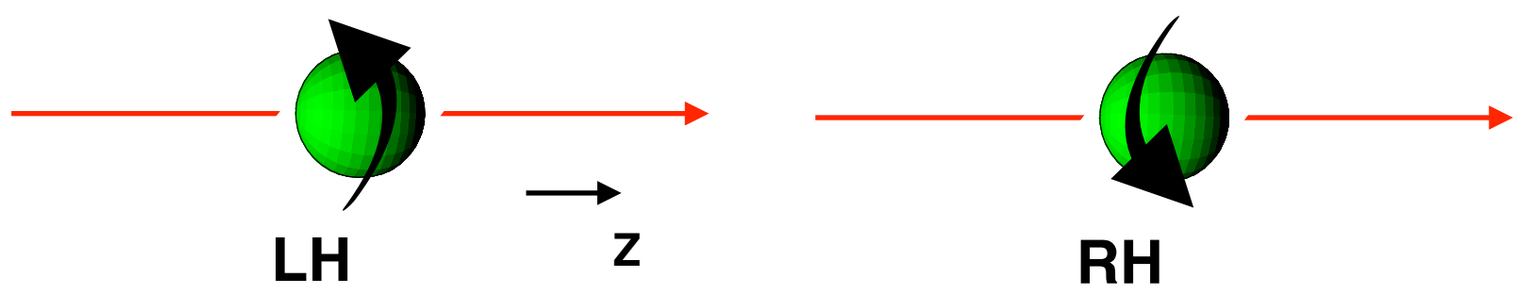
- **The weak integration violates parity. And one can determine that the neutrino is left handed and antineutrino is right handed. Their mirror images are not produced in weak interactions.**
- **The value of G_F is determined by muon lifetime. Calculate G_F from the measured muon lifetime and mass. What could be missing ?**
- **The three parameters of electro-weak interactions are: G_F, α, Z^0 mass.**

Parity Violation

$$P = \frac{N^+ - N^-}{N^+ + N^-} = \pm(\vec{\sigma} \bullet \vec{p}) / E = \pm\beta$$

Weak Interaction

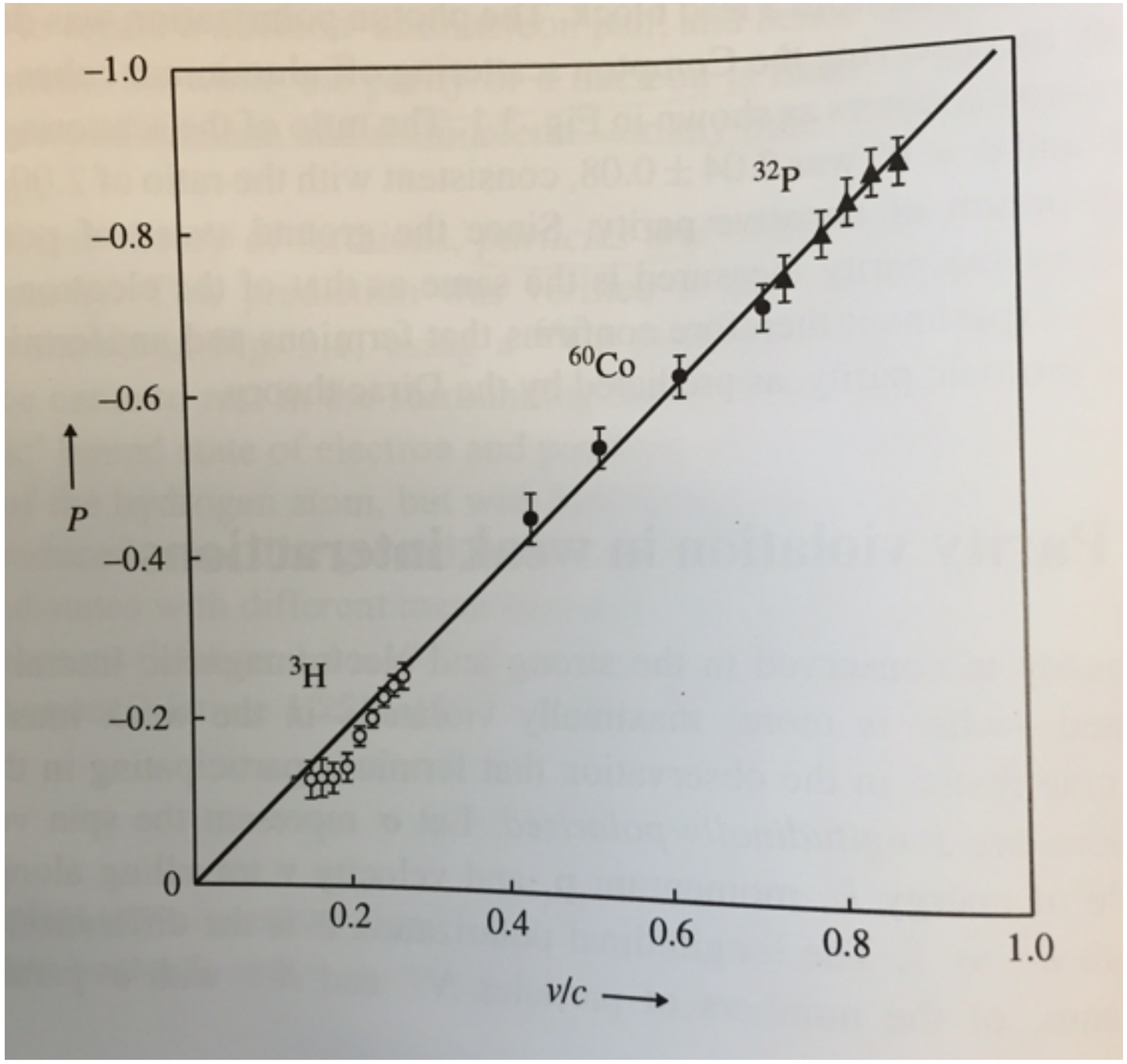
negative for fermions
positive for anti-fermions



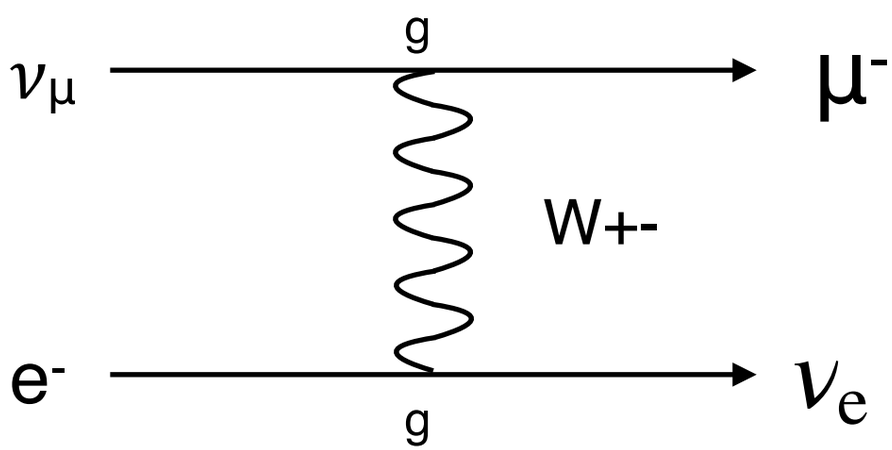
- *P* is polarization in which N^+ are particles with spin aligned with velocity and N^- are with spin opposite. σ is the spin of the particle with absolute value of 1.
- Weak charged current interactions violate parity maximally. They are chiral. Only particles that have $P=-\beta$ and anti-particles with $P=+\beta$ have weak interactions.
- Since neutrinos have only weak interactions and no mass, neutrinos are only left-handed. $P = -1$. now called helicity. Anti-neutrinos then have $P=+1$.
- In extremely high energy collisions (mass is neglected) Helicity is conserved if interactions are vector or axial. Scalar interactions do not conserve helicity.
- Mass is an example of an interactions that flips helicity !

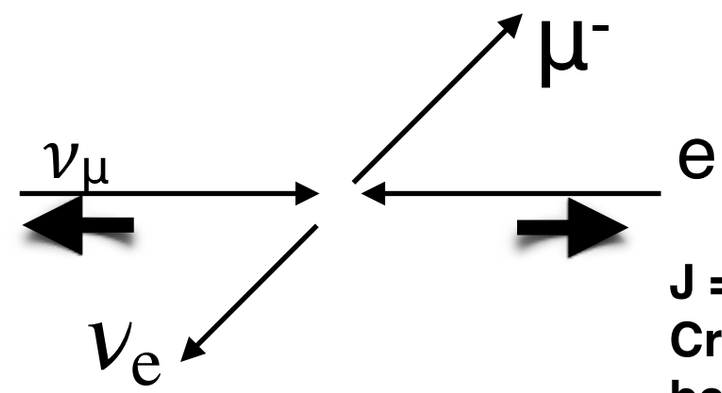
Experimental data on V-A.

Polarization in beta decay versus velocity.



Neutrino Electron Scattering





J = 0
Cross section should be isotropic with no angular dependence in CM

$$s = (p_{\nu_\mu} + p_e)^2$$

$$t = (p_{\nu_\mu} - p_\mu)^2 = q^2 = -Q^2$$

$$u = (p_{\nu_\mu} - p_{\nu_e})^2$$

$$y = \frac{p_e . q}{p_e . p_\nu} = \frac{E_\nu - E_\mu}{E_\nu}$$

y is the inelasticity

At high energy -s < t < 0

$$E_{thrs} = (m_\mu^2 - m_e^2)/2m_e \approx 11\,GeV$$

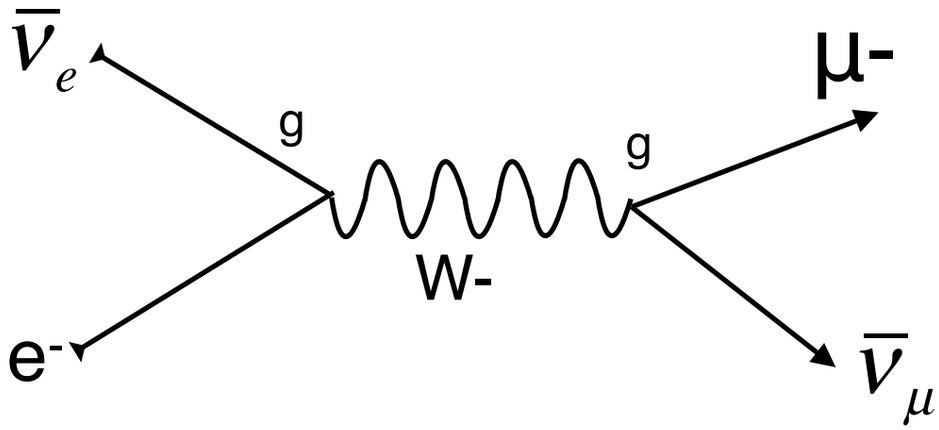
$$\theta^2 < 2m_e/E_\mu$$

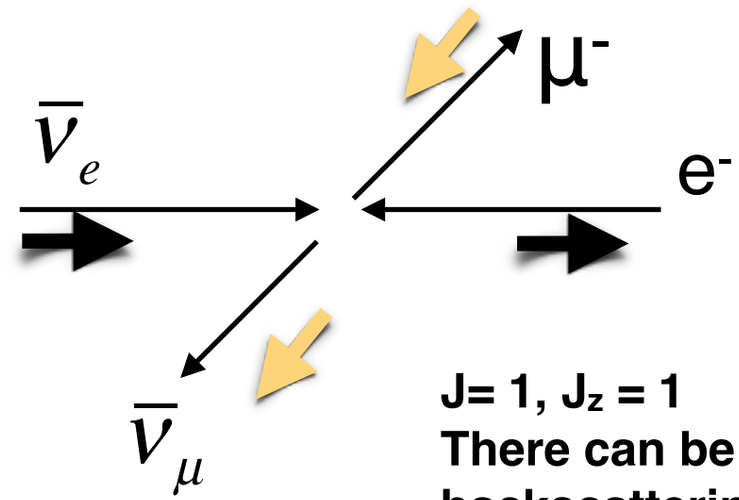
$$\frac{d\sigma(\nu_l e \to \nu_e l)}{dy} = \frac{2m_e E_\nu G_F^2}{\pi}\left(1 - \frac{m_\mu^2 - m_e^2}{2m_e E_\nu}\right)$$

$$\sigma \approx G_F^2 s/\pi$$

- What is the threshold for the similar process for tau neutrino production ?
- Why do neutrino cross sections increase with energy while Electromagnetic cross sections decrease as ~1/s?

Anti-Neutrino Electron Scattering





J= 1, J_z = 1
There can be no backscattering. Only one spin projection is allowed

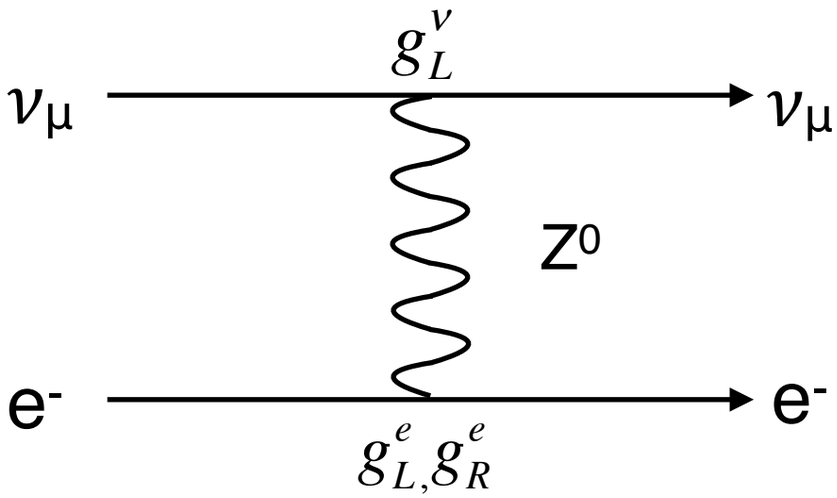
$$\frac{d\sigma(\bar{\nu}_e e \to \bar{\nu}_l l)}{dy} = \frac{2m_e E_\nu G_F^2}{\pi}\left((1-y)^2 - \frac{m_\mu^2 - m_e^2}{2m_e E_\nu}(1-y)\right)$$

$$\sigma \approx G_F^2 s / 3\pi$$

When y=1 all incoming kinetic energy is lost from the beam neutrino.

- Helicity plays a central role in determining cross sections of neutrinos versus anti-neutrinos. Cross section is reduced by factor 3.
- Think through the cross sections if the scattering is off positrons.

Neutrino Electron Elastic



Fermion	gL	gR
Neutrino	+1/2	0
electron, muon, tau	-1/2+Sin²θ_W	Sin²θ_W
up type quarks	+1/2-2/3Sin²θ_W	-2/3Sin²θ_W
down type quarks	-1/2+1/3Sin²θ_W	+1/3Sin²θ_W

$$\mathrm{Sin}^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

$$g_V = 2g_L^\nu(g_L^e + g_R^e)$$

$$g_A = 2g_L^\nu(g_L^e - g_R^e)$$

J = 0 term

J = 1 term

$$\frac{d\sigma}{dy}(\nu_\mu e) = \frac{m_e G_F^2 E_\nu}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2(1-y)^2 - (g_V^2 - g_A^2)\frac{m_e y}{E_\nu}\right]$$

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e) = \frac{m_e G_F^2 E_\nu}{2\pi}\left[(g_V - g_A)^2 + (g_V + g_A)^2(1-y)^2 - (g_V^2 - g_A^2)\frac{m_e y}{E_\nu}\right]$$

- In neutral current scattering both Left and Right charged particles contribute. The contribution of the Right is proportional to the weak mixing: Sin²θ_W.
- Here y is now defined as the fraction of energy lost to the electron.
- Plot the angular distribution of the first two terms. Recall ($\theta^2 < 2m_e/E_e$)

$$\nu_e + e^- \rightarrow \nu_e + e^- \Rightarrow \frac{1}{4} + \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W = 0.54$$

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^- \Rightarrow \frac{1}{12} + \frac{1}{3}\sin^2\theta_W + \frac{4}{3}\sin^4\theta_W = 0.22$$

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_\mu + \mu^- \Rightarrow \frac{1}{3}$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \Rightarrow 1$$

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^- \Rightarrow \frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W = 0.093$$

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \Rightarrow \frac{1}{12} - \frac{1}{3}\sin^2\theta_W + \frac{4}{3}\sin^4\theta_W = 0.075$$

$$\sigma_0 = \frac{2G_F^2 m_e E_\nu}{\pi} = 1.72 \times 10^{-41}\ \mathrm{cm}^2\mathrm{GeV}^{-1} \times (E_\nu / GeV)$$

The cross sections are small due to the target mass of the electron.
The first two cross sections have both charged and neutral current components.

Inverse Beta Decay (IBD)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

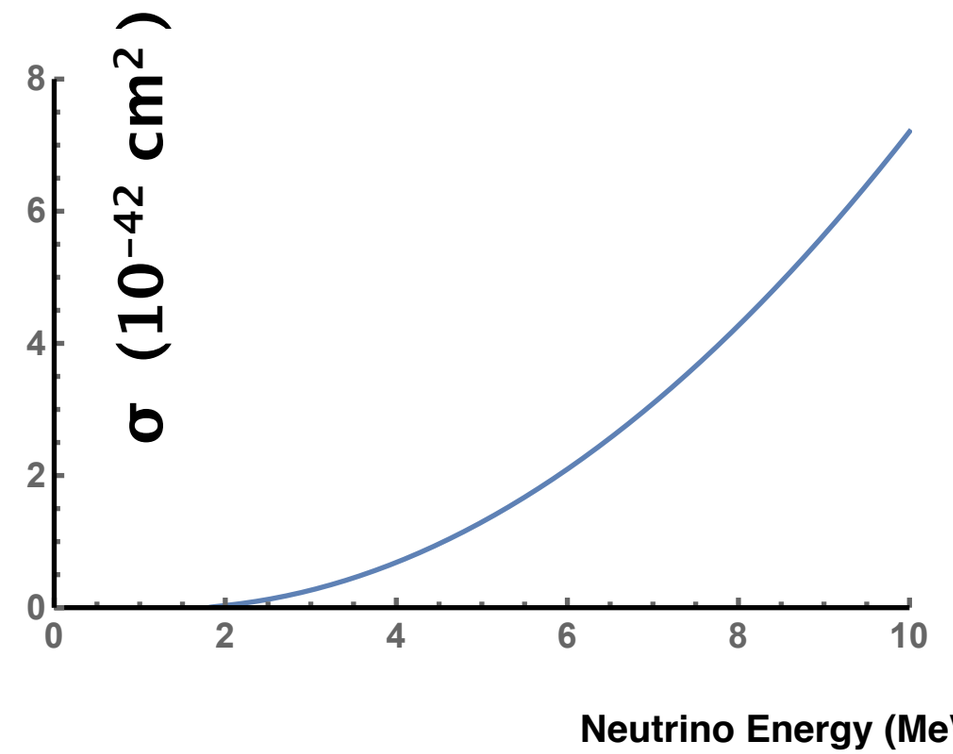
Constants can be obtained from neutron beta decay

Vector/ Fermi

Axial/Gamow-Teller

$$\sigma(E_\nu) = \frac{G_F^2 |V_{ud}|^2}{2\pi}(f_V^2 + 3f_A^2)E_e p_e$$

$$= 0.0952 \times 10^{-42} E_e p_e (cm^2 / MeV^2)$$



- **Vector term is forward in positron angle, Axial term is backwards. Overall slightly backwards.**
- **Threshold is due to (n-p) mass difference and positron mass. E_{th} = 1.806 MeV**
- **Only works for free proton targets and electron anti-neutrinos. No analog for electron neutrinos since there are no free neutrons !**
- **Neutrino Energy = Detected Energy - 2 electron mass + 1.806 MeV.**

Very low energy nuclear cross sections

$$\nu_e + (N,Z) \rightarrow e^- + (N-1,Z+1)$$

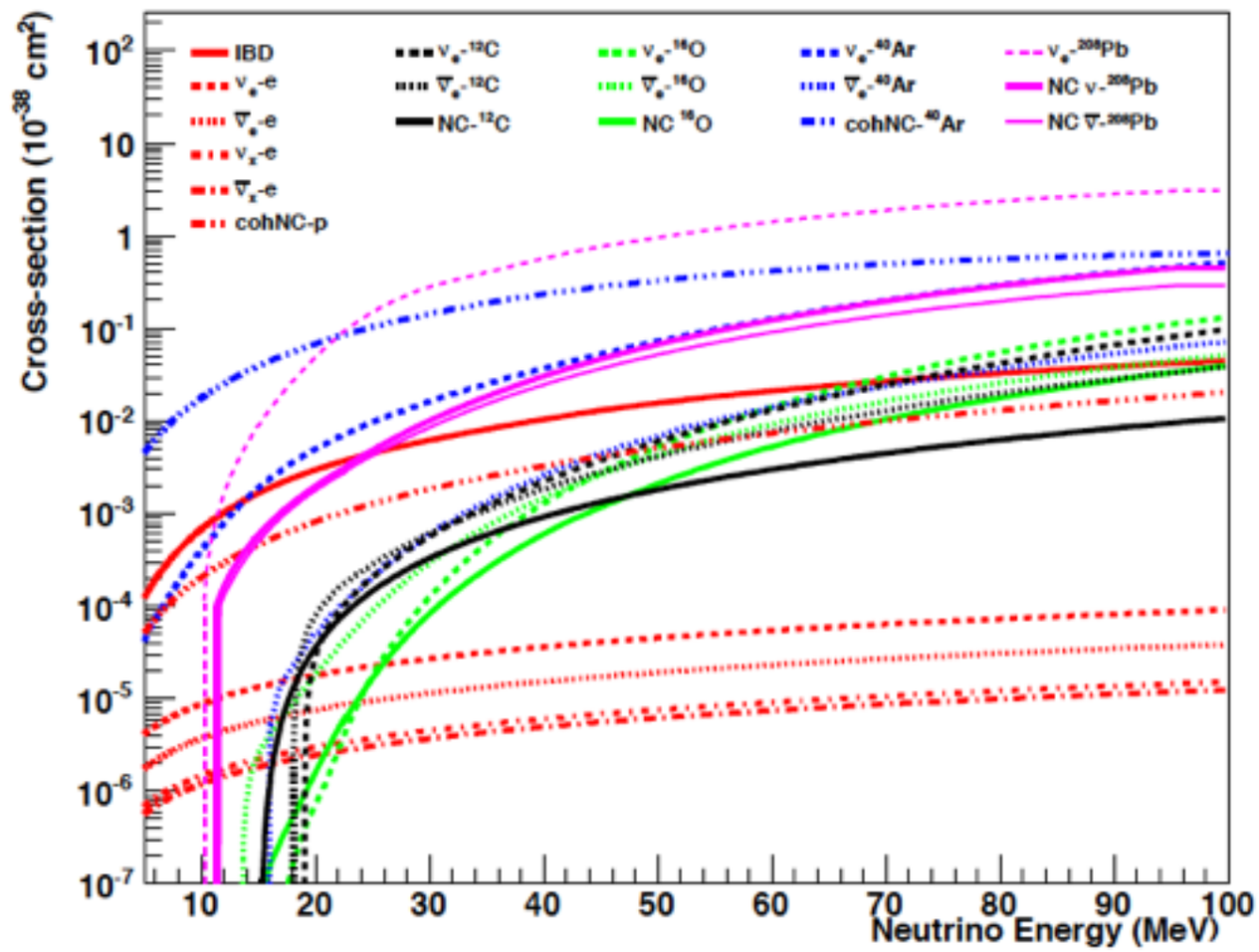
$$\bar{\nu}_e + (N,Z) \rightarrow e^+ + (N+1,Z-1)$$

$$E_{thr} \sim M_f - M_i + m_e$$

$$\sigma \sim 10^{-43} E_e p_e F(Z_f, E_e)(2J_f + 1)$$

Coulomb corrections

Exact calculation depends on type of transition: Fermi, G-T, forbidden, etc.



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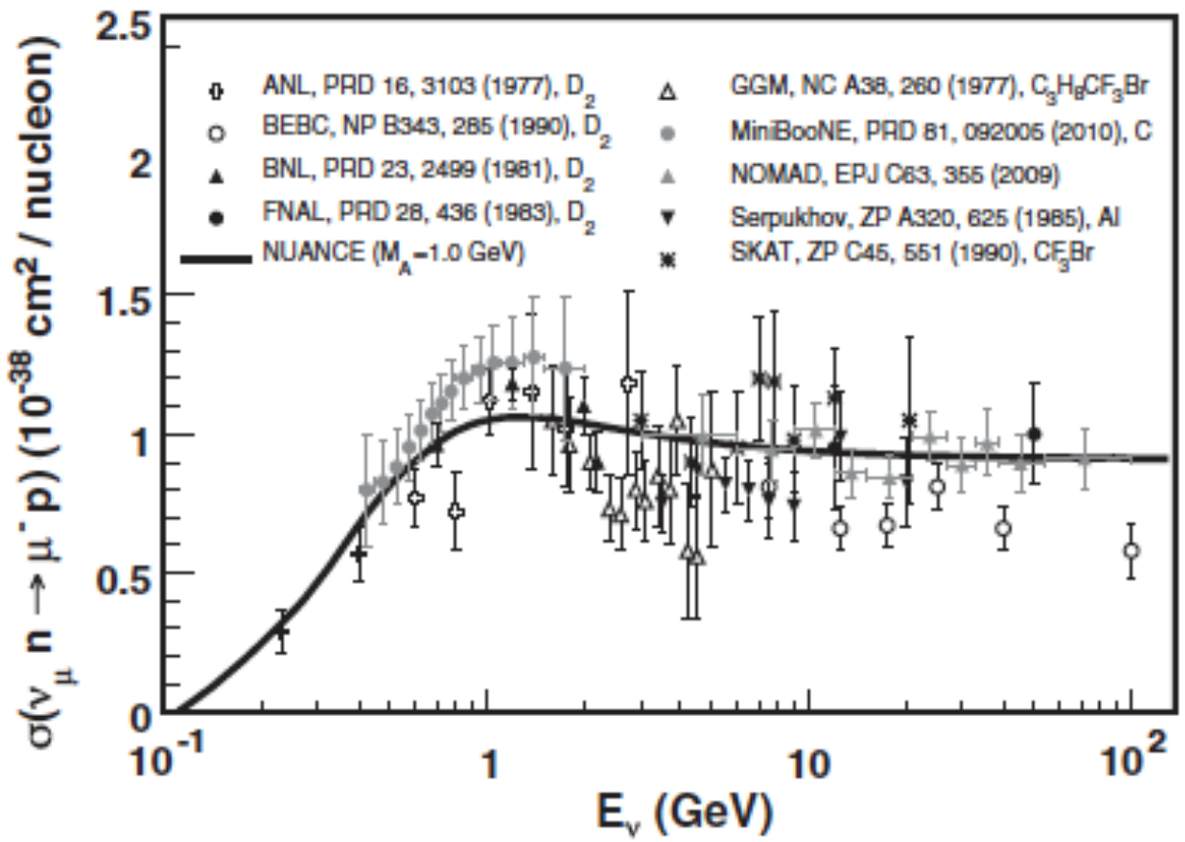
- **Low energy cross sections on nuclear targets poorly measured (errors ±20%)**
- **Needed for elemental abundances and supernova physics.**
- **Muon and Tau neutrinos below threshold for CC reactions <100 MeV**
- **There can be gammas from de-excitation of nuclear final states.**
- **NC reactions only have de-excitation gammas as signature.**

Quasi-elastic Scattering

$$\nu_\mu + n \rightarrow \mu^- + p$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

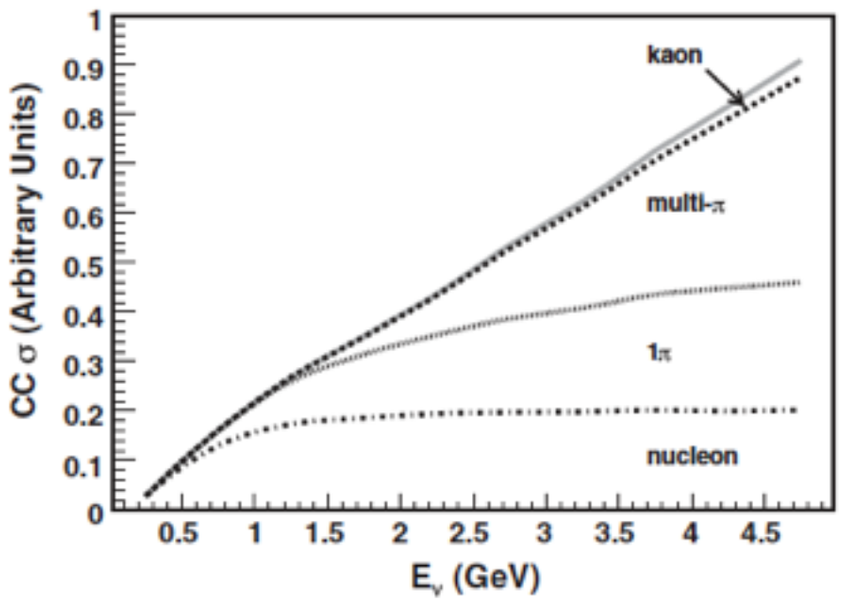


The data have inconsistencies because nuclear effects are still a work-in-progress.

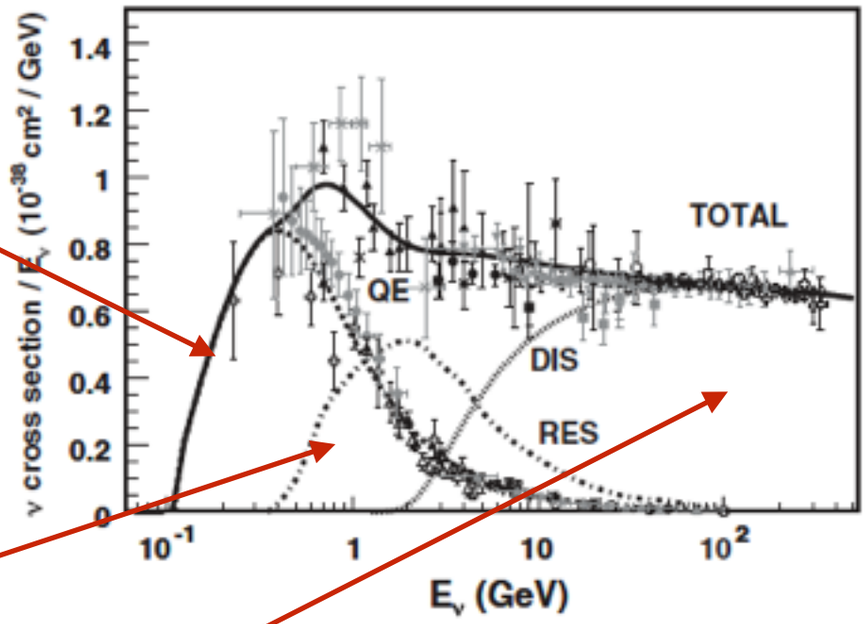
A single parameter is used to characterize $M_A \sim 1$ GeV.

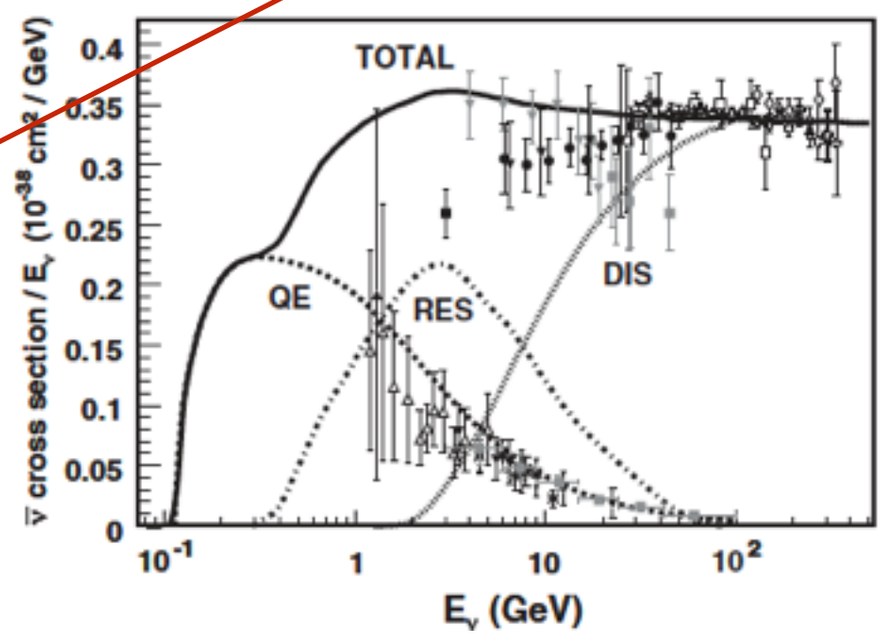
- The vector(L+R) and axial(L-R) scattering amplitudes become much more complicated on nuclei at low energies because of the internal structures.
- The simple g_V , g_A are replaced by form factors that depend on the Q^2 or the momentum transfer of the reaction.
- The cross sections are much larger because they are now proportional to the nucleon mass.
- If the nucleons can be treated as free, calculate the neutrino energy in terms of muon E,θ

Cross sections at a few GeV



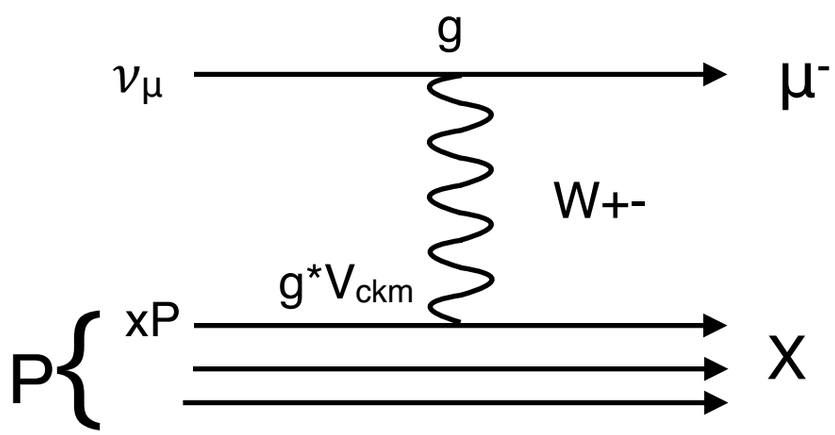
Cross section prediction for various processes add up to exhibit a linear dependence of the total cross section on E





- The intermediate energy range has many final states. Many of these have not been measured well.
- This modeling is an active part of the field as new data becomes available. (List all ν and anti-ν nucleon single pion producing modes.)

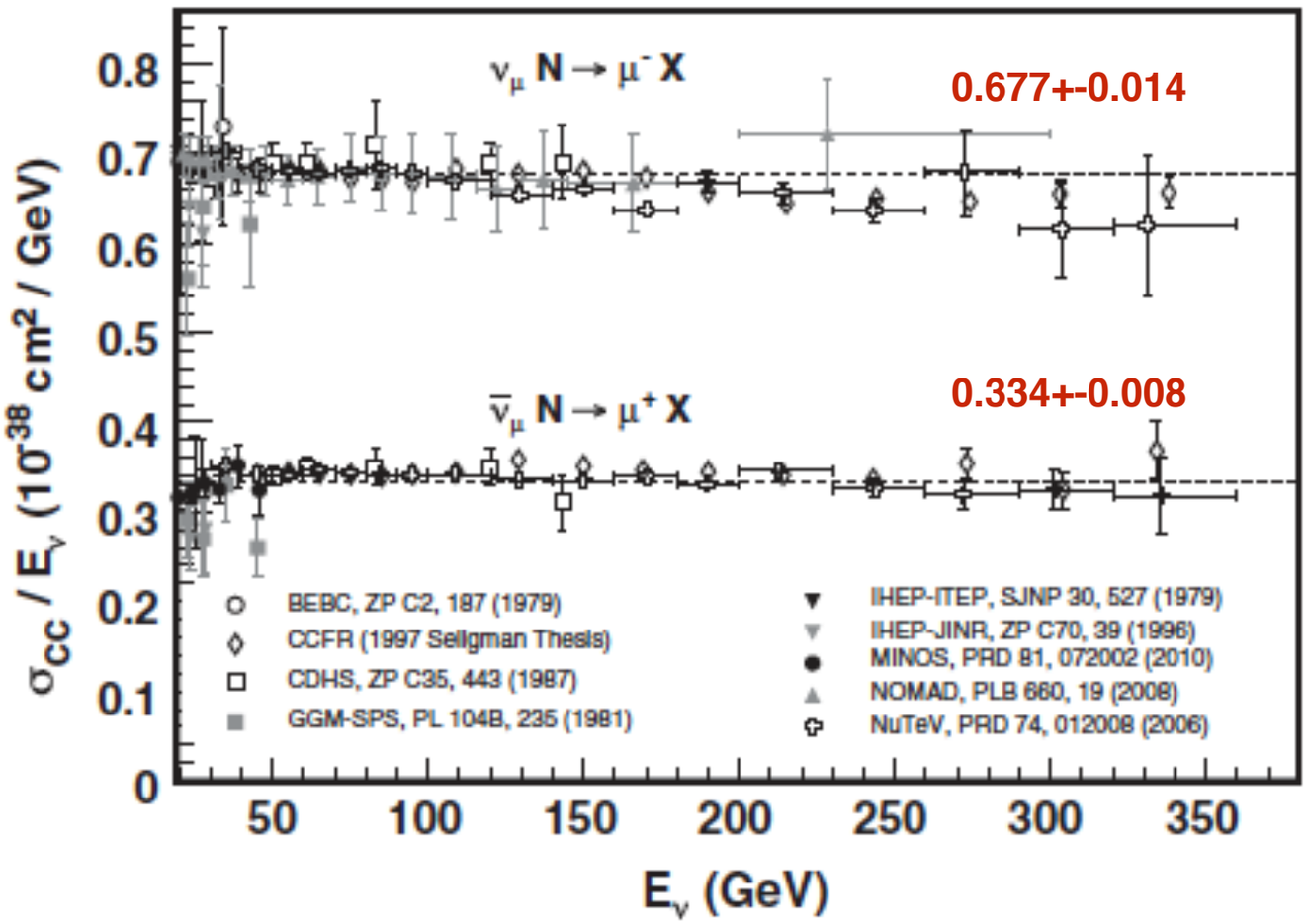
Deep Inelastic Scattering



$$y = E_{had} / E_\nu$$

$$x \equiv \frac{-q^2}{2P \bullet q} = \frac{Q^2}{2M_N E_{had}}$$

$$Q^2 = -m_\mu^2 + 2E_\nu \bullet (E_\mu - p_\mu \operatorname{Cos}\theta_\mu)$$



$$d\sigma(\nu q)/dy \propto G_F^2 M_N x E_\nu \quad d\sigma(\bar{\nu} q)/dy \propto G_F^2 M_N x E_\nu (1-y)^2$$

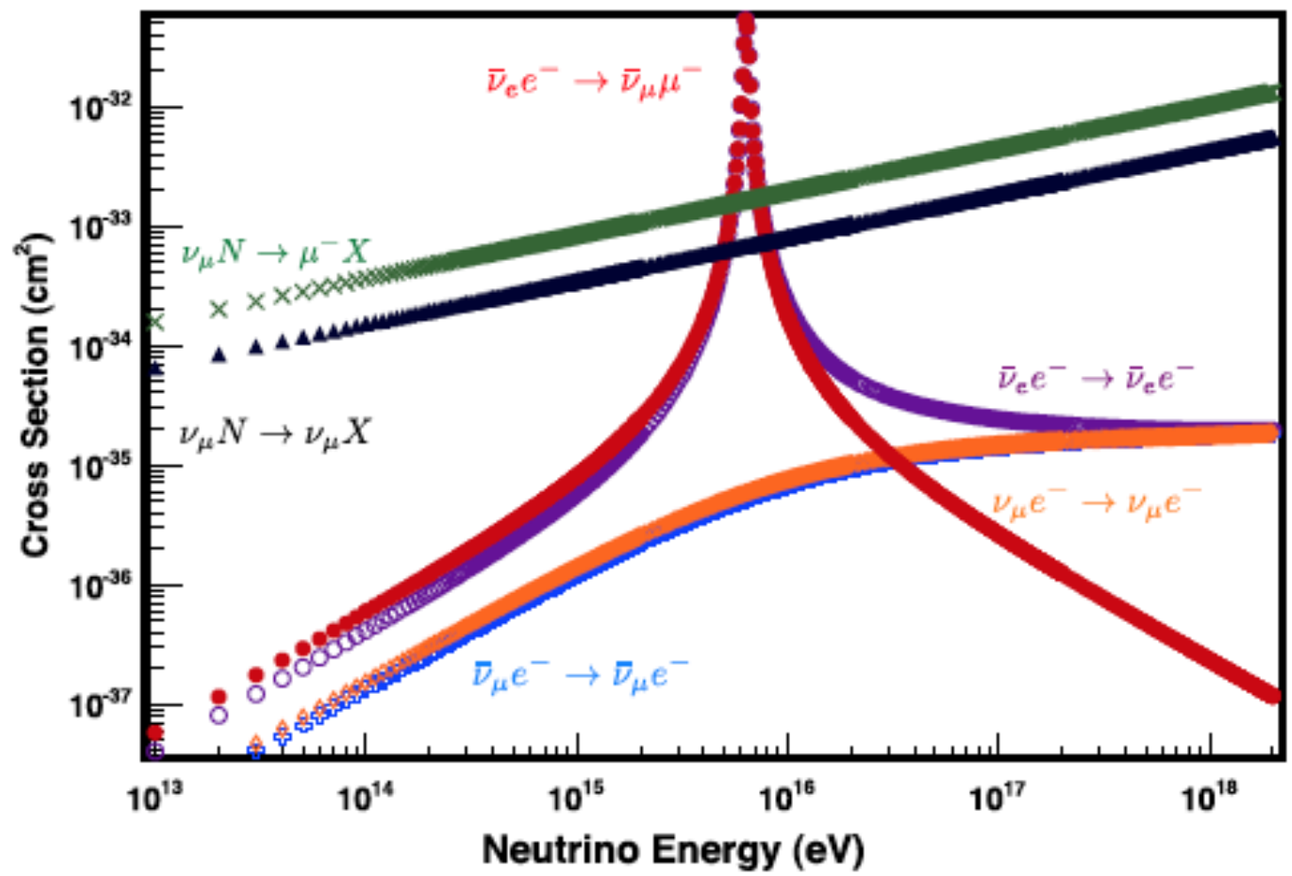
- In case of high energies, the scattering is off quarks in the nucleons.
- But quarks carry only some fraction of the nucleon momentum. In the 'infinite momentum frame' it is shown to be ~*x*.
- Since scattering is off mostly quarks, the y distributions follow the usual rule for ν and anti-ν, but the total ratio differs from 1/3 because of the sea quark-anti-quarks.

Very High Energies

$$\sigma_{\nu N}^{CC} \approx 5.53\times10^{-36}\, cm^2 (E_\nu / GeV)^{0.363}$$
$$\sigma_{\nu N}^{NC} \approx 2.31\times10^{-36}\, cm^2 (E_\nu / GeV)^{0.363}$$

There is a resonance due to formation of the W boson only for

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_l + l^-$$
$$E_{res} = M_W^2 / 2m_e = 6300 TeV$$



Gandhi et al, Astroparticle Phys. 5, 81, 1996

- **> 10^6 GeV, the cross sections do not grow linearly.**
- **ν and anti-ν cross sections are nearly identical. Why?**
- **Uncertainties arise from quark/antiquark distributions.**

Tau neutrino interactions

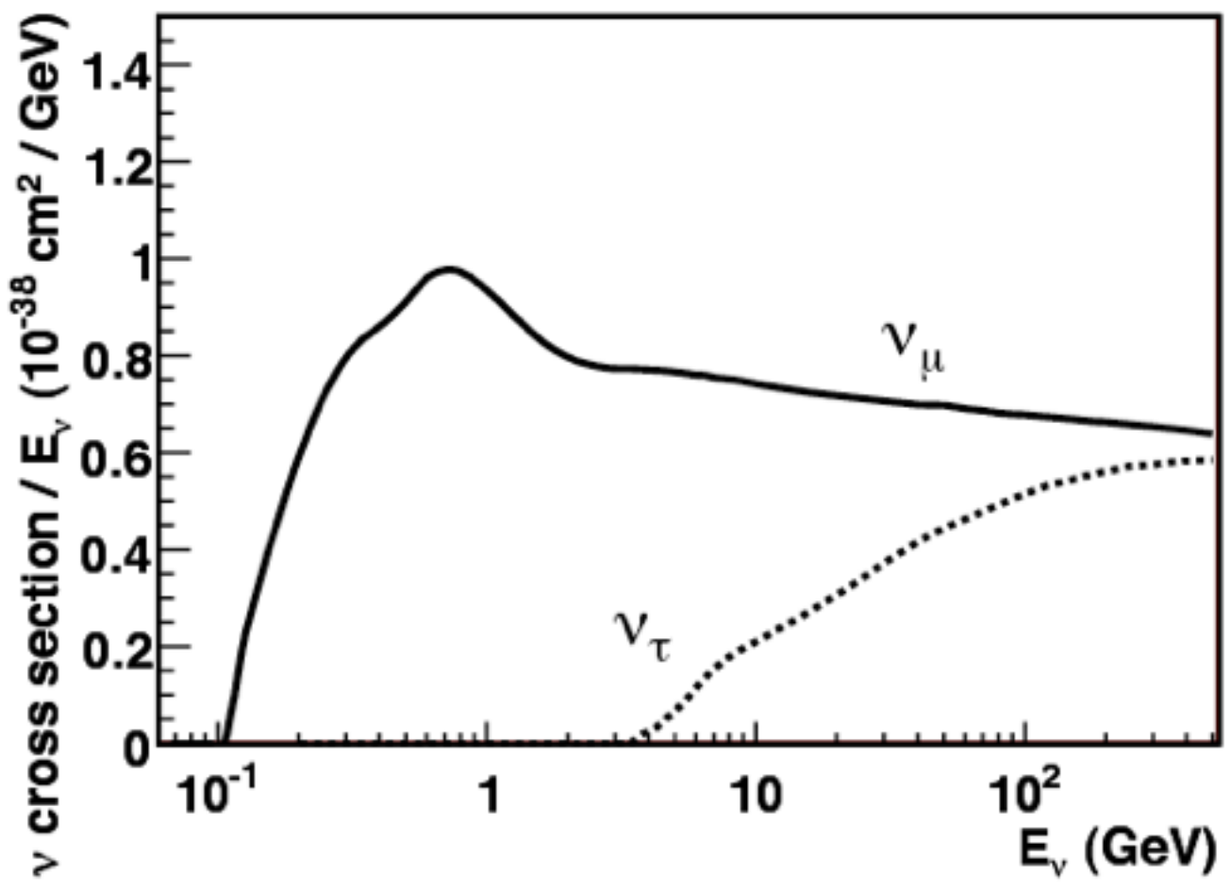
$$\nu_\tau + N \rightarrow \tau + X$$

$$E_{th} = 3.5 GeV$$

Tau lepton properties.

$$mass = 1.777\ GeV$$

$$\tau_{life} = 2.3 \times 10^{-13}\ \text{sec}$$



~17% of the decays are to a single muon +neutrinos and ~17% to electron and neutrinos.

Formaggio/Zeller RevModPhys.84.1307

- **CC interactions of the tau neutrino are suppressed due to the high energy threshold. (Calculate the threshold).**
- **Tau decays to low multiplicity states making it difficult to identify.**

Conclusion

- This lecture was about the basics of neutrino interactions.
- The basics have been known since the 1980s.
- Knowledge of interactions on complex nuclei is not yet precise. This will be needed in the future.
- Very low energy interactions also have very important applications for astrophysics.
- In the next lecture I will describe the technology of detectors for these measurements.