A pure integrator that gives step pulses (DC coupled) eventually will have divergent noise. This is easy to see as below.

fix the output at 0 at $t = 0$, and let the rate of white noise pulses be $r = N / T$. Each white noise pulse gives ± 1.

Let each elementary pulse give an integral of 1. Then the value of the output at any time t is

$$V(t) = n_p - n_m$$

Where n_p are positive pulses and n_m are the negative ones.

$$\langle V(t) \rangle = 0,$$

but the variance diverges as $t \to \infty$.

$$\langle V(t)^2 \rangle = n_p + n_m \approx r \bullet t$$

There is a formal way to understanding this in probability theory, but we will leave it at that.