# Superbeam vs. Neutrino Factory

## correlations, uncertainties degeneracies

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based on

hep-ph/0204352

# Motivation

neutrinos oscillate $\Longrightarrow$ (at least) 6 additional fundamental parameters

LBL experiments offer the unique possibility to study in detail:

- atmospherical parameters $\Longrightarrow$ $\Delta m^2_{31}$ and $\theta_{23}$
- subleading oscillations $\Longrightarrow$ $\theta_{13}$
- mass hierarchy $\Longrightarrow$ $\mathrm{sign}\Delta m^2_{31}$
- three neutrino effects $\Longrightarrow$ $\delta_{\mathrm{CP}}$

But:

The size of $\theta_{13}$ will determine if any experiment can answer those fundamental questions

# Outline

Comparison of different experiments

- Superbeams vs. Neutrino Factories
- initial and advanced setups
- identical boundary conditions
  - running time, target power, confidence levels etc.
  - equal level of sophistication in the analysis

Impact factors - which controllable factors determine the physics potential

- luminosity (i.e. money)
- systematical effects
- quality of external input

Full three flavour analysis

- 6 physical parameters
- systematical uncertainties
- external input
- correlations
- degeneracies

# Appearance Probability

Analytic approximation of the appearance probability – expansion in powers of $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$

$$
\begin{aligned}
P_{\nu_e\to\nu_\mu} \approx\ & \sin^2\theta_{23}\frac{\sin^2 2\theta_{13}}{(\hat{A}-1)^2}\sin^2((\hat{A}-1)\Delta) \\
\pm\ & \alpha\ \frac{\sin\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\sin(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\
+\ & \alpha\ \frac{\cos\delta_{\rm CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\cos(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\
+\ & \alpha^2\ \frac{\cos^2\theta_{23}\sin^2 2\theta_{12}}{\hat{A}^2}\sin^2(\hat{A}\Delta)
\end{aligned}
$$

$$\Delta = \Delta m^2_{31}L/4E;\ \hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31};\ V = \sqrt{2}G_F n_e.$$

$P_{\nu_e\to\nu_\mu}$ depends on solar parameters only via the product $\sin 2\theta_{12}\cdot\Delta m^2_{21}$

M. Freund, hep-ph/0103300.

A. Cervera *et al.*, Nucl. Phys. **B579** (2000) 17.

# Simulation of LBL Experiments

**Source $\otimes$ Oscilation $\otimes$ Detector**

## Source

- neutrino energy , flux and spectrum
- neutrino flavour composition (and contamination)
- operation of neutrino and anti-neutrino modes

## Oscillation

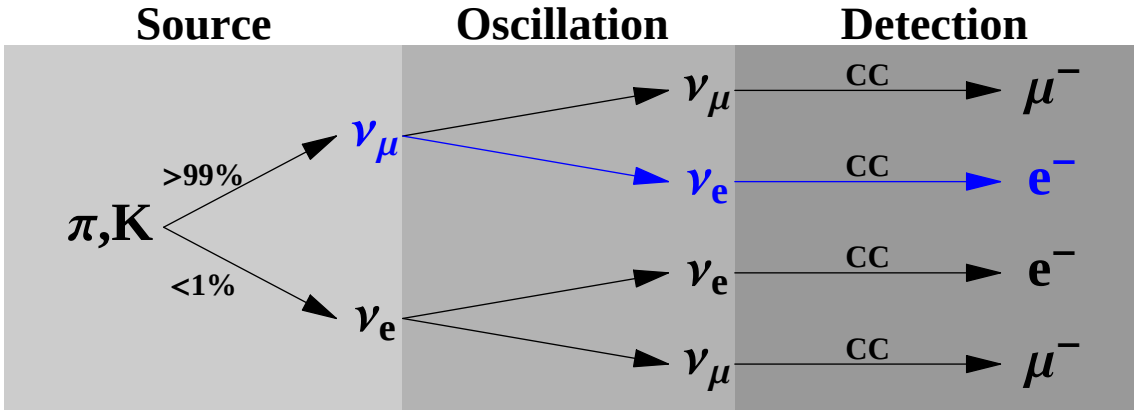
- initial and final flavour $\Leftrightarrow$ oscillation channels
- oscillation parameters: mass splittings, angles and phase
- baseline, matter profile, ...

## Detector

- fiducial mass, material, energy threshold and resolution
- particle ID (flavour, charge, event reconstruction, ...)
- x-sections

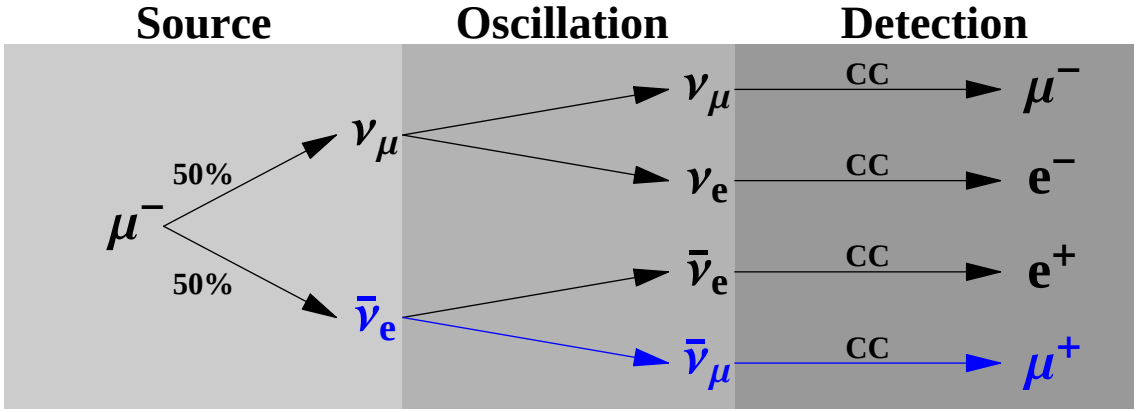
# Possible Signals

## Superbeams




$\nu_\mu \to \nu_e$ oscillation most interesting
$\nu_e$ contamination $\Leftrightarrow$ off-axis
good electron detection efficiency
good NC background rejection
near detector
$\overline{\nu}$-beam $\simeq$ different experiment

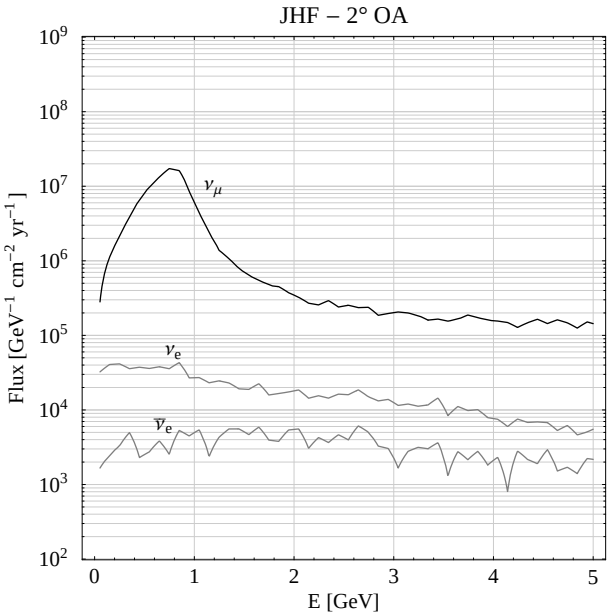
## Neutrino Factories




$\overline{\nu}_e \to \overline{\nu}_\mu$ oscillation most interesting
excellent beam properties
very good charge ID required
good NC background rejection
$\mu^+$ mode very symmetric

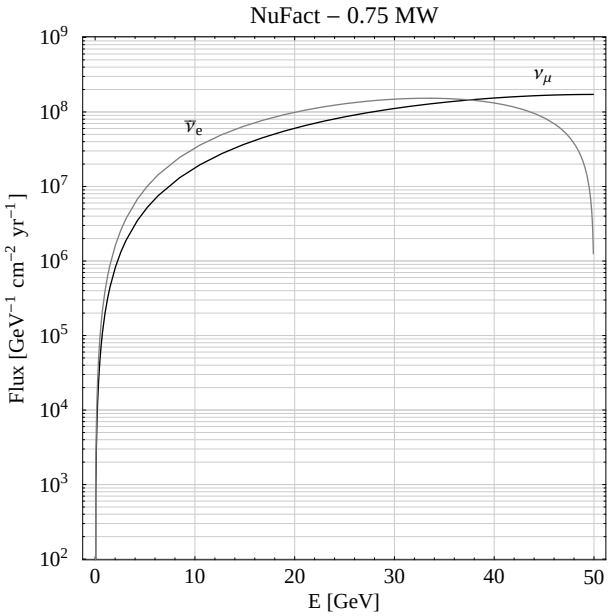
# Our Analysis – Source

## JHF




- mean energy $0.51\,\mathrm{GeV}$
- peak intensity
  $1.7 \cdot 10^7\,\mathrm{GeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{yr}^{-1}$
  at $0.78\,\mathrm{GeV}$
- $\nu_\mu/\nu_e$-ratio at peak $0.2\%$

## NuFact




- mean energy $30\,\mathrm{GeV}$
- peak intensity
  $1.5 \cdot 10^8\,\mathrm{GeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{yr}^{-1}$
  at $3.33\,\mathrm{GeV}$
- $\nu_\mu/\nu_e$-ratio at peak $83\%$

Uncertainties in flux and $\nu_e$–background

# Our Analysis – Oscillation

JHF
- $295\,\mathrm{km}$ baseline
- $L/E_{\text{peak}} = 378\,\mathrm{km\,GeV^{-1}}$
- matter density $\rho = 2.8\,\mathrm{g\,cm^{-3}}$

NuFact
- $3\,000\,\mathrm{km}$ baseline
- $L/E_{\text{peak}} = 90\,\mathrm{km\,GeV^{-1}}$
- matter density $\rho = 3.5\,\mathrm{g\,cm^{-3}}$

$P_{\nu_\mu \to \nu_e}$ depends on solar parameters via the product

$$\pi_{\text{sol}} := \sin 2\theta_{12} \cdot \Delta m^2_{21}$$

thus we use this $\pi_{\text{sol}}$ as parameter –
this is the only approximation in the treatment of the oscillation

- full 3 neutrino calculation in matter
- we allow for errors on all parameters, including the matter density
- for solar parameters we assume KamLand input
- CP-phase $\delta_{\mathrm{CP}}$ is unconstrained

Uncertainties on all parameters

# Our Analysis – Detector

JHF

- water Cherenkov detector – SuperK
- fiducial mass $22.5\,(1\,000)\,\mathrm{kt}$
- running period of $5\,(8)$ years
- NC-rejection $\sim 0.01$
- energy range $0.4 - 1.2\,\mathrm{GeV}$
- constant efficiency
- efficiency for $\nu_e \sim 0.5$
- $61\%$ QE-interactions at $E_{\mathrm{peak}}$
- energy resolution $\delta E = 5\%$
- $5\%$ uncertainty on all backgrounds

NuFact

- magnetized iron calorimeter
- fiducial mass $10\,(50)\,\mathrm{kt}$
- running period of $5\,(8)$ years
- NC-rejection $< 10^{-5}$
- energy range $4 - 50\,\mathrm{GeV}$
- linear rise of efficiency from $4 - 20\,\mathrm{GeV}$
- efficiency for $\nu_\mu \sim 0.4$
- DIS
- energy resolution $\delta E = 15\%$
- $5\%$ uncertainty on all backgrounds

Y. Itow *et al.*, hep-ex/0106019.

A. Blondel *et al.*, Nucl. Instrum. Meth. **A451**, 102 (2000).

# Setups

| JHF-SK | JHF-HK | NuFact-I | NuFact-II |
|---|---|---|---|
| $22.5\,\mathrm{kt}$ water Cherenkov | $1\,000\,\mathrm{kt}$ water Cherenkov | $10\,\mathrm{kt}$ magnetized iron calorimeter | $50\,\mathrm{kt}$ magnetized iron calorimeter |
| $0.75\,\mathrm{MW}$ target power | $4\,\mathrm{MW}$ target power | $0.75\,\mathrm{MW}$ target power | $4\,\mathrm{MW}$ target power |
| $5$ years | $8$ years | $5$ years | $8$ years |
| $139.0$ signal | $13\,180.0$ signal | $1\,522.8$ signal | $64\,932.6$ signal |
| $23.3$ background | $2\,204.6$ background | $4.2$ background | $18.3$ background |

# Statistics

- Poissonian statistics $\chi^2 = \sum_{i=1}^{b} \left( 2[\langle x_i \rangle - x_i] + 2\, x_i \log \frac{x_i}{\langle x_i \rangle} \right)$
- parameterization of systematical uncertainties

$$
\begin{aligned}
s_i &= s_i(n_s, t_s) = s_i^0\,(1 + n_s + t_s \cdot E_i/(E_{\text{max}} - E_{\text{min}})) \\
b_i &= b_i(n_b, t_b) = b_i^0\,(1 + n_b + t_b \cdot E_i/(E_{\text{max}} - E_{\text{min}})) \\
x_i &= x_i(n_s, t_s, n_b, t_b) = s_i(n_s, t_s) + b_i(n_b, t_b) \\
\chi^2_\alpha &= \chi^2_\alpha(\lambda, \rho; n_s^\alpha, t_s^\alpha, n_b^\alpha, t_b^\alpha)
\end{aligned}
$$

- integrating out the nuisance parameters

$$
\hat{\chi}^2_\alpha(\lambda, \rho) = \min_{n_s^\alpha, t_s^\alpha, n_b^\alpha, t_b^\alpha} \left( \chi^2_\alpha(\lambda, \rho; n_s^\alpha, t_s^\alpha, n_b^\alpha, t_b^\alpha) + \frac{(n_s^\alpha)^2}{\sigma^2_{n_s^\alpha}} + \frac{(t_s^\alpha)^2}{\sigma^2_{t_s^\alpha}} + \frac{(n_b^\alpha)^2}{\sigma^2_{n_b^\alpha}} + \frac{(t_b^\alpha)^2}{\sigma^2_{t_b^\alpha}} \right)
$$

# Statistics

- projection on the parameter of interest – taking into account additional information

$$\chi^2_{\mathrm{F}}(\eta) \quad = \quad \min_{\lambda,\rho} \left( \sum_{\alpha} \hat{\chi}^2_{\alpha}(\lambda,\rho) + \frac{(\rho-\rho^0)^2}{\sigma^2_{\rho}} + \frac{(\pi_{\mathrm{sol}}-\pi^0_{\mathrm{sol}})^2}{\sigma^2_{\pi_{\mathrm{sol}}}} \right)$$

- external information from KamLand and geophysics

$$\sigma_{\pi_{\mathrm{sol}}} = 0.15\,\pi^0_{\mathrm{sol}} \quad \text{and} \quad \sigma_{\rho} = 0.05\,\rho^0$$

- 6 parameters remain

$$\lambda = (\rho, \pi_{\mathrm{sol}}, \Delta m^2_{31}, \theta_{23}, \theta_{13}, \delta_{\mathrm{CP}})$$

R. J. Geller and T. Hara, hep-ph/0111342.

M. C. Gonzalez-Garcia and C. Peña-Garay, Phys. Lett. **B527**, 199 (2002).

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4×10$^{-6}$

# Systematical Errors

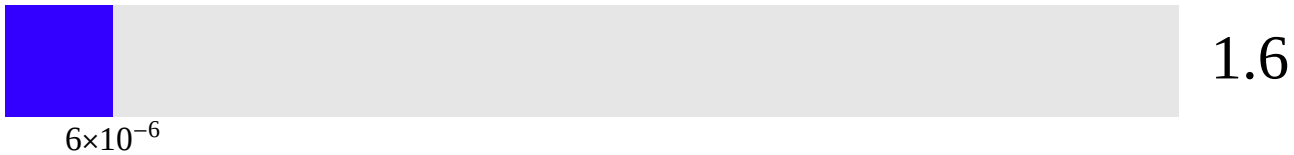
Signal

- use the energy information
- use appearance and disappearance
- appearance channel is a relative measurement
  - overall normalization error of $5\%$ is okay
  - energy calibration error of $5\%$ is okay

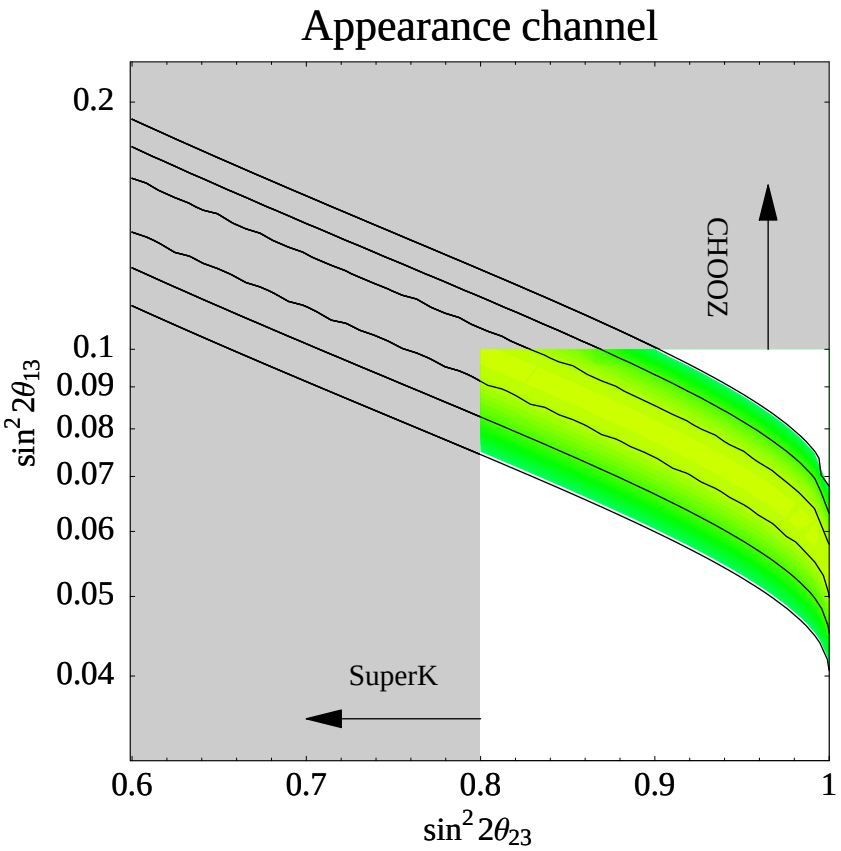
Background

- magnitude of backgrounds limits sensitivity for Superbeams
- energy information helps to reduce the impact of the background
- background uncertainty becomes important when $\sigma_{b_n} \geq 1/\sqrt{b}$
  for $\sigma_{b_n} = 5\% \Leftrightarrow b = 400$

Background uncertainty important for high statistics experiments



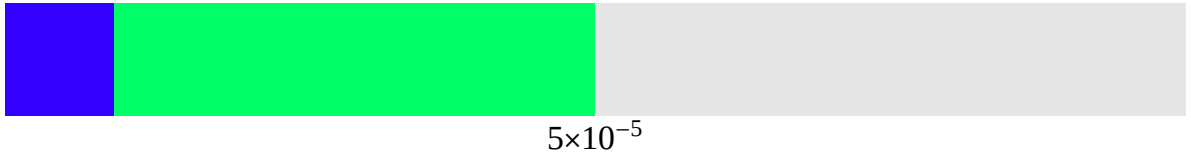
# Correlations




- multi-parameter problem
- highly non-linear
- complex topology

$\Rightarrow$ use all available information

- appearance & disappearance channels
- energy information
- external input

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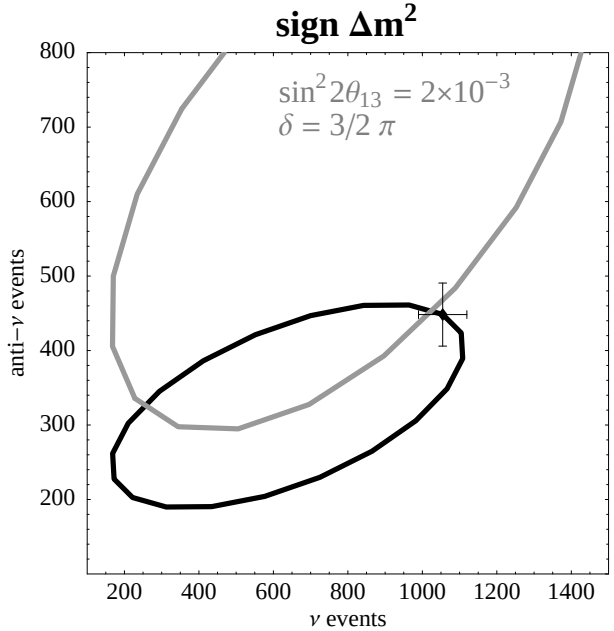
# Degeneracies

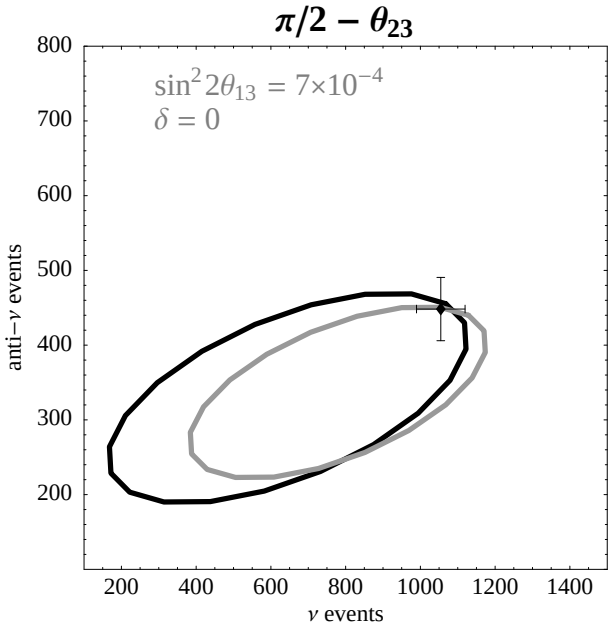
Starting values: $\sin^2 2\theta_{13} = 1 \cdot 10^{-3}$ and $\delta_{\rm CP} = 0$




$\sin\delta$ and $\mathrm{sign}\Delta m^2_{31}$ act different on $\nu$ and $\overline{\nu}$

- matter effect

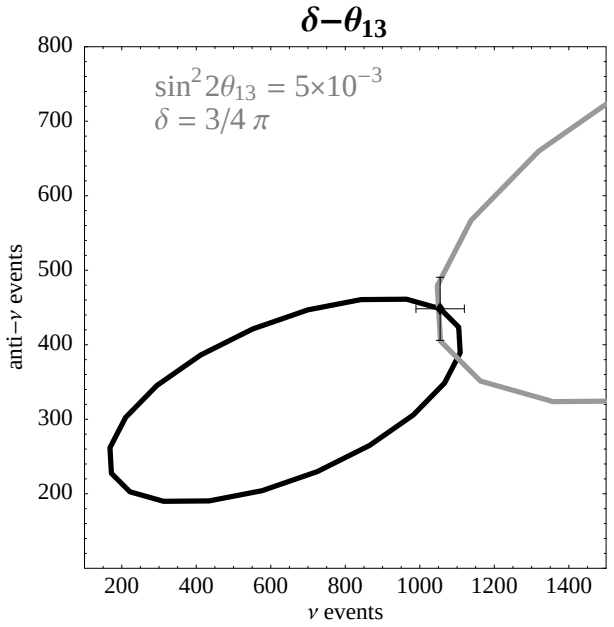
$\Longrightarrow$ $\delta'$ and opposite sign




$\sin^2 2\theta_{23}$ in $\nu_\mu \to \nu_\mu$ is symmetric around $\pi/4$

$\sin^2\theta_{23}$ in $\nu_e \to \nu_\mu$ is not

$\Longrightarrow$ $\theta'_{13}$ and $\theta'_{23}$




$\sin 2\theta_{13}, \cos\delta, \sin\delta$

- two solutions with identical event rates for $\nu$ and $\overline{\nu}$

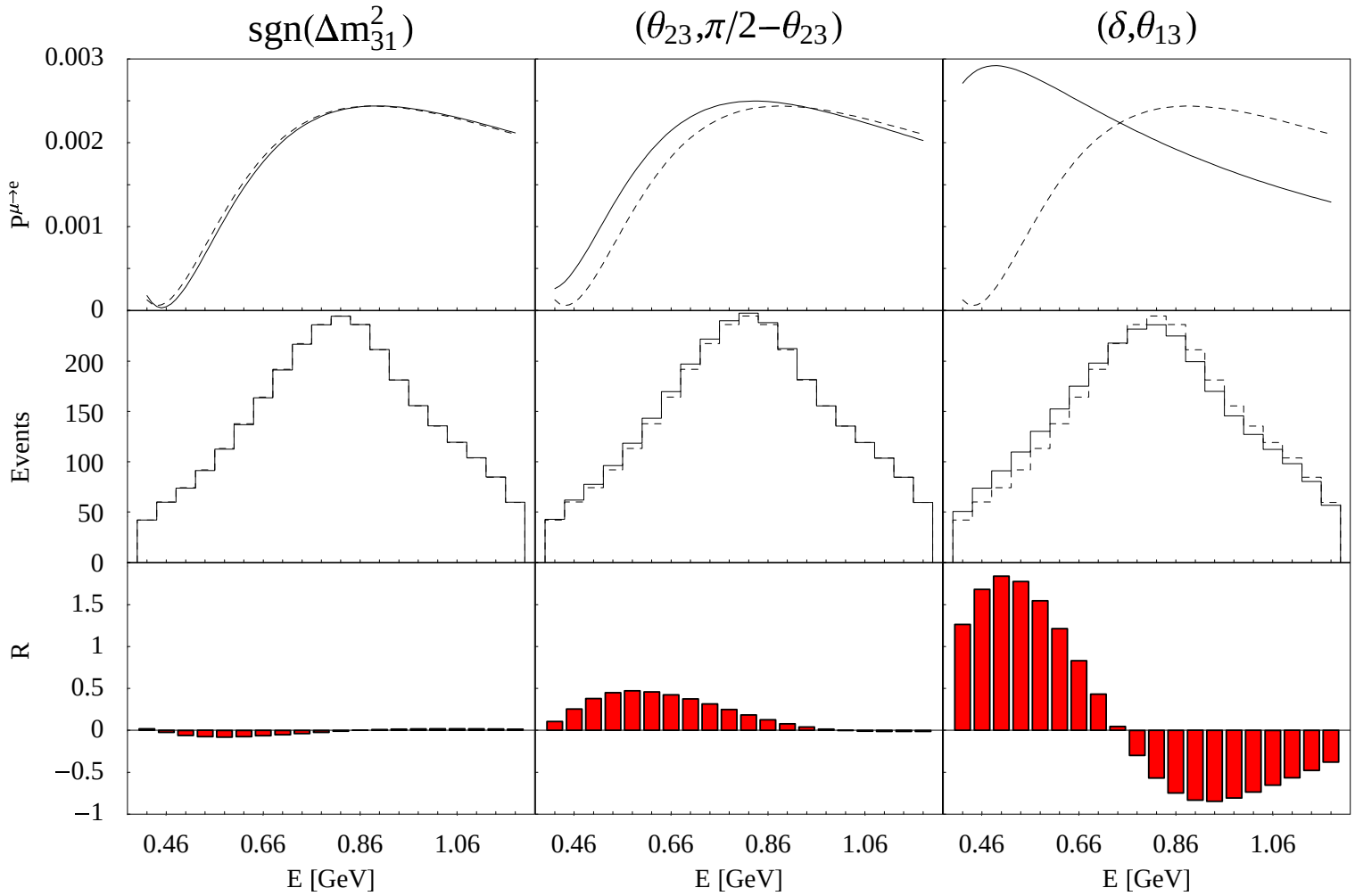
$\Longrightarrow$ $\delta'$ and $\theta'_{13}$


J. Burguet-Castell *et al.*, Nucl. Phys. **B608**, 301 (2001).

V. Barger *et al.*, Phys. Rev. **D65**, 073023 (2002).

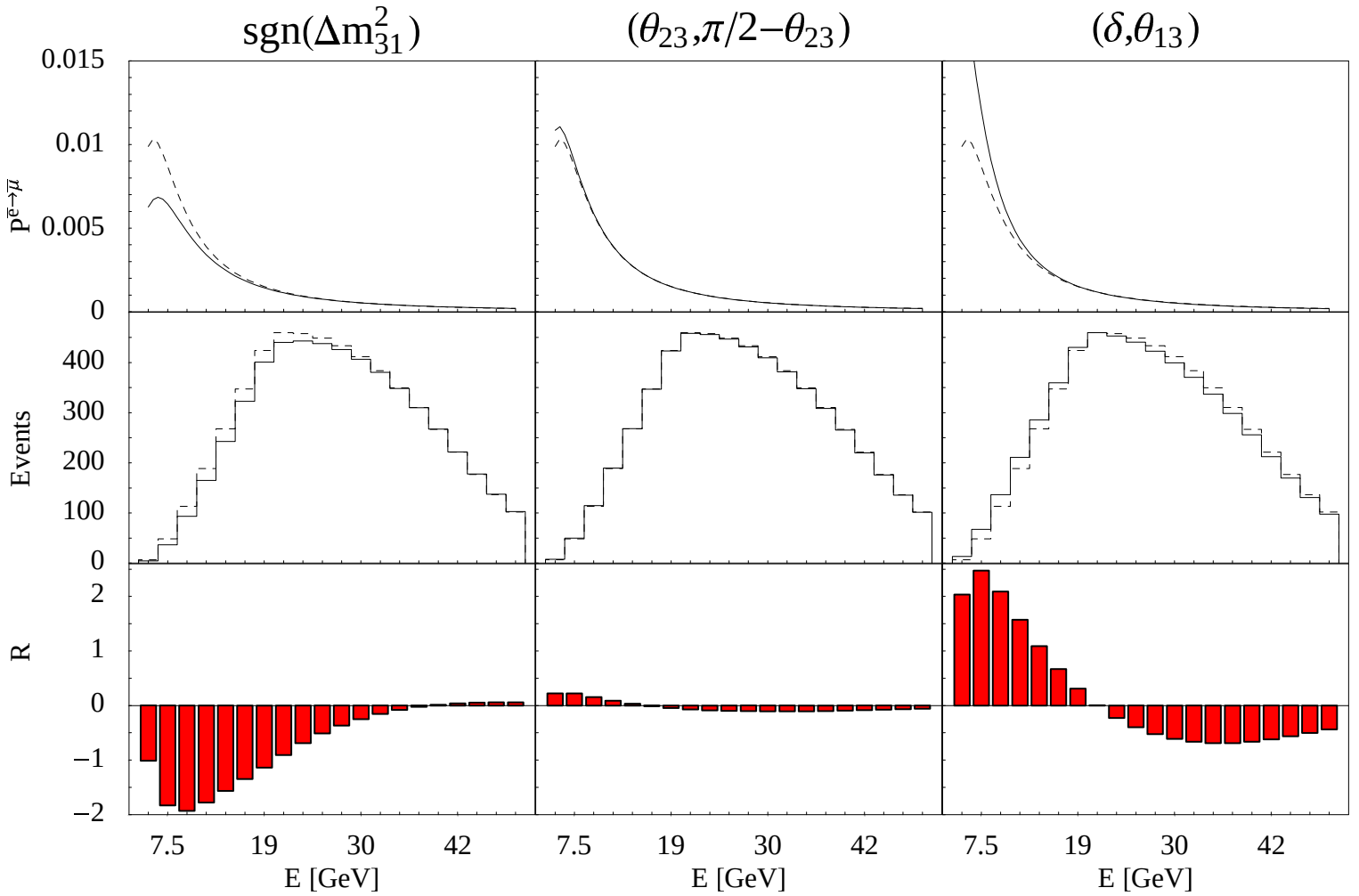
# Degeneracies

Superbeam

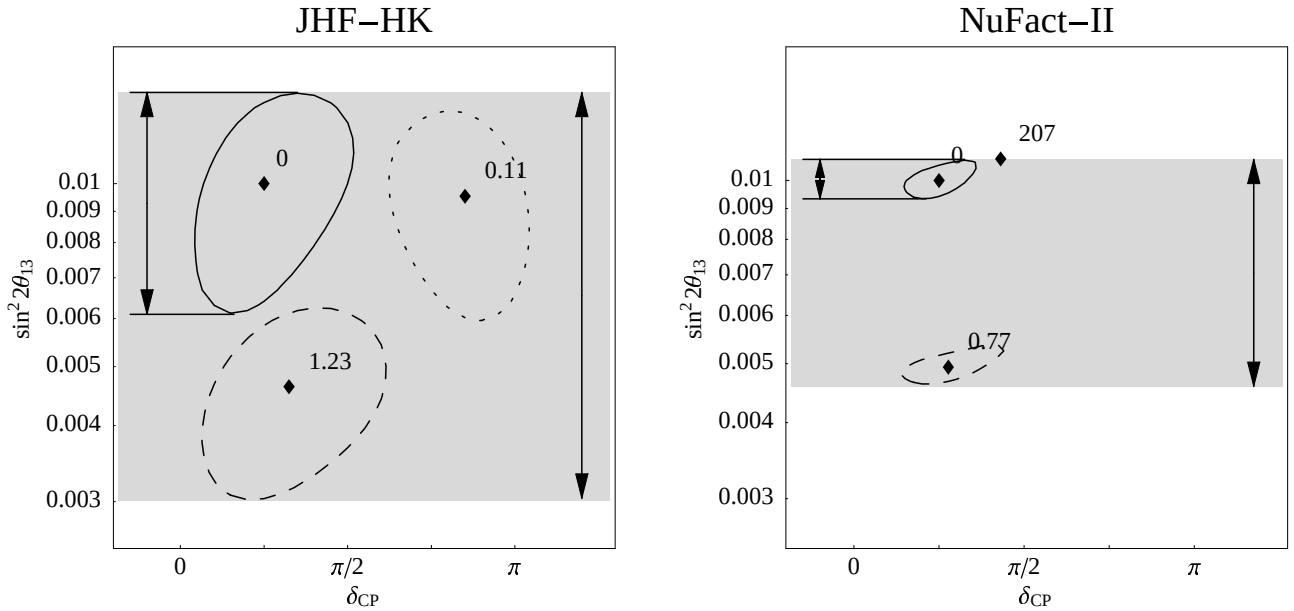


# Degeneracies

NuFact



# Do They Really Exist?



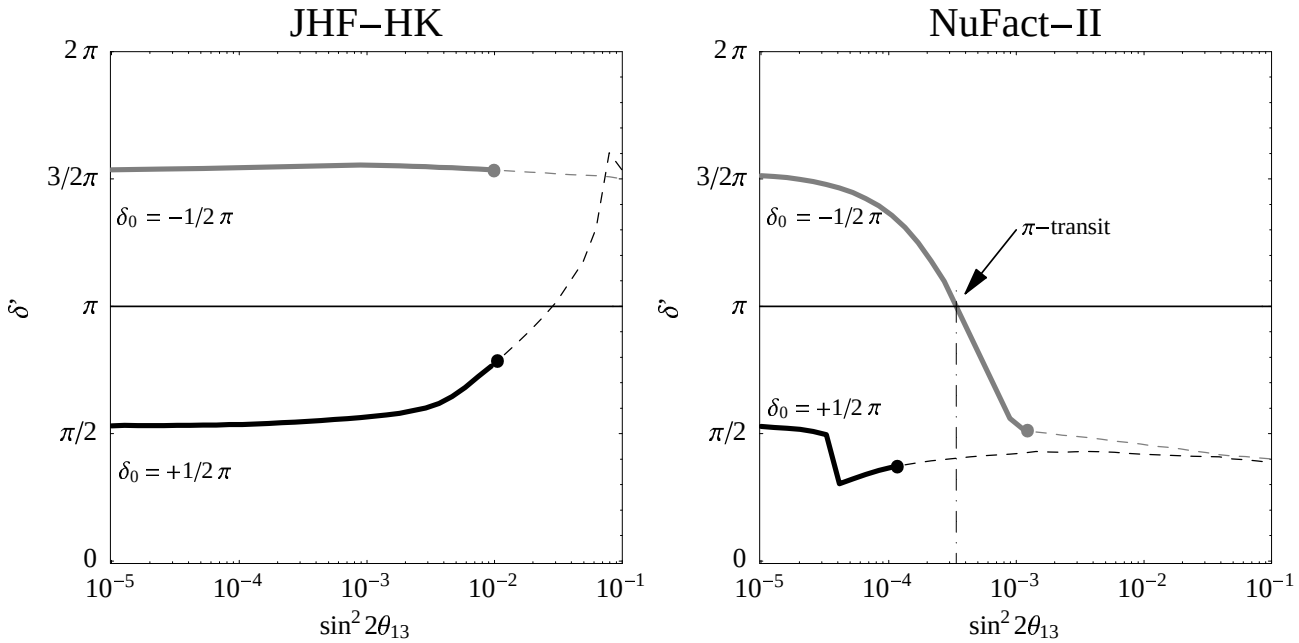

- some show up in the fit
- depends on statistics
- depends on detector performance
- depends on strength of MSW-effect
- depends on energy resolution

disconnected, multiple solutions

No general rule how to quote an error or sensitivity limit –

appropriate choice on case by case base

# The Mass Hierarchy Problem



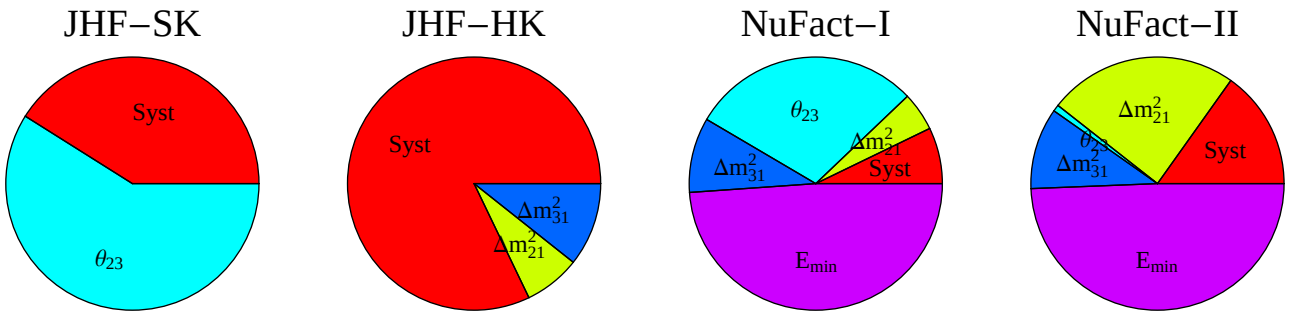

- in general two solutions with opposite sign of $\Delta m^2_{31}$ and different $\delta_{\rm CP}$
- sufficient matter-effects allow to reject the wrong solution

$\delta_{\rm CP}$ gets continously mapped to any $\delta_{\rm CP}$‘

“$\pi$–transit” $\Longleftrightarrow$ maximal $\not{CP}$ is mapped to $CP$–conservation



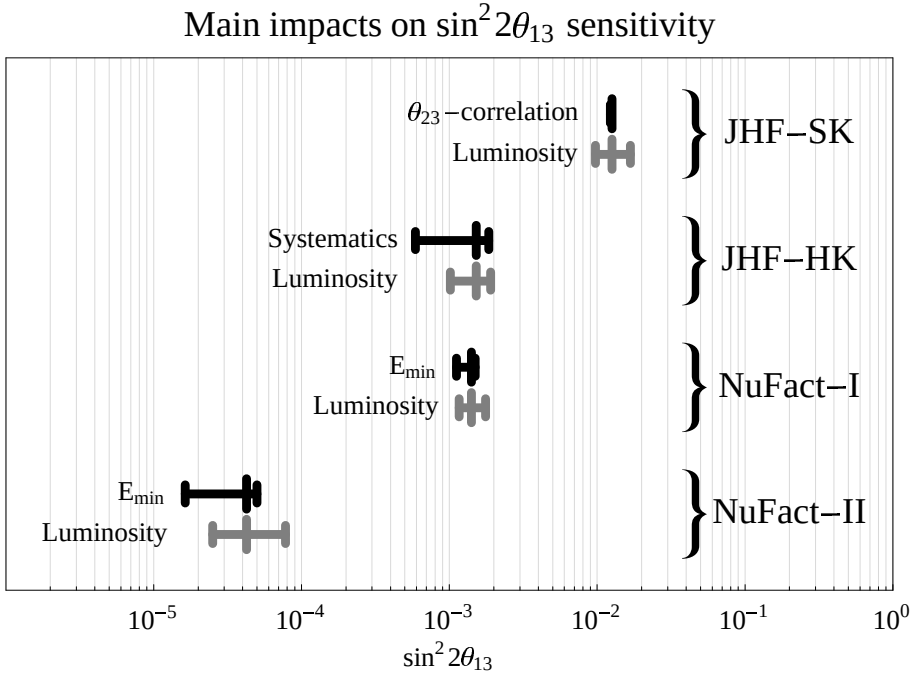
# $\sin^2 2\theta_{13}$ – Impact Factors




- relative contribution of controllable factors
- background uncertainty for Superbeams
- threshold effects for NuFact

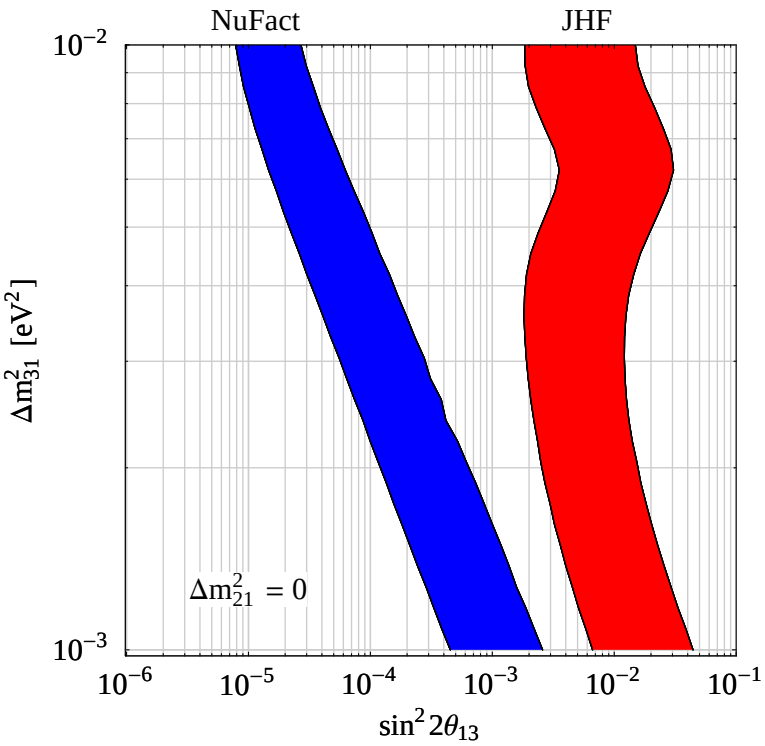
variation of main impact factors
$\Longleftrightarrow$ variation of luminosity

- Superbeams – systematics
- NuFact – low energies



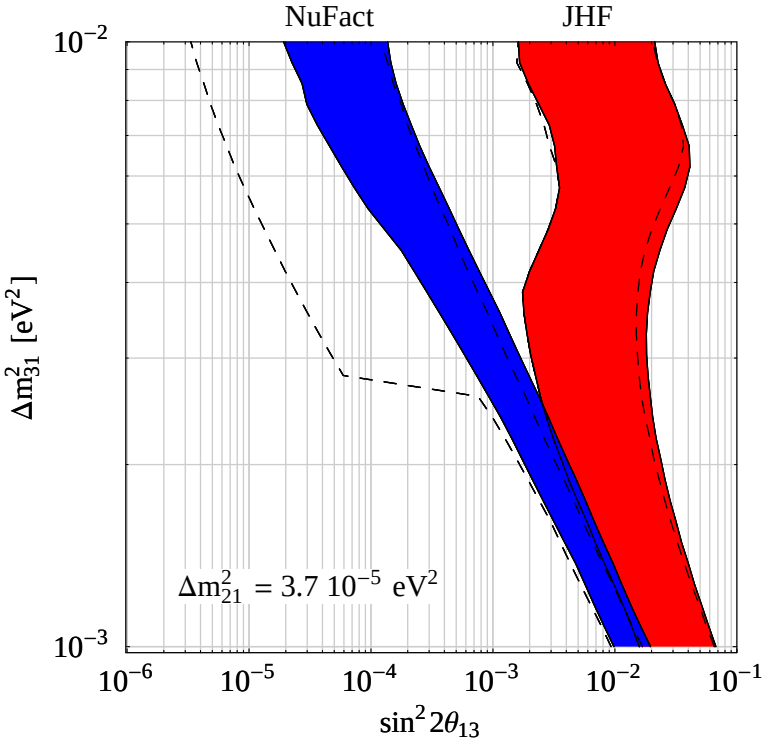
# $\sin^2 2\theta_{13}$ – Sensitivity

$\Delta m^2_{21} = 0$




- no problems due to $\delta_{\mathrm{CP}}$

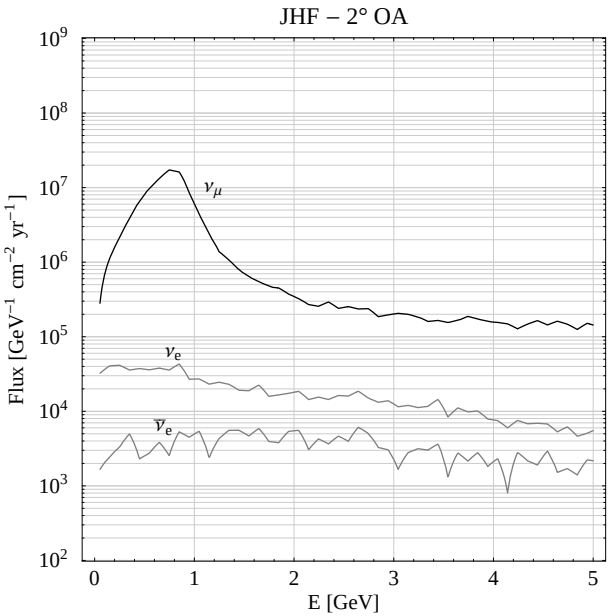
$\Delta m^2_{21} = 3.7 \cdot 10^{-5}\,\mathrm{eV}^2$




- dashed line shows result neglecting $\pi/2 - \theta_{23}$ degeneracy

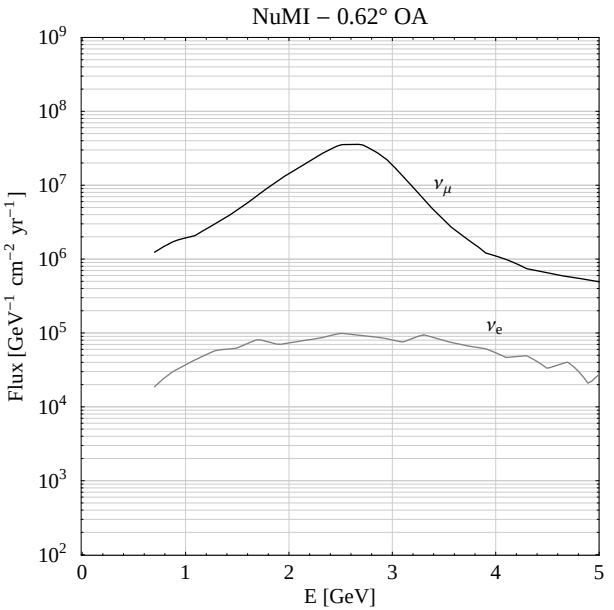
# JHF vs. NuMI

## JHF




- mean energy $0.51\,\mathrm{GeV}$
- peak intensity
  $1.7 \cdot 10^7\,\mathrm{GeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{yr}^{-1}$
  at $0.78\,\mathrm{GeV}$
- $\nu_\mu/\nu_e$-ratio at peak $0.2\%$

## NuMI




- mean energy $2.78\,\mathrm{GeV}$
- peak intensity
  $3.6 \cdot 10^7\,\mathrm{GeV}^{-1}\,\mathrm{cm}^{-2}\,\mathrm{yr}^{-1}$
  at $2.18\,\mathrm{GeV}$
- $\nu_\mu/\nu_e$-ratio at peak $0.2\%$

Uncertainties in flux and $\nu_e$–background

# JHF vs. NuMI

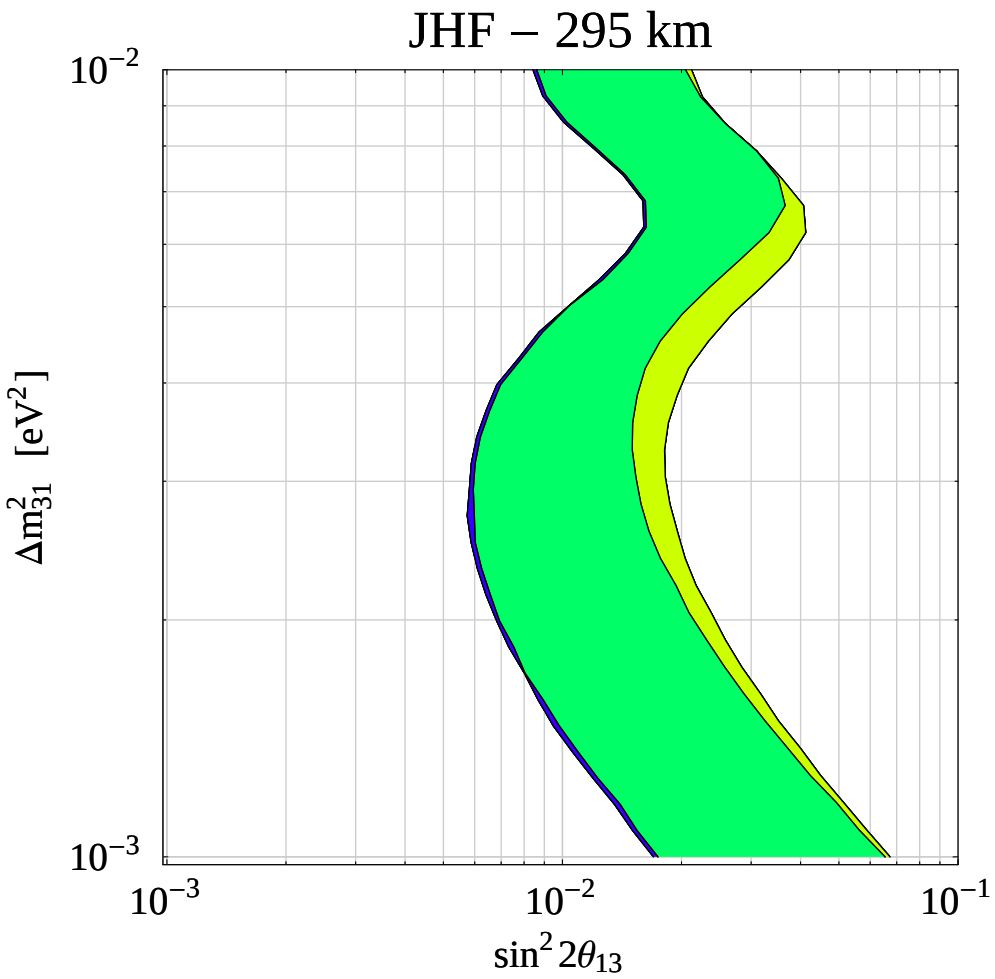
JHF
- $295\,\mathrm{km}$
- water Cherenkov detector – SuperK
- fiducial mass $22.5\,\mathrm{kt}$
- running period of $5$ years
- NC-rejection $\sim 0.01$
- efficiency for $\nu_e \sim 0.5$
- $61\%$ QE-interactions at $E_{\mathrm{peak}}$
- energy range $0.4 - 1.2\,\mathrm{GeV}$
- energy resolution $\delta E = 5\%$
- $5\%$ uncertainty on all backgrounds

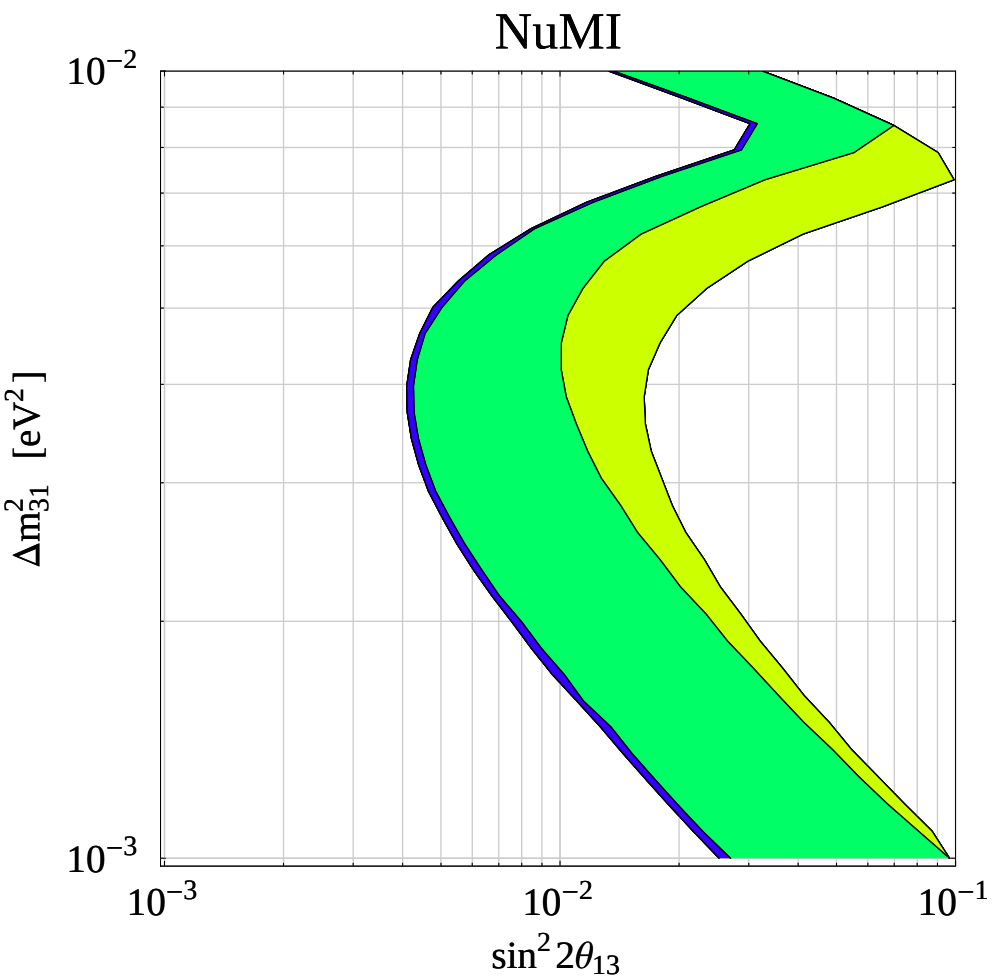
NuMI
- $712\,\mathrm{km}$
- low-Z calorimeter
- fiducial mass $17\,\mathrm{kt}$
- running period of $5$ years
- NC-rejection $\sim 0.001$
- efficiency for $\nu_e \sim 0.4$
- $22\%$ QE-interactions at $E_{\mathrm{peak}}$
- energy range $1.6 - 2.8\,\mathrm{GeV}$
- energy resolution $\delta E = 16\%$
- $10\%$ uncertainty on all backgrounds

Y. Itow *et al.*, hep-ex/0106019.

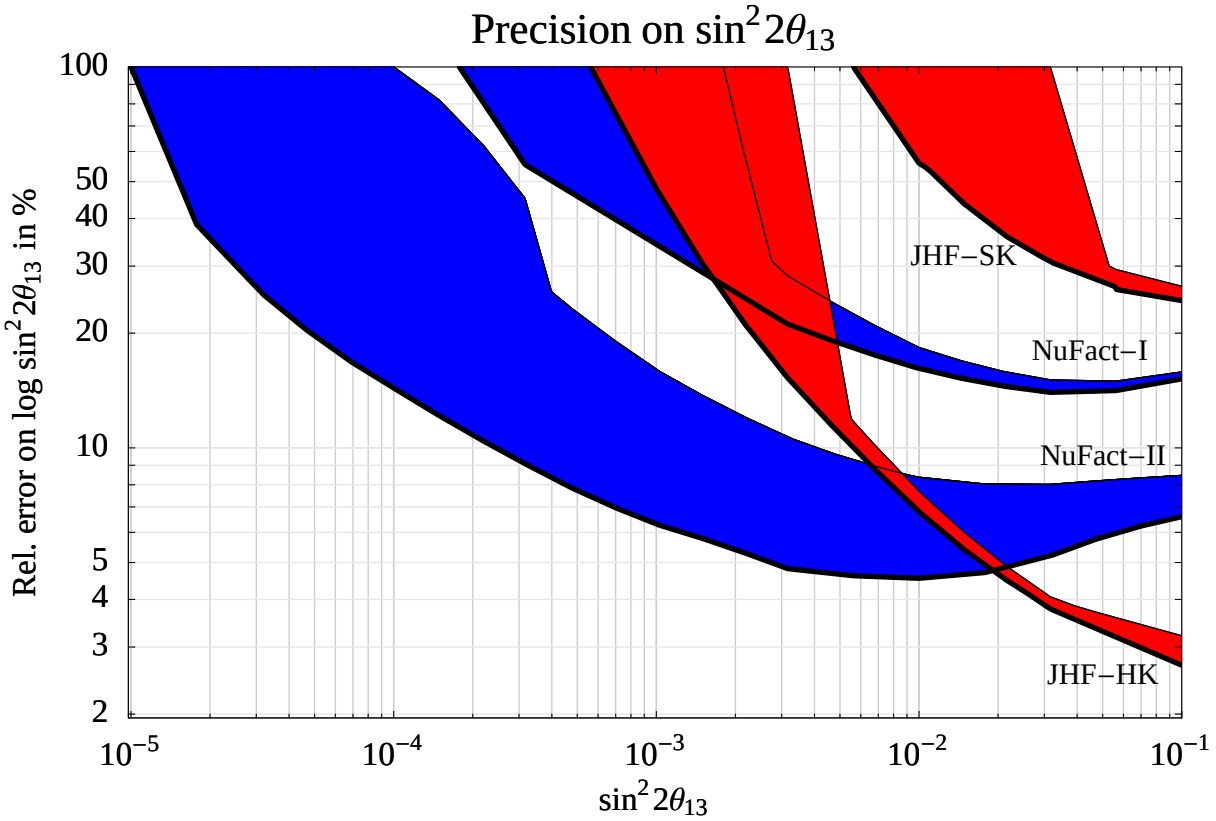
A. Para and M. Szleper, hep-ex/0110001, hep-ex/0110032.

# JHF vs. NuMI


JHF – 295 km
$10^{-2}$
$10^{-3}$
$\Delta m^2_{31}$ [eV$^2$]
$10^{-3}$
$10^{-2}$
$10^{-1}$
$\sin^2 2\theta_{13}$

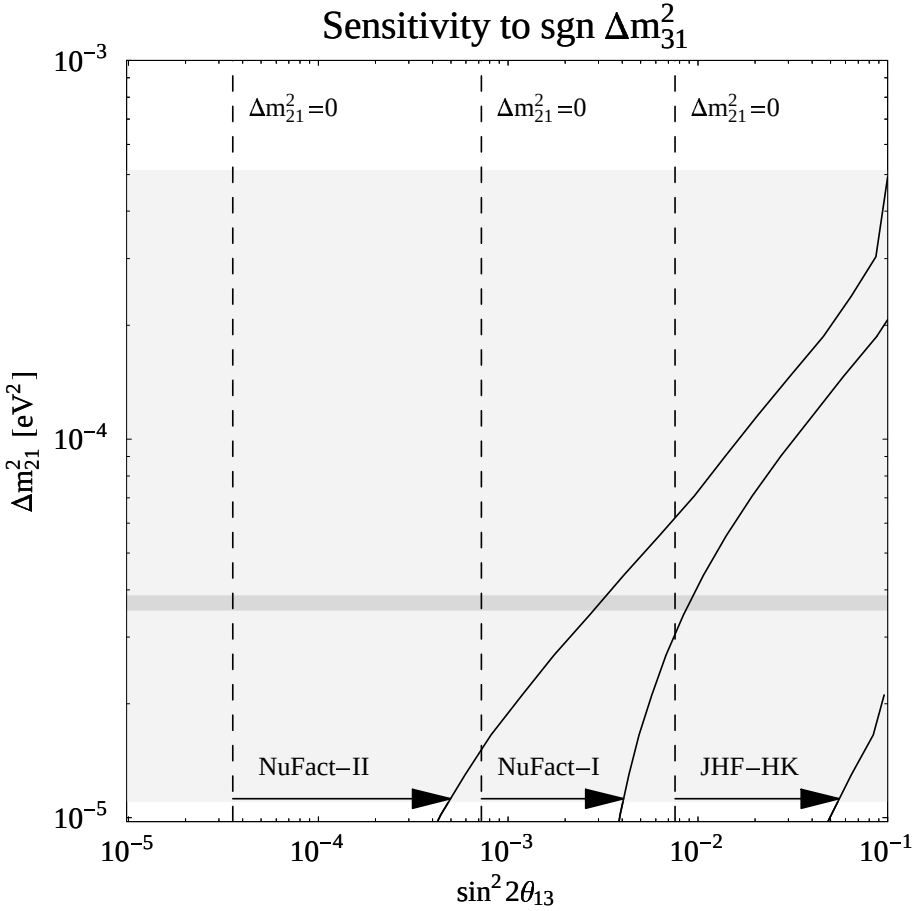

NuMI
$10^{-2}$
$10^{-3}$
$\Delta m^2_{31}$ [eV$^2$]
$10^{-3}$
$10^{-2}$
$10^{-1}$
$\sin^2 2\theta_{13}$

# $\sin^2 2\theta_{13}$ – Accuracy




- local minima only
- bands due to $\delta_{\mathrm{CP}}$
- upward bending
  for $\sin^2 2\theta_{13} > 10^{-2}$
  due to matter uncertainty

# $\mathrm{sign}\Delta m^2_{31}$ – a Difficult Task




- degeneracy with $\delta_{\mathrm{CP}}$ determines the sensitivity
- bigger $\Delta m^{21}_2$

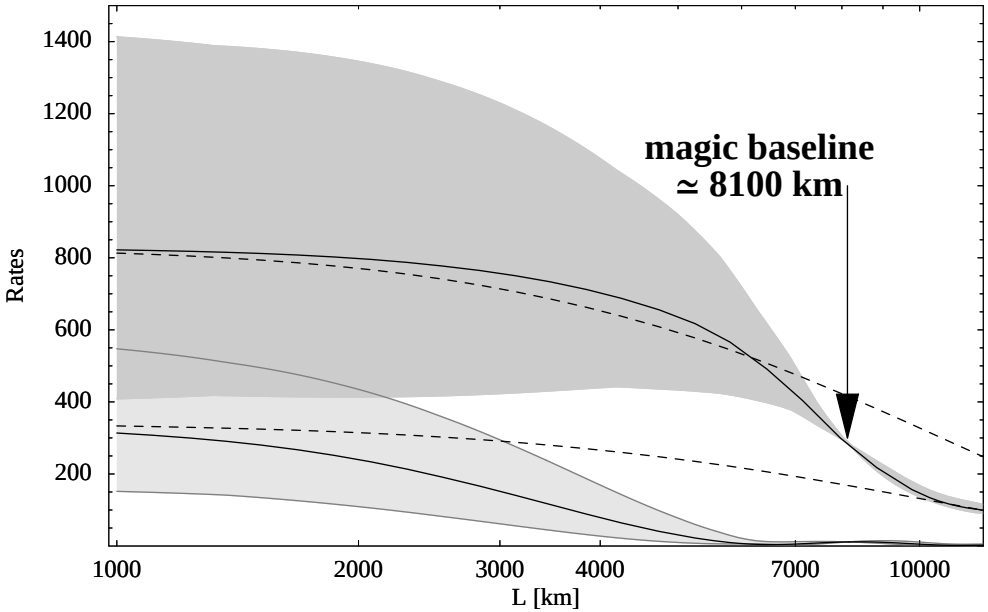
$\Longrightarrow$ more CP–effects

$\Longrightarrow$ more confusion

$\Longrightarrow$ several orders of magnitude loss in sensitivity

Very long baseline $> 7\,000\,\mathrm{km}$ is needed

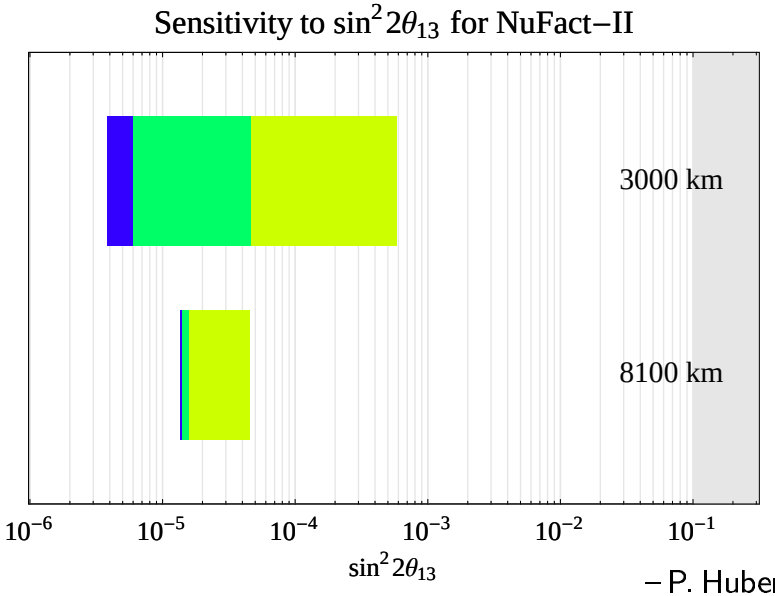
# Magic Baseline




$$P_{\nu_e\to\nu_\mu} \approx \sin^2\theta_{23}\frac{\sin^2 2\theta_1{}^3}{(\hat{A}-1)^2}\sin^2((\hat{A}-1)\Delta)$$
$$\pm\ \alpha\ \sin(\hat{A}\Delta)\sin\delta\times f_1$$
$$+\ \alpha\ \sin(\hat{A}\Delta)\cos\delta\times f_2$$
$$+\ \alpha^2\ \sin^2(\hat{A}\Delta)\times f_3$$

$$\sin(\hat{A}\Delta)=0 \Leftrightarrow \hat{A}\Delta=n\pi \Leftrightarrow VL/2=n\pi$$

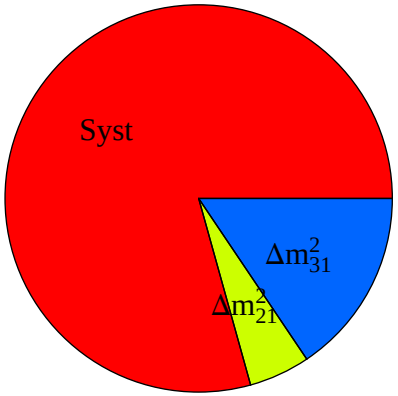
this relation gives a magic baseline $\simeq 8\,100\,\mathrm{km}$ where all effects due to $\alpha$ and $\delta_{\mathrm{CP}}$ vanish independent of energy and mass splittings



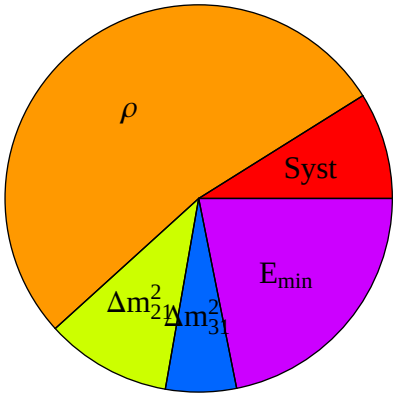

V. Barger *et al.*, Phys. Rev. **D65**, 073023 (2002).

# $\delta_{CP}$ – Impact Factors

JHF–HK




NuFact–II

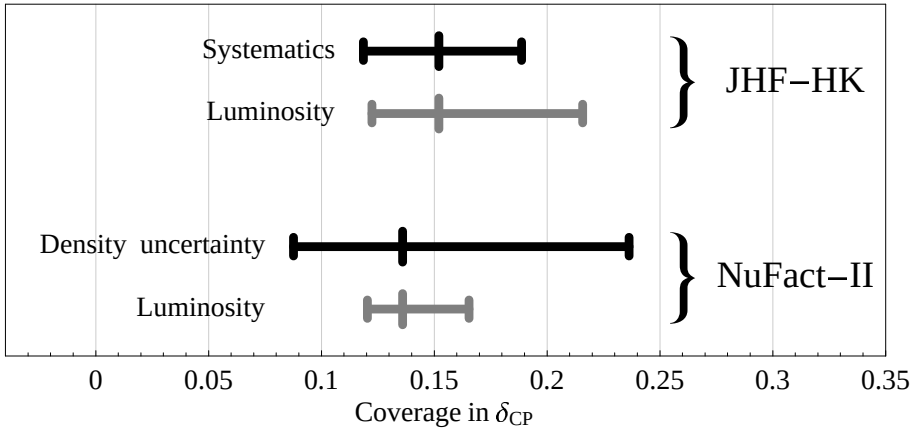



- relative contribution of controllable factors
- background uncertainty for Superbeams
- matter uncertainty for NuFact

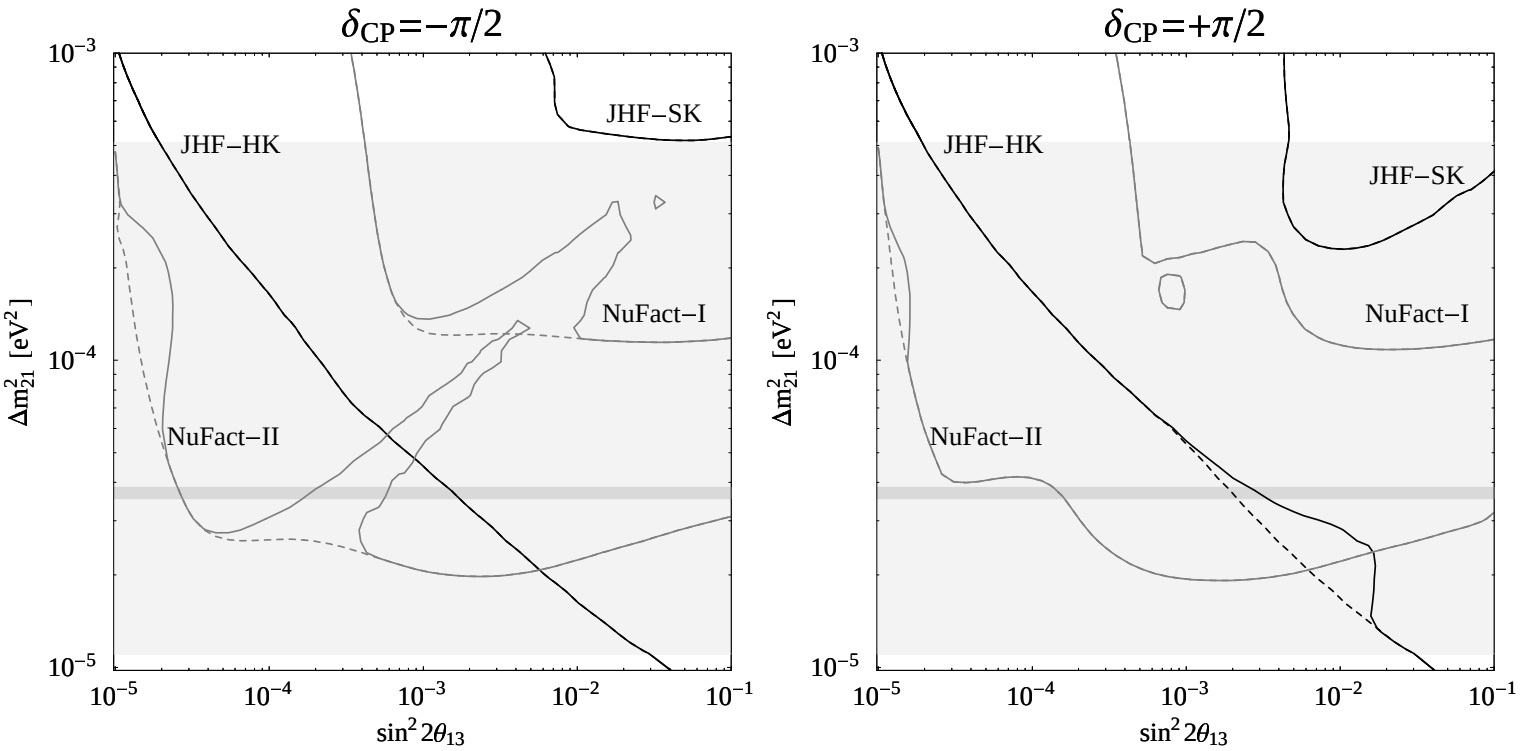
variation of main impact factors
$\Longleftrightarrow$ variation of luminosity

- Superbeams – systematics
- NuFact – matter uncertainty

Main impacts on CP measurement precision

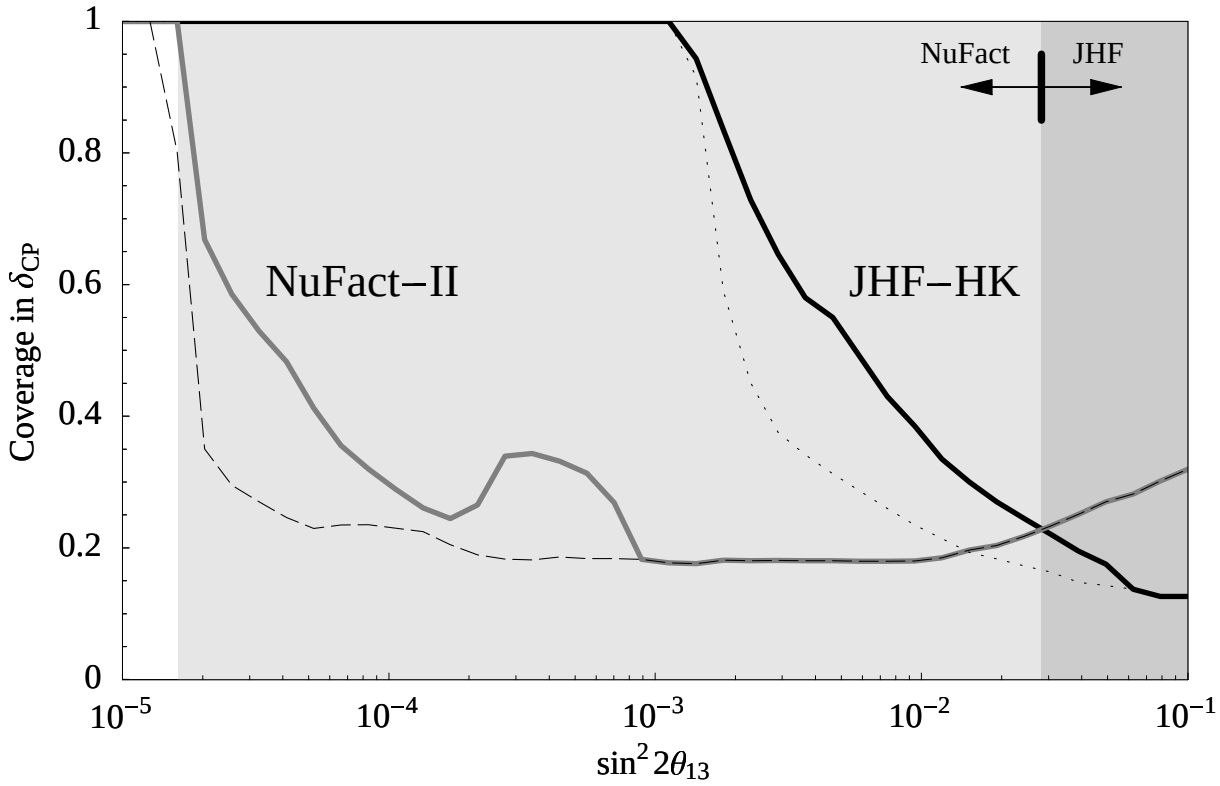


# $\delta_{\mathrm{CP}}$ – Sensitivity



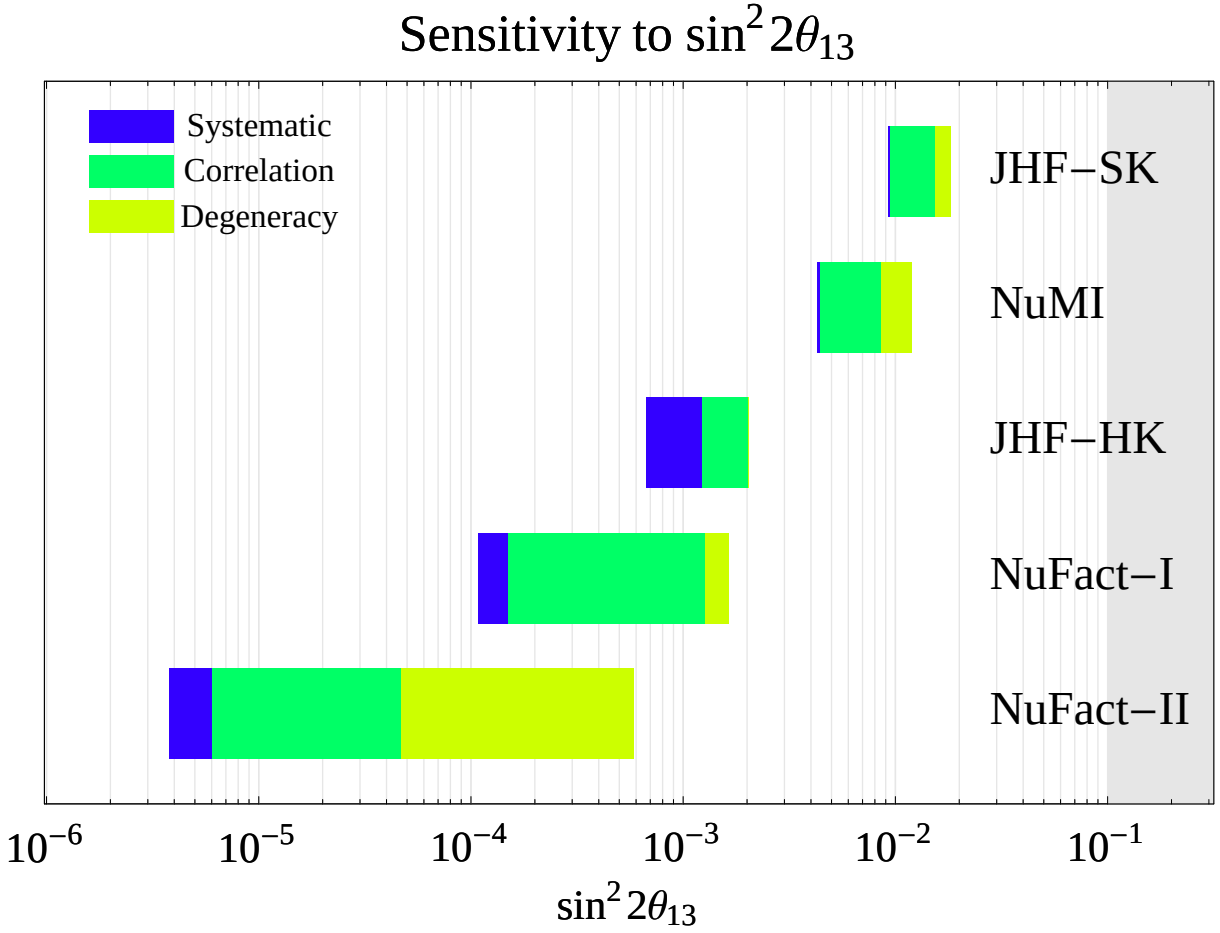

- degeneracies taken into account
- valleys due to $\pi$–transit
- high luminosity mandatory to cover LMA–range
- even if Superbeams do not see $\theta_{13}$ ($\sin^2 2\theta_{13} < 10^{-3}$) NuFact can go up to two orders of magnitude lower !

# $\delta_{\rm CP}$ – Accuracy




- with degeneracies
- upward bending for $\sin^2 2\theta_{13} > 10^{-2}$ due to matter uncertainty
- bump due to $\pi$–transit
- NuFact is better in most cases

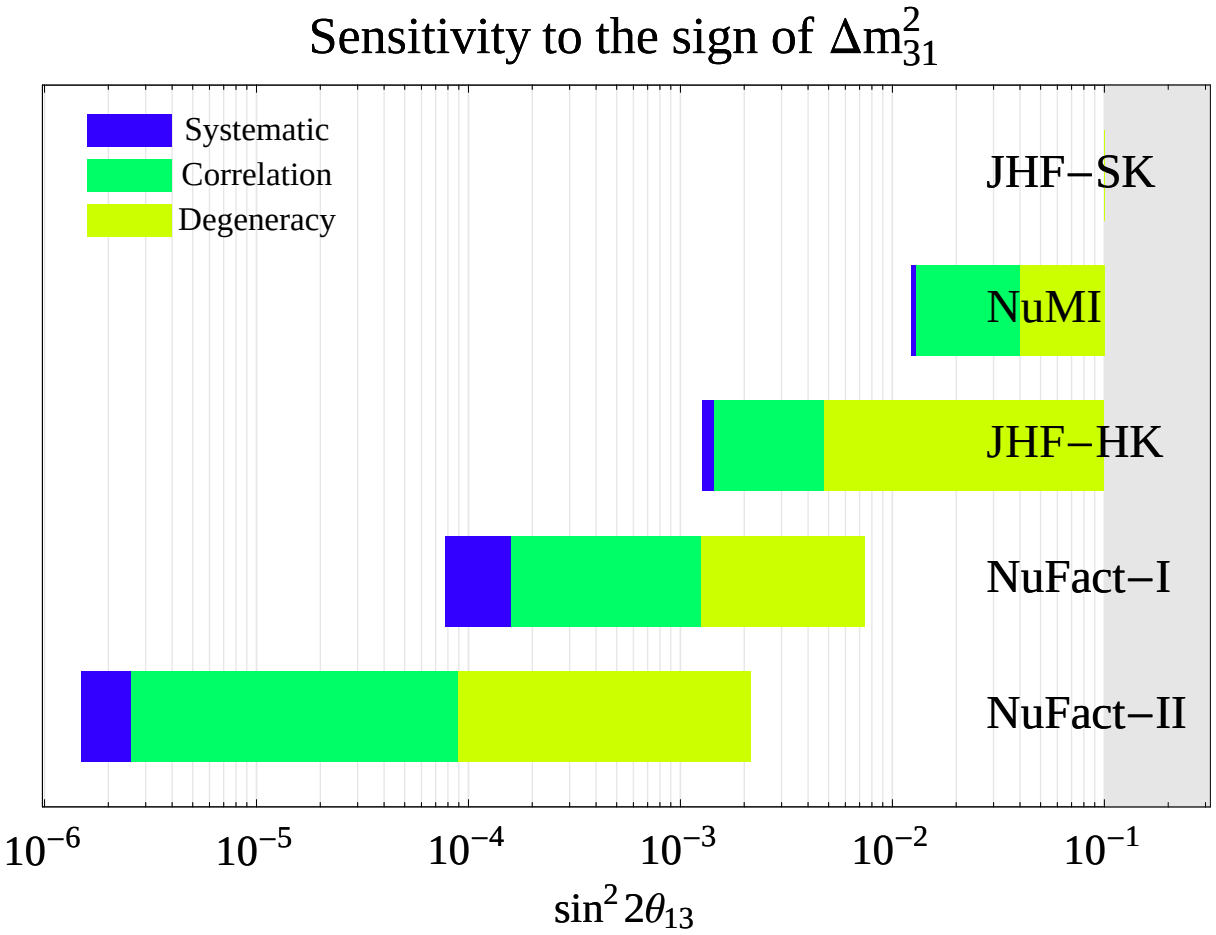
# Summary – $\sin^2 2\theta_{13}$




- JHF-SK and NuMI statistics limited
- JHF-HK systematics limited
- NuFact far away from systematic limit
- correlations and degeneracies require a clever setup

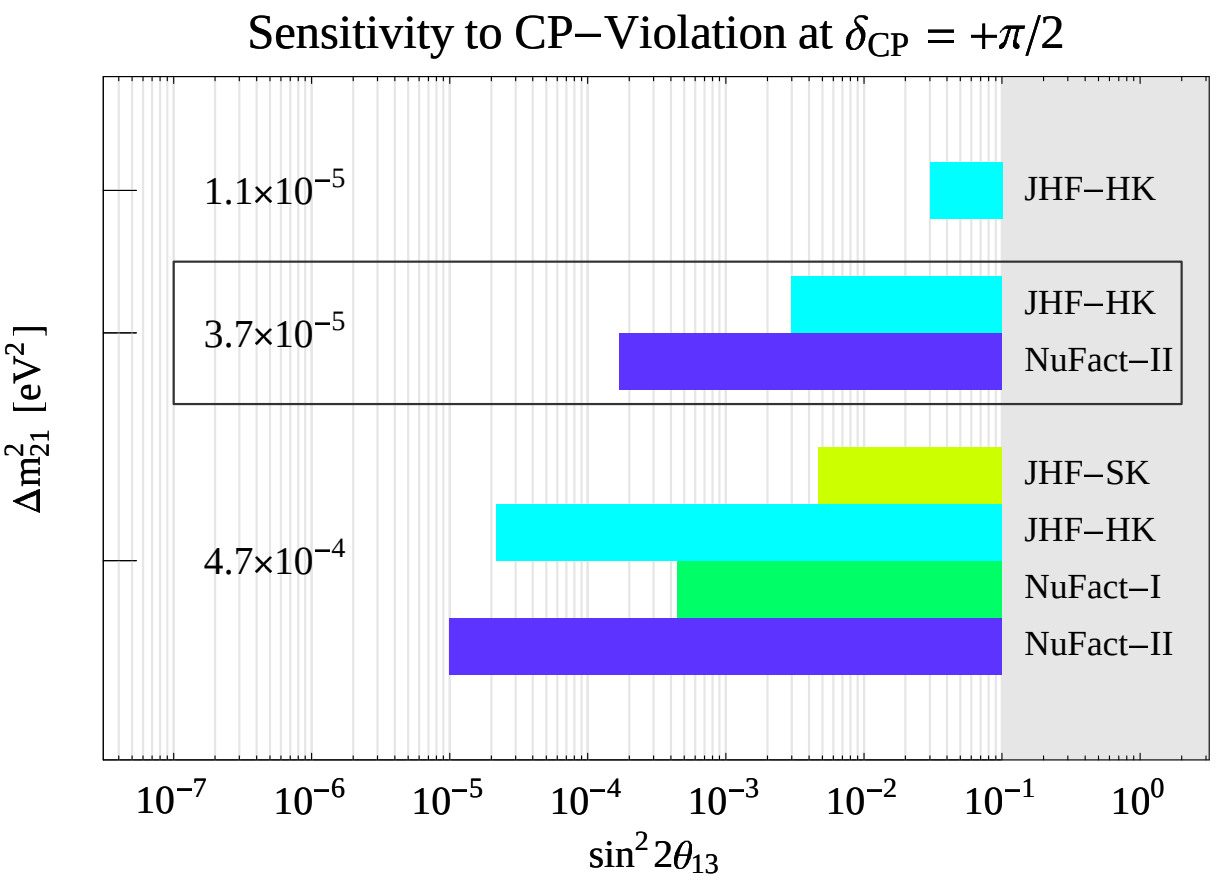
$\Longrightarrow$ e.g. magic baseline

# Summary – $\mathrm{sign}\Delta m^2_{31}$




- hard with Superbeams
- $\delta$-degeneracy has to be resolved
- NuFact $\Longrightarrow$ magic baseline

# Summary – $\delta_{\mathrm{CP}}$




- needs luminosity (expensive)
- JHF-HK has a real potential
- NuFact-II is the tool

# Conclusion

Analysis

- $\theta_{13}$ determination is already involved
- correlations extremely important esp. with $\delta_{\mathrm{CP}}$
- degeneracies exist also in a full analysis
- strong interplay between mass hierarchy and $\delta_{\mathrm{CP}}$

further clever ideas needed

see e.g. talk by O. Mena

Comparison

- next generation of LBL experiments one order of magnitude below CHOOZ
- Superbeam can be pushed one further order of magnitude
- NuFact can go much further down to $\geq 10^{-5}$
- mass hierarchy requires high energies and long baselines
- $CP$ needs luminosity

$U_{\mathrm{MNS}}$ can be measured