SUSY GUT Predictions for Neutrino Oscillation

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Deficiencies in the Standard Model

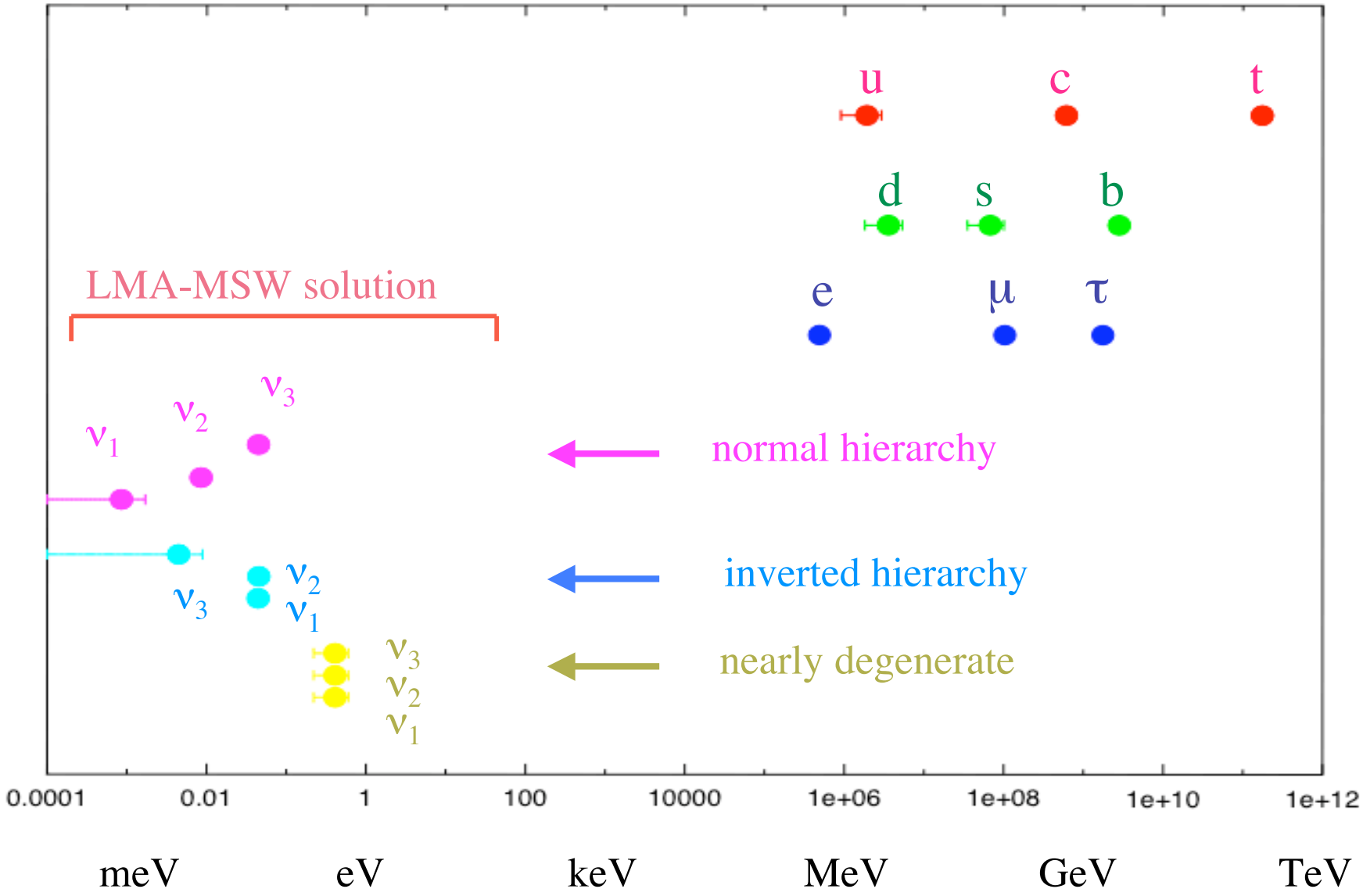
- Many free parameters:

  fermion masses, CKM matrix elements and CP violating phases are completely arbitrary; they are parameterized by three arbitrary 3x3 complex Yukawa matrices

- Neutrinos are massless, because there are no RH neutrinos and lepton number is conserved

- The observation of non-zero neutrino masses has given the first solid evidence of physics beyond the standard model

Mass spectrum of elementary particles



CKM Matrix $\Longleftrightarrow$ PMNS Matrix

- Quark mixings are small

$$V_{CKM} \sim \begin{pmatrix} 0.9745-0.9757 & 0.219-0.224 & 0.002-0.005 \\ 0.218-0.224 & 0.9736-0.9750 & 0.036-0.046 \\ 0.004-0.014 & 0.034-0.046 & 0.9989-0.9993 \end{pmatrix}$$

- Lepton mixings are large

$$U_{LM} \sim \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.0-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix}$$

Origin of Fermion Mass Hierarchy and Flavor Mixing

- No fundamental origin of mass hierarchy and flavor mixing has been found or suggested

- The less ambitious aim is to reduce the number of parameters in the Yukawa sector which parameterizes the masses and mixing angles

- Parameter reduction obtained by imposing symmetry:
  - grand unified gauge symmetry:

    allows relations between up, down, charged lepton and neutrino masses
  - family symmetry:

    leads to further reduction of parameters; allows relations among three families.
  - supersymmetry:

    though leads to an increase in number of parameters, required by data otherwise many incorrect predictions

Small neutrino mass: See-saw Mechanism

Gell-Mann, Ramond, Slansky, 1981

- Mixture of light fields and heavy fields:

$$(\nu_L \quad \nu_R)\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$\nu_R\,\nu_R$: sterile (singlets under ALL gauge groups in SM); mass term allowed

- Diagonalize the mass matrix:

$$m_\nu \sim m_{light} \sim \frac{m_D^2}{M_R} << m_D$$

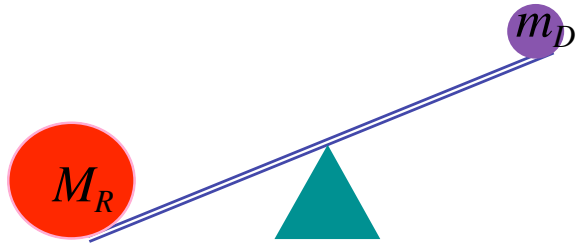
$$m_{heavy} \sim M_R$$

- Smallness of neutrino masses suggests a high mass scale

For $m_\nu \sim \sqrt{\Delta m^2_{atm}}$

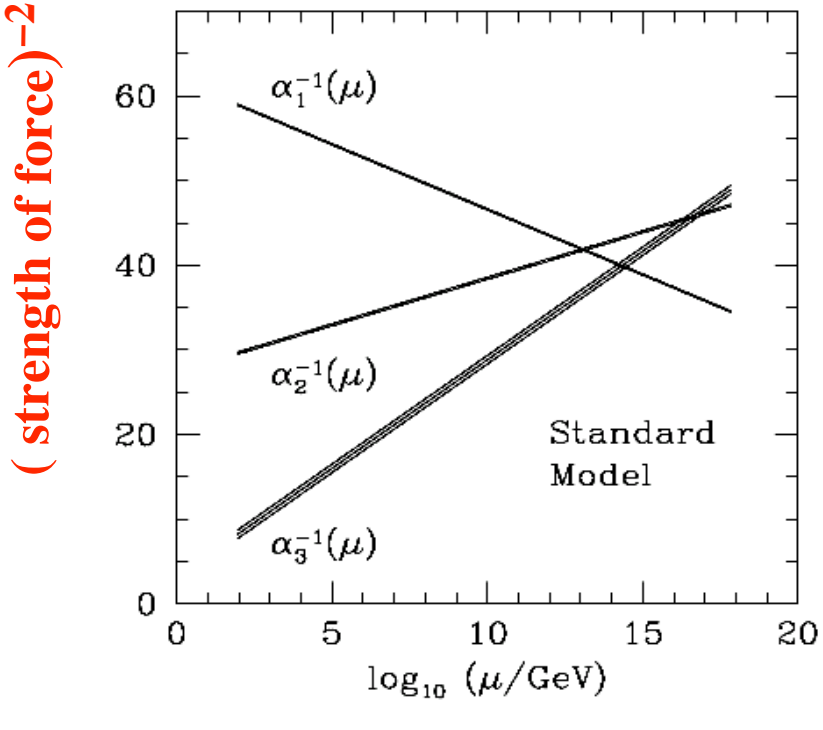
$$m_D \sim m_t \sim 180 \;\; GeV$$

$\Rightarrow$ $M_R \sim 10^{15}\ GeV$ (GUT !!)

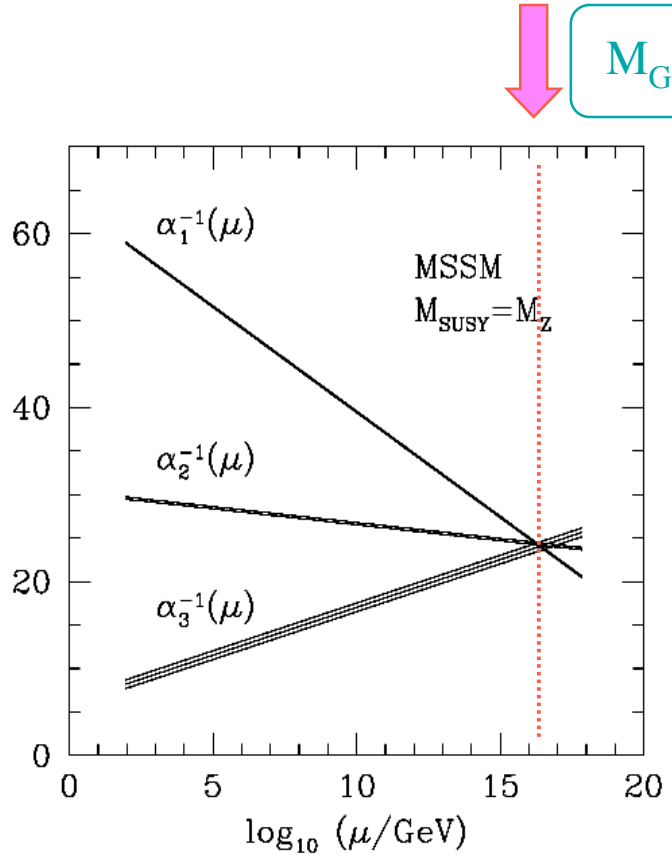


SUSY GUT

- Unification of the three gauge coupling constants



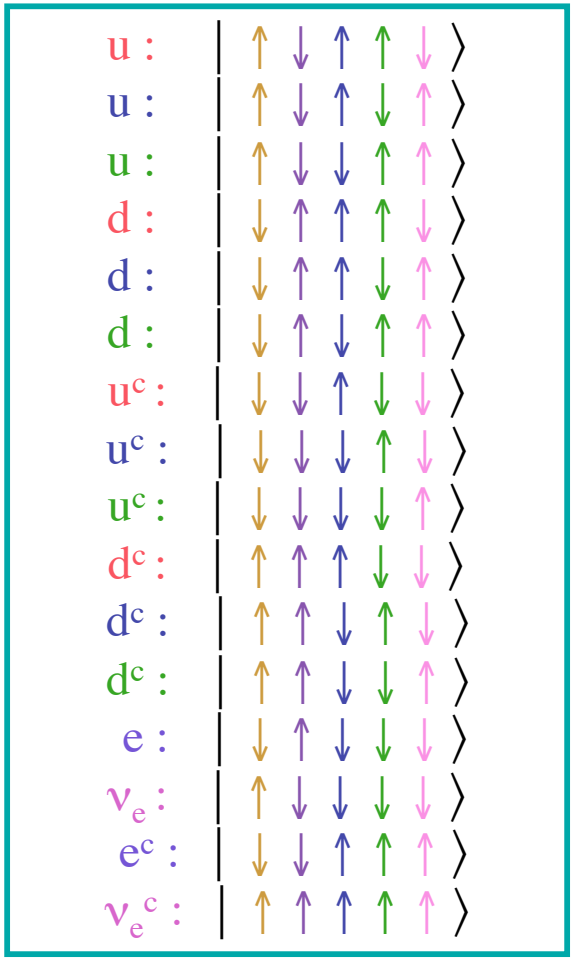
Energy scale



To obtain $m_\nu \sim (\Delta m^2_{\text{atm}})^{1/2}$,
$m_D \sim m_t \Rightarrow M_3 \sim 10^{15}$ GeV!

SO(10) Grand Unification

- Matter unification: All 15 known fermions and the RH neutrino are unified into one single spinor representation
- RH neutrino predicted in the theory!!
- Charge quantization explained
- Quarks and leptons can transform into one another by exchanging extremely heavy particles
- These heavy particles would mediate proton decay



A Realistic SUSY SO(10) Model

M.-C.C and K.T. Mahanthappa, Phys. Rev. D62, 113007 (2000)

Left-right symmetry breaking route:

$$SO(10) \rightarrow SU(4) \times SU(2)_L \times SU(2)_R$$
$$\rightarrow SU(3) \times SU(2)_L \times U(1)_Y$$

$\Rightarrow$ *symmetric* mass matrices

$\Rightarrow$ Intra-family mass relations:

Up-type quarks $\Leftrightarrow$ Dirac neutrinos
Down-type quarks $\Leftrightarrow$ charged leptons

$\Rightarrow$ These intra family relations greatly reduce the number of parameters in the Yukawa sector

A Realistic SUSY SO(10) Model

M.-C.C and K.T. Mahanthappa, Phys. Rev. D62, 113007 (2000)

- Resulting mass matrices:

  Quark and lepton mass matrices are related:

$$M_U = M_{\nu_L \nu_R} = \begin{pmatrix} 0 & 0 & r_2\varepsilon' \\ 0 & r_4\varepsilon & \varepsilon \\ r_2\varepsilon' & \varepsilon & 1 \end{pmatrix} m_t \qquad 1 >> \varepsilon >> \varepsilon' \sim \varepsilon^2$$

$$M_D\,,\, M_E \;\; = \begin{pmatrix} 0 & \varepsilon' & 0 \\ \varepsilon' & (1,-3)\,p\varepsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} m_b$$

- The factor of (-3) is needed to satisfy the Georgi-Jarlskog relations at GUT scale

- Arise as CG factor due to coupling to <126>

A Realistic SUSY SO(10) Model

- RH neutrino mass term: coupling to 126

$$M_{\nu_L \nu_R} \sim \begin{pmatrix} 0 & 0 & \varepsilon^2 \\ 0 & \varepsilon & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} m_t \qquad M^{-1}_{\nu_R \nu_R} = \begin{pmatrix} a & b & c \\ b & d & 0 \\ c & 0 & 0 \end{pmatrix} \mathrm{M_R^{-1}}$$

- Type I see-saw:

$$M_\nu^{eff} = M_{LR} M_{RR}^{-1} M_{LR}^T \sim \begin{pmatrix} 0 & 0 & c\varepsilon^4 \\ 0 & d\varepsilon^2 & d\varepsilon^2 + c\varepsilon^3 \\ c\varepsilon^4 & d\varepsilon^2 + c\varepsilon^3 & d\varepsilon^2 + 2c\varepsilon^2 + 2b\varepsilon^3 + a\varepsilon^4 \end{pmatrix} \frac{\upsilon_u^2}{M_R}$$

Zeros are exact!!

If parameter d dominates => bi-large mixing (from seesaw)

A Realistic SUSY SO(10) Model

- RH neutrino mass term:

$$M_{\nu_R \nu_R} = \begin{pmatrix} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \\ \delta_1 & \delta_3 & 1 \end{pmatrix} \mathrm{M_R}$$

$$\delta_1 : \delta_2 : \delta_3 : 1 = 10^{-3} : 10^{-3} : 10^{-2} : 1 \qquad M_{R_1} : M_{R_2} : M_{R_3} = 10^{-6} : 10^{-4} : 1$$

- Type I seesaw:

$$M_\nu^{eff} = M_{LR} M_{RR}^{-1} M_{LR}^T \sim \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1+t' \\ t & 1+t' & 1 \end{pmatrix} \frac{d^2 v_u^2}{M_R}$$

$$t' << t << 1$$

A Realistic SUSY SO(10) Model

- The mass eigenvalues and diagonalization matrix

$\Rightarrow$ hierarchical masses:

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = \sqrt{tt'} : \sqrt{tt'} : 1$$

$$\left|\Delta m^2_{sun}\right| = \left| m^2_{\nu_2} - m^2_{\nu_1} \right| = O(tt')\frac{\upsilon^2}{M_R}$$

$$\left|\Delta m^2_{atm}\right| = \left| m^2_{\nu_3} - m^2_{\nu_2} \right| = O(1)\frac{\upsilon^2}{M_R}$$

$\Rightarrow$ Bi-large mixing angle

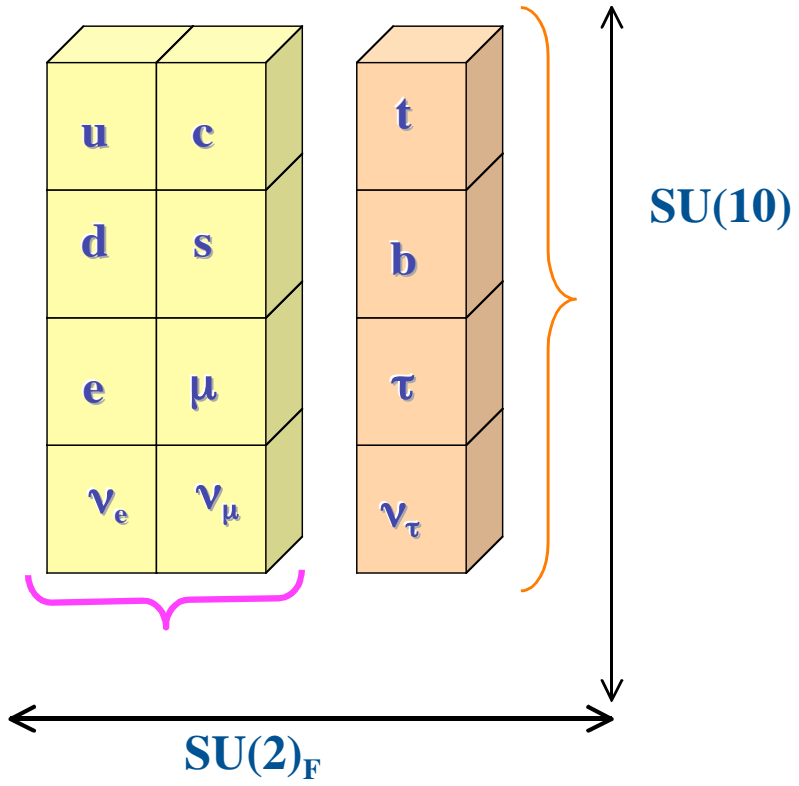
$$U_{LM} = \begin{pmatrix} \frac{1}{\sqrt{2}} + O\left(\sqrt{t}\right) & -\frac{1}{\sqrt{2}} + O\left(\sqrt{t}\right) & \frac{O\left(\sqrt{tt'}\right)}{2\sqrt{2}} \\ -\frac{1}{2} + O\left(\sqrt{t}\right) & -\frac{1}{2} - O\left(\sqrt{t}\right) & \frac{1}{\sqrt{2}} - O\left(t^2\right) \\ \frac{1}{2} - O\left(\sqrt{t}\right) & \frac{1}{2} + O\left(\sqrt{t}\right) & \frac{1}{\sqrt{2}} + O\left(t^2\right) \end{pmatrix}$$

Predictions for sin(theta13) is related to

$$\sin\theta_{13} \sim \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2} \sim O(0.1) \Rightarrow LMA$$

The Horizontal Symmetry

- What these zeros and hierarchy among the elements in mass matrices imply is an underlying family symmetry
- Zeros in the mass matrices are protected by a family symmetry $SU(2)_F$
- Three families are the same under vertical GUT symmetry; but they are different under horizontal symmetry



The Horizontal Symmetry

- top quark (3rd family) is heavy $\Rightarrow$ mass allowed by family symmetry

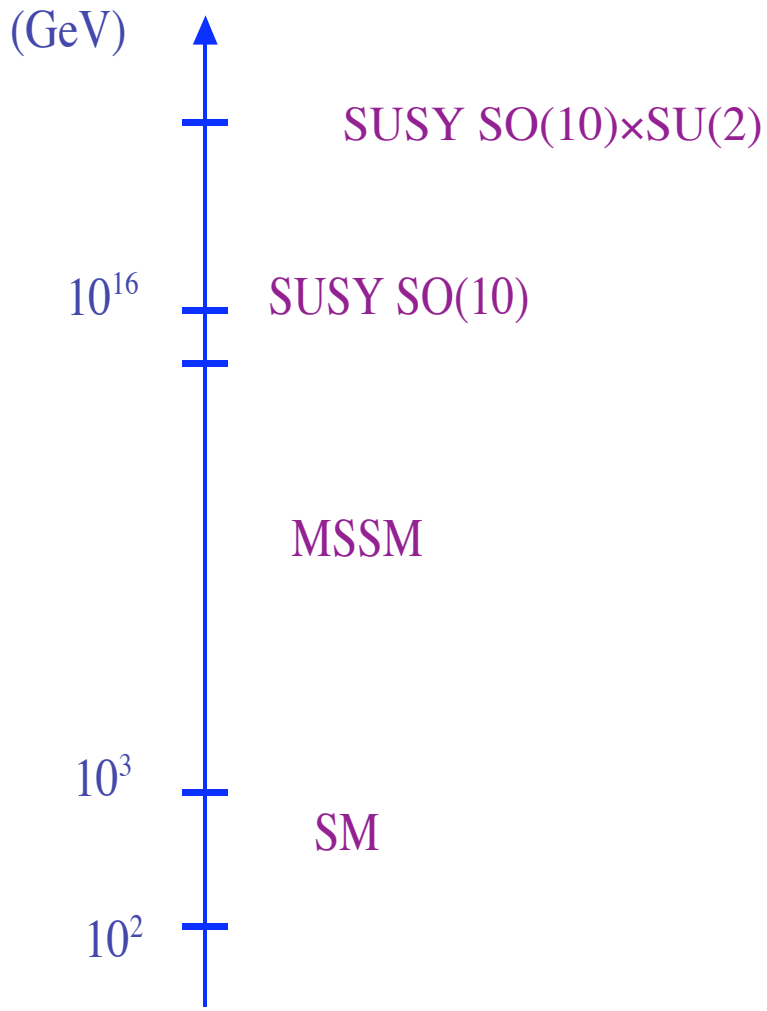
- up and charm quarks (1st & 2nd families) are light $\Rightarrow$ masses generated after breaking of family symmetry

- To understand the hierarchy among the matrix elements
  multiple steps of symmetry breaking

$$SU(2)_F \xrightarrow{M_1} U(1)_F \xrightarrow{M_2} \text{nothing}$$

If $M_1 >> M_2 \Rightarrow$ hierarchical matrix elements

Characteristic Scales in the Model



Predictions for quark masses and mixing

- Input parameters at Mz:

$m_u = 2.21\ \ MeV(2.33^{+0.42}_{-0.45})$

$m_c = 682\ \ MeV(677^{+56}_{-61})$

$m_t = 181\ \ GeV(181^{+}_{-}13)$

$m_e = 0.486\ \ MeV(0.486847)$

$m_\mu = 103\ \ MeV(102.75)$

$m_\tau = 1.74\ \ GeV(1.7467)$

$|V_{us}| = 0.225(0.221 - 0.227)$

$|V_{ub}| = 0.00368(0.0029 - 0.0045)$

$|V_{cb}| = 0.0392(0.039 - 0.044)$

TABLE I: The predictions for the charged fermion masses, the CKM matrix elements and the CP violation measures.

	experimental results extrapolated to M_Z	predictions at M_z
m_s/m_d	$17 \sim 25$	25
m_s	$93.4^{+11.8}_{-13.0} MeV$	$86.0 MeV$
m_b	$3.00 \pm 0.11 GeV$	$3.03 GeV$
$\|V_{ud}\|$	$0.9739 - 0.9751$	0.974
$\|V_{cd}\|$	$0.221 - 0.227$	0.225
$\|V_{cs}\|$	$0.9730 - 0.9744$	0.973
$\|V_{td}\|$	$0.0048 - 0.014$	0.00801
$\|V_{ts}\|$	$0.037 - 0.043$	0.0386
$\|V_{tb}\|$	$0.9990 - 0.9992$	0.999
J^q_{CP}	$(2.88 \pm 0.33) \times 10^{-5}$	2.87×10^{-5}
$\sin 2\alpha$	-0.16 ± 0.26	-0.048
$\sin 2\beta$	0.736 ± 0.049	0.740
γ	$60^0 \pm 14^0$	64^0
$\overline{\rho}$	0.20 ± 0.09	0.173
$\overline{\eta}$	0.33 ± 0.05	0.366

All agree with data within 1 sigma

Predictions for neutrino masses and mixing

- Input parameters:

$$\Delta m^2_{atm} = 2.33 \times 10^{-3} eV^2 \quad , \quad \Delta m^2_{sun} = 8.14 \times 10^{-5} eV^2$$

- Predictions: **Agree with all exp. data within 1σ !!**

$$m_{\nu_1} = 0.00262\ eV$$
$$m_{\nu_2} = 0.00939\ eV$$
$$m_{\nu_3} = 0.0492\ eV$$

Hierarchical mass spectrum

$$\sin^2 2\vartheta_{atm} = 1.00$$
$$\tan^2 2\vartheta_{sun} = 0.36$$
$$\sin\vartheta_{rea} = 0.116$$

Predicted value in an interesting range

- will be probed by next generation experiments
- leptonic Dirac CP violating phase measurable

Predictions for neutrino masses and mixing

- Heavy RH neutrino masses:

$$M_{R_1} = 1.72 \times 10^{7}\ GeV$$
$$M_{R_2} = 2.44 \times 10^{9}\ GeV$$
$$M_{R_3} = 5.94 \times 10^{12}\ GeV$$

- Leptonic CP violation phases:

$$J_{CP}^{lep} = -0.00690$$
$$\phi_{12} = -2.29\ rad$$
$$\phi_{13} = 0.490\ rad$$



J_{CP} in quark sector ~ 10^{-5}

2 orders of magnitude larger in lepton sector due to large leptonic mixing

- Neutrinoless double beta decay:

$$\left| \langle\ m\ \rangle \right| = 2.22 \times 10^{-3} eV$$

	$\sin^2 2\theta_{13}$	$\sin\theta_{13}$
current limit	10^{-1}	0.16
reactor	10^{-2}	0.05
Conventional beam	10^{-2}	0.05
superbeam	3×10^{-3}	2.7×10^{-2}
Neutrino factory	$(5\text{-}50) \times 10^{-5}$	$(3.5\text{-}11) \times 10^{-3}$

model	Flavor symmetry	$\sin\theta_{13}$
Albright-Barr	U(1)	0.014
Blazek-Raby-Tobe	$U(2)xU(1)^n$	0.049
Ross-Velasco-Sevilla	SU(3)	0.070
Chen-Mahanthappa	SU(2)	0.116
Kitano-Mimura	SU(3)xU(1)	0.22
Maekawa	U(1)	0.22
Mohapatra-Parida-Rajasekaran	RG enhanced	0.08-0.10
Raby	3x2 seesaw	0.1
Goh-Mohapatra-Ng	Minimal model	0.16

(SUSY) GUT Signatures

- Signatures of any GUT model: baryon number violating processes
  --- Δ(B-L) = 0 : proton decay
  --- Δ(B-L) = 2 : neutron-anti-neutron oscillation

- Proton decay

  --- dim-4 operators: if R-parity exact, no dim-4 operators

  --- dim-6 operators (mediated by X, Y gauge bosons):
  * Dominant decay mode $\tau\left(p \to e^+\pi^0\right)$ with decay amplitude

$$M_{p\to e^+\pi^0} \cong \frac{4\pi\alpha_{GUT}}{M_{GUT}^2} \qquad \tau(p \to e^+\pi^0) \cong 4.5\times 10^{29\pm 0.7}\left(\frac{M_{GUT}}{2.1\times 10^{14}\,GeV}\right)^4$$

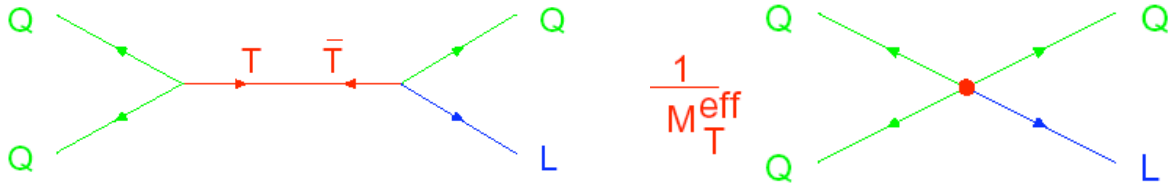
  * Theory prediction $\tau_p \sim 10^{37}\,years$
  * Far above the current capability of SuperK (~10^{34} years)

Proton decay

- dim-5 operators:

  generated by the exchange of color triplet higgsinos

$\frac{QQQL}{M_H}$



  dominant decay mode $p \to K^+ \tilde{\nu}$ with decay amplitude

$$M_{p\to K^+\tilde{\nu}} \cong \frac{h_u h_d}{M_H}\frac{m_{gaugino} g_{GUT}^2}{16\pi^2 M_{\tilde{Q}}^2} \qquad \tau\left(p \to K^+\tilde{\nu}\right) \sim m_{\tilde{h}}^2$$

  require heavy color triplet Higgsinos to suppress the rate

  need light MSSM doublets to have gauge coupling unification (doublet-triplet splitting)

Proton decay

- Predictions of minimal SUSY SO(10) model with 16-dim Higgs:

$$\tau\left(p \to e^+\pi^0\right) \sim 10^{35\pm1}\, yrs$$

$$\tau\left(p \to K^+\tilde{\nu}\right) \le (1/3-3)\times 10^{34}\, yrs$$

to suppress dim-5 operators requires light gauginos and heavy 1st and 2nd families of squarks and sleptons

mode	lifetimes ($\times 10^{32}$ yrs)
$p \to K^+\bar{\nu}$	19
$p \to K^0 e^+$	5.4
$p \to K^0 \mu^+$	10
$p \to \pi^+\bar{\nu}$	0.2 [27], 0.16 [28]
$p \to \pi^0 e^+$	50
$p \to \pi^0 \mu^+$	37
$n \to \pi^0\bar{\nu}$	4.4
$n \to K^0\bar{\nu}$	1.8

- In models with 126-dim Higgs, proton life-time can be prolonged by an order of magnitude

- Predictions of minimal SUSY SO(10) model with 126-dim Higgs:

when the bound on $\tau\left(p \to K^+\tilde{\nu}\right)$ is satisfied, found upper bound for the modes

$$\tau\left(n \to \pi^0\tilde{\nu}\right) = 2\tau\left(p \to \pi^+\tilde{\nu}\right) \le (5.7-13)\times 10^{32}\, yr$$

$$\tau\left(n \to K^0\tilde{\nu}\right) \le 2.97\times 10^{33}\, yr$$

for ave. squark mass of a TeV and a wino mass of 200 GeV.

(SUSY) GUT Signatures: neutron-anti-neutron oscillation

Neutron-anti-Neutron oscillation:

-- theoretical predictions based on GUT for oscillation time can naturally satisfy the experimental lower limit $\tau_{N-\bar{N}} \geq 0.86 \times 10^{8}\,\text{sec}$ if a high (B-L) scale is assumed.

-- in SUSY left-right symmetric model without embedded into a GUT group, UPPER bound of oscillation time $10^{9\text{-}10}$ sec very close to current experimental limit is predicted. As operators for both n-nbar oscillation and neutrino mass are related to (B-L) breaking scale => n-nbar oscillation time related to tau-neutrino mass

Babu, Mohapatra, Phys. Lett. B518, 269 (2001)

-- observation of n-nbar oscillation indicates an explicit $U(1)_{B\text{-}L}$ symmetry all the way to Planck (String) scale [as opposed to being embedded in a SU(4) or SO(10)]

-- new experimental searches important for distinguishing new Physics beyond the SM

Leptogenesis

- CP violation in the quark sector is not sufficient to explain the observed matter-antimatter asymmetry in the Universe

- Neutrino oscillation opens up the possibility that CP violation in the lepton sector may be responsible, through leptogenesis, for the observed BAU
  - Depend on parameters probed by neutrino oscillations

$$\delta,\ \theta_a,\ \theta_s,\ \theta_{rea}$$

  - Also depend on parameters not probed by neutrino oscillations

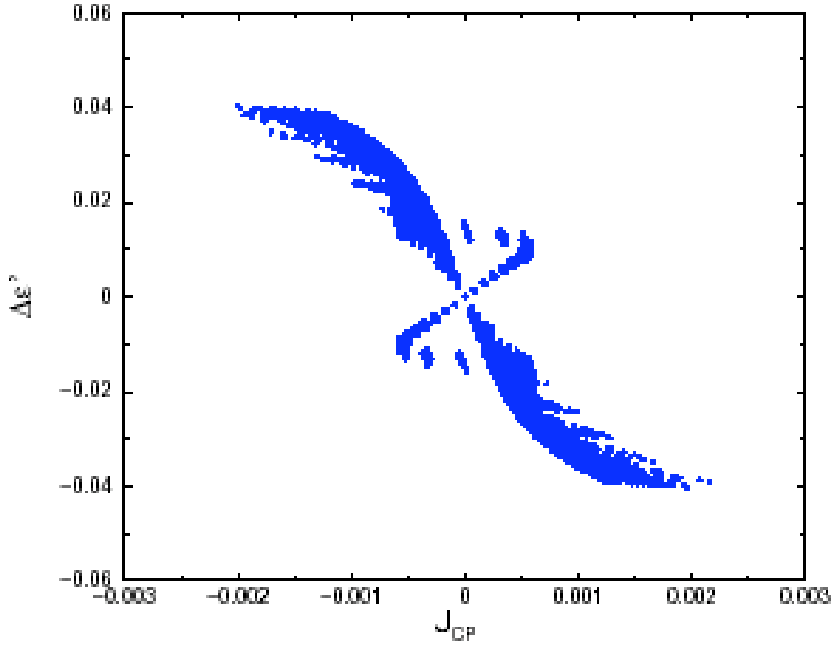
$$\phi_{12},\ \phi_{13} \qquad M_1, M_2, M_3$$

- In general it is not possible to relate leptogenesis with low energy CP violating processes due to the presence of extra phases and mixing angles in the heavy neutrino sector

Leptogenesis

- In minimal left-right symmetric models with spontaneous CP violation, a single phase is responsible for all leptonic CP violation. LH and RH Majorana mass terms are proportional to each other. Pronounced relation between leptongenesis and low energy CPV processes does exist

M.-C.C and K.T. Mahanthappa, hep-ph/0411158



with $J_{CP} \sim 10^{-5}$, sufficient amount of BAU can be generated

Summary

- Neutrino oscillation is the first direct evidence of physics beyond the Standard Model
- Supersymmetric SO(10) Grand Unified Theories are by far the most attractive candidates as they alleviate many deficiencies in the Standard Model of particle Physics
- DUSEL is indispensable for testing signatures of GUT models (neucleon decay, …)