Recovering longitudinal diffusion constant and its systematics.

Using responses relevant to SBND from closed-form calculations.

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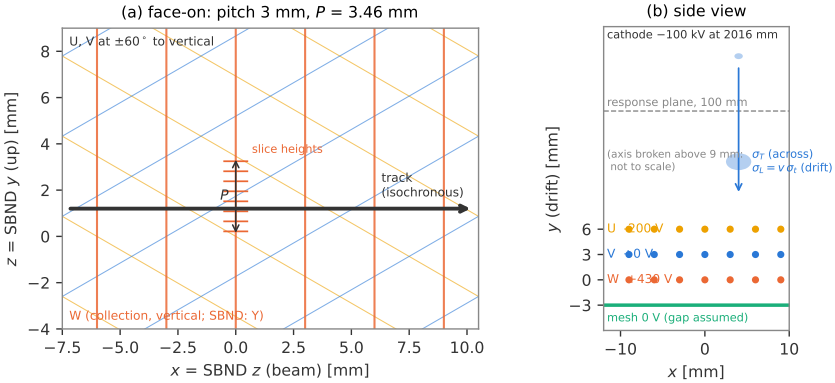
Topics

- The parameter lattice.
- Recovering longitudinal diffusion constant
- Varying drift HV

Multi-dimensional parameter lattice: $4 \times 3 \times 3 \times 3$

- Four field responses:
  - **Garfield 2D**: thin wire, no dipole moments.
  - **firep aligned-2D**: thin wire, with dipole moments.
  - **firep 2D phase slices**: averaged over many different U/V vs W alignments.
  - **firep full-3D**: thin wire, cross-wire moments.
- Three waveforms
  - **raw collection ADC**: non-Gaussian shape, subject to forward FR dependence.
  - **sigproc signals**: Gaussian shape, subject to sim/real + sigproc FR mismatch.
  - **splat signals**: smeared truth modeling ideal sim+sigproc, soft FR dependence.
- Three width measures
  - **RMS**, **Gaussian fit**, and **FWHM**.
- Three methods
  - **M1** fit raw.
  - **M2** fit signals.
  - **M3** fit via forward model.

SBND 2D and 3D detector and source models

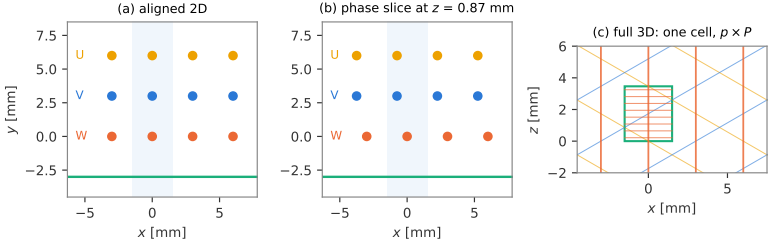


The SBND anode in firep's coordinates. (a) Face-on: vertical collection (W) wires and U, V wires at $\pm 60^\circ$, all at 3 mm pitch, with SBND's lattice registration: U–V crossings are pair-wise, halfway between W wires. The U/V crossing pattern repeats along a W wire every $P = p/\sin 60^\circ = 3.46$ mm. The selected track is isochronous and crosses the W wires. The marks on the central wire are the heights of the phase slices of note Sec. 4. (b) Side view, with the biases, the grounded mesh, the response plane and the cathode (the drift axis is not to scale above the wires). A deposit drifting toward the anode spreads along the drift (σ_L, seen as σ_t) and across it (σ_T).

Field responses

firep calculates FRs with closed-form solutions and a least-squares fit to boundary data.

- Continuous and differentiable function, 2D or 3D, large domains.
- firep **aligned-2D**, **2D phase-slices**, **full 3D**.

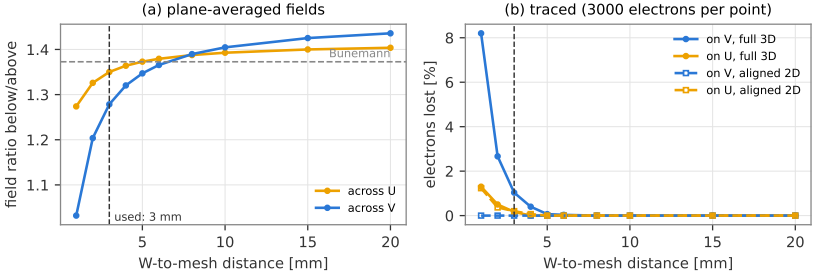


The geometry models. (a) Aligned 2D: the three planes parallel and stacked, one periodic cell (shaded). (b) A phase slice: a 2D cell perpendicular to the W wires, with the U and V lattices shifted to where their wires cross the central W wire at height z. (c) Full 3D: electrons start on a grid over one W pitch and one crossing period P (box). The lines across the box are the slice heights.

Also include GARFIELD 2D thin wire model.

- GARFIELD deficiencies were uncovered and are shown in backups.

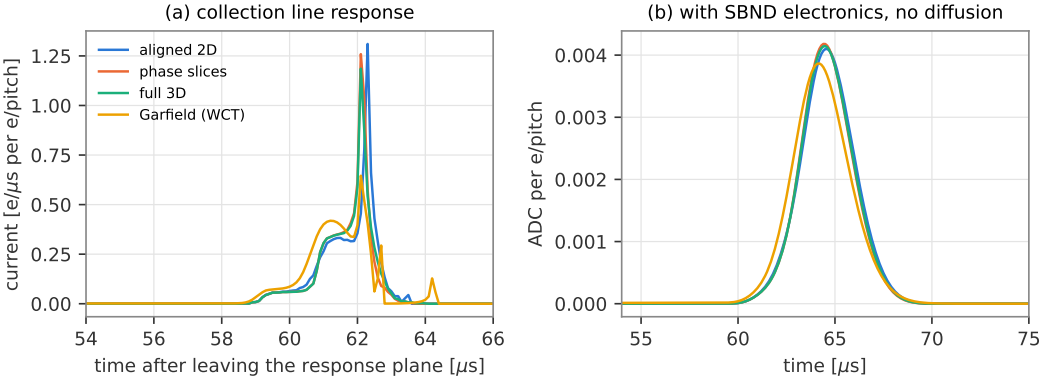
FR: transparency vs mesh plane location - 2D vs 3D



Transparency against the distance from the W plane to the mesh, nominal biases. (a) Ratio of the plane-averaged fields below and above each induction plane, against the Bunemann requirement (this does not depend on the geometry model). (b) Fraction of 3000 electrons from the response plane that end on V or U, in full 3D (filled) and aligned 2D (open, dashed). The vertical dashed line marks the 3 mm gap used in the rest of the note.

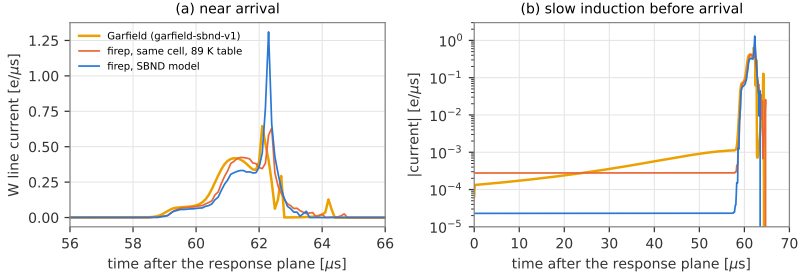
- Aligned-2D: U shadows V so never any V collection. 3D shows otherwise.
- Assumed mesh gap of 3 mm breaks transparency at 1% level in 3D. Best to have at least 6 mm.

FR: mean collection responses



Height-averaged collection line responses for the nominal run. (a) Current against the time since the electrons left the response plane. (b) The same with the SBND electronics, before diffusion. The RMS widths in (b) are 1.383, 1.341, 1.352 and 1.516 μs for aligned 2D, phase slices, full 3D and Garfield.

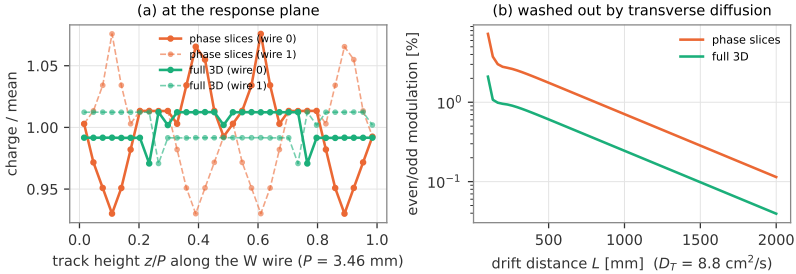
FR: near and far fields



Collection line responses: `garfield-sbnd-v1`, firep solving the same Garfield cell with the 89 K velocity table, and firep's SBND model (mesh at 3 mm, cathode at 2016 mm). (a) Near arrival. (b) On a log scale: the slow induction before arrival.

- For drift, the "cathode" position/potential doesn't matter as long as E-field is correct.
- But, for weighting, the "cathode" is absolutely grounded, so its position matters.
  - All early weighting is compressed into a shorter drift (eg 204 mm compared to "SBND model" 2016 mm).
  - Limited transverse domain size plays a role here as well.

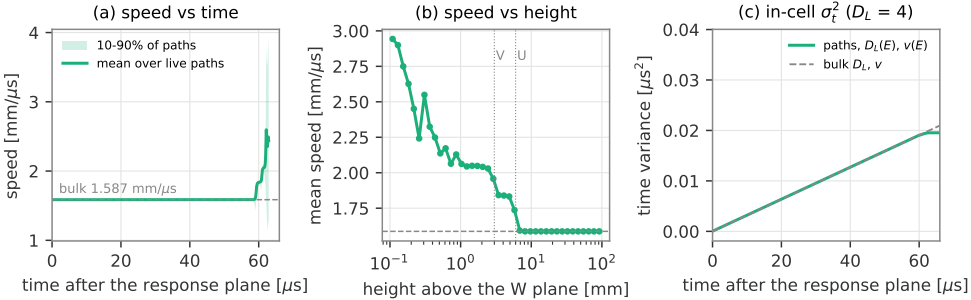
FR: 3rd dimension effects



(a) Charge collected by two neighbouring W wires from a track at fixed height z, relative to the mean, at the response plane. The crossed geometry steers electrons across the pitch and the steering alternates from wire to wire. (b) The even/odd modulation against drift distance: transverse diffusion along the wire averages over heights and washes it out.

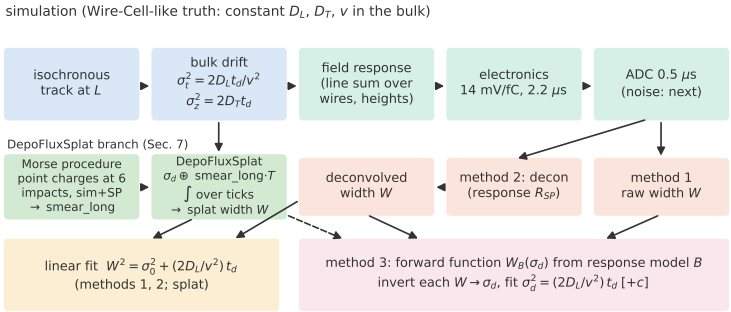
We likely can not observe 3rd dimension FR variance.

FR: Velocity averaged over drift paths



Electrons from the response plane through the full-3D field. (a) Speed against time since leaving the response plane: mean over the paths still drifting, and the 10–90% band. (b) Mean speed against height above the W plane. (c) Time variance accrued inside the cell, $\int 2D_L(E)/v(E)^2\,dt$ along each path (mean over paths), compared with the bulk value continued at constant speed.

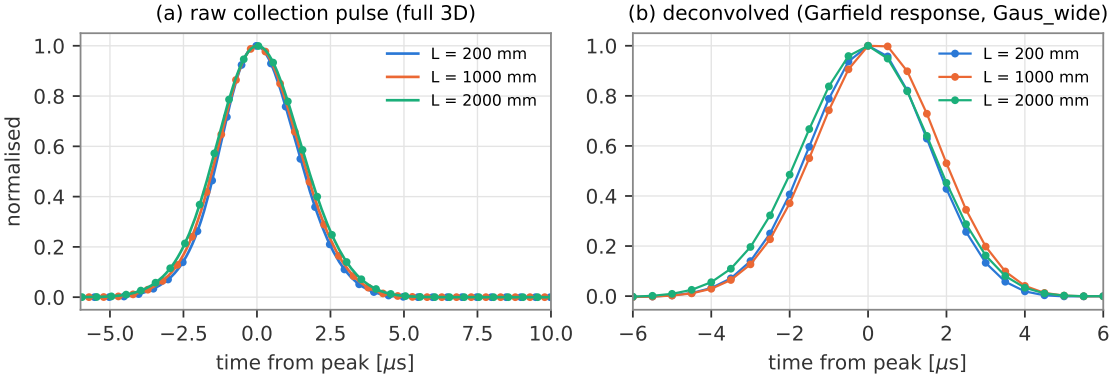
Analysis DAG



The simulation chain (top) and the three analyses. Method 1 measures the width of the raw ADC pulse and method 2 the width after deconvolution with a response R_{SP}. Both fit note Eq. 1. Method 3 inverts each width, raw or deconvolved, with a forward function $W_B(\sigma_d)$ computed from a response model B. The branch on the left (green) is Wire-Cell's DepoFluxSplat (note Sec. 7). The Morse procedure runs point charges at the six field-response impacts through sim+SP (here the idealised versions of note Sec. 5.1) and fits the extra smearing `smear_long`. Splat adds it in quadrature to the post-drift width and integrates over ticks. Its widths are fitted like those of methods 1 and 2, and splat can also serve as a forward function for method 3 (dashed).

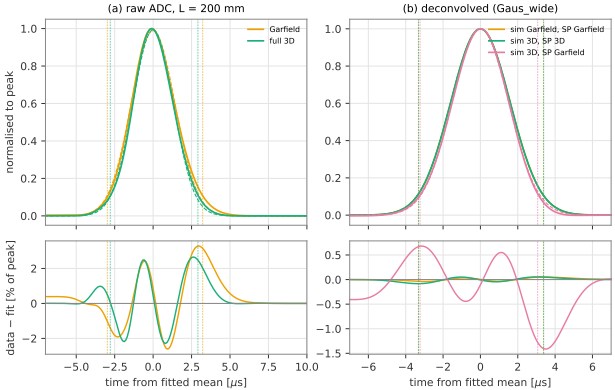
An **idealized** sim/convolution and sigproc/deconvolution is used, no noise, no Wire-Cell Toolkit.

Raw vs deconvolved



Collection pulses from the full-3D line response, **normalised and aligned on the peak**, at three drift distances. (a) The raw ADC samples. (b) After the idealised deconvolution with the Garfield response and SBND's Gaus_wide filter. The **diffusion broadening is small compared with the detector response** in both.

How Gaussian is the pulse?



How Gaussian the pulses are, and how the shape depends on the field response, at $L = 200$ mm. (a) Raw ADC pulses simulated with the Garfield and the full-3D responses. (b) Deconvolved pulses for three combinations of the response used in the simulation and in the signal processing. Solid: the pulse (five sampling phases interleaved to 0.1 μs), from the idealised sim and SP of note Sec. 5.1; dashed: a Gaussian fitted to the contiguous samples above 10% of the peak (between the dotted lines). Times are relative to the fitted mean. Bottom: data minus fit, in percent of the peak.

- Mismatching FRs with sim/real + sigproc adds non-Gaussian shape.
- But, raw ADC is less Gaussian than SP with any mismatched FR.

Gaussian fits to pulse and the effect of mismatched FR

pulse	sim	SP	mean [μs]	σ [μs]	χ^2/ndf	RMS [μs]	FWHM [μs]
raw	Garfield	–	127.277	1.421	235/60	1.524	1.395
raw	3D	–	127.587	1.316	175/55	1.360	1.295
deconvolved	Garfield	Garfield	63.221	1.592	0/65	1.593	1.592
deconvolved	3D	3D	63.214	1.591	0/65	1.591	1.592
deconvolved	3D	Garfield	63.533	1.509	16/61	1.443	1.513

Truncated-Gaussian fits to the pulses of note Fig. 15, $L = 200$ mm. Mean: time from the track time. The raw mean includes the transit from the response plane and the electronics delay; the deconvolution refers the charge back to the response plane. χ^2 assumes an uncertainty of 1% of the peak on each sample (samples every 0.1 μs), so it scales as the inverse square of a real noise level. RMS and FWHM (divided by 2.355) are computed on the pulse itself. "sim" and "SP": the responses used in the simulation and in the deconvolution.

This χ^2 is not statistical.

Matched FRs give almost perfect fit, $\chi^2 \approx 10^{-4}$.

Recovering D_L

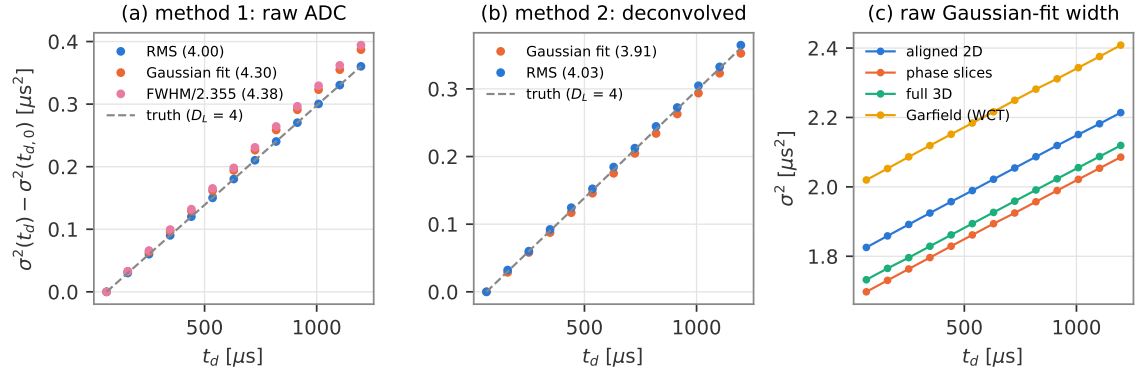
$$\sigma_t^2(t) = \sigma_0^2 + \frac{2D_L}{v^2}\, t$$

- Use $D_L = 4.0\,\mathrm{cm}^2/\mathrm{s}$ for the simulations.
- Muon started at different distance.
- Bulk drift velocity for v and to convert distance to t.

Methods

1. **Raw widths.** Measure the width W of the raw ADC collection pulse at each drift time and fit $W^2 = \sigma_0^2 + (2D_L/v^2)\, t_d$, with σ_0^2 and D_L free. Needs no field response and no signal processing; exact only for the RMS width.
2. **Signal-processed widths.** The same fit after deconvolution with an assumed response R_{SP} and SBND's Gauss filter (idealised here). When R_{SP} matches the true response the pulse is Gaussian and any width estimator works.
3. **Forward function.** A response model B (field response, electronics, and the deconvolution for processed widths) predicts the width $W_B(\sigma_d)$ for a diffusion width σ_d, with the same estimator. Each measured width is inverted to σ_d, and $\sigma_d^2 = (2D_L/v^2)\, t_d + c$ is fitted with c free or $c = 0$.

Results from methods 1 and 2



Squared width against drift time to the response plane. (a) Method 1 on raw ADC pulses, full-3D truth, for three width estimators, each relative to its value at the shortest drift. The fitted D_L is in parentheses. The dashed line is the truth. (b) Method 2, after the idealised deconvolution with the Garfield response. (c) Raw Gaussian-fit widths for each truth: the intercepts σ_0^2 differ by more than the diffusion adds over the whole drift.

RMS vs Gaussian, matched vs mismatched FRs

method	width	aligned	slices	3D	Garfield
1: raw	RMS	4.00	4.00	4.00	3.94
1: raw	Gaussian fit	4.31	4.30	4.30	4.30
1: raw	FWHM/2.355	4.52	4.39	4.38	4.46
2: SP Garfield	Gaussian fit	3.91	3.92	3.91	4.00
2: SP Garfield	RMS	4.05	4.03	4.03	4.00
2: SP aligned	Gaussian fit	3.99	3.99	4.00	4.05
2: SP aligned	RMS	4.00	4.00	4.00	3.96
2: SP 3D	Gaussian fit	3.99	4.00	4.00	4.05
2: SP 3D	RMS	4.00	4.00	4.00	3.97
σ_0^2 [μs^2]	raw RMS	1.915	1.799	1.830	2.316
σ_0^2 [μs^2]	raw Gaussian	1.805	1.677	1.711	2.000
σ_0^2 [μs^2]	SP Garfield, Gaussian	2.352	2.227	2.258	2.514

D_L [cm^2/s] from a linear fit of note Eq. 1, for each truth response (columns), method and width. The simulated value is 4.0. The last rows give the fitted intercepts.

- RMS is most robust for raw but only if full domain used.
- Mismatch FR: RMS and Gaussian error has different signs.
- σ_0^2 strongly soaks up bias from FR mismatch.

Method 3 results - raw collection

truth ↓ / model →	Garfield (WCT)		aligned 2D		full 3D	
	free	origin	free	origin	free	origin
aligned 2D	3.93	1.61	4.01	4.00	4.04	5.36
phase slices	3.86	0.59	3.94	2.31	3.99	3.52
full 3D	3.88	0.82	3.97	2.71	4.00	4.00
Garfield (WCT)	4.00	4.00	4.06	6.78	4.08	8.15

Method 3 on raw Gaussian-fit widths: D_L [cm^2/s] when the analysis model (columns) differs from the truth (rows). "free" fits $\sigma_d^2 = (2D_L/v^2)t_d + c$, "origin" fixes $c = 0$.

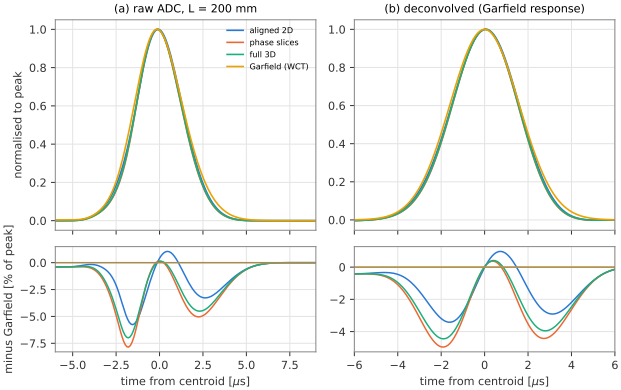
Method 3 results - ideal deconvolution

truth ↓ / model →	Garfield (WCT)		aligned 2D		full 3D	
	free	origin	free	origin	free	origin
aligned 2D	3.90	1.69	4.00	4.00	3.99	5.46
phase slices	3.91	0.65	4.01	2.20	4.00	3.53
full 3D	3.97	0.89	4.02	2.61	3.99	4.00
Garfield (WCT)	4.00	4.00	4.08	6.61	4.08	8.06

As note Table 8, for Gaussian-fit widths after deconvolution with the Garfield response.

Allowing c to float soaks up bias from mismatched FRs

Pulse shape vs FR model



Collection pulses of a horizontal track at $L = 200$ mm for every response model, normalised to the peak and aligned on the centroid, with the differences from Garfield (bottom). (a) Raw ADC (the analogue waveform before sampling). (b) After deconvolution with the Garfield response and Gaus_wide (five sampling phases interleaved to 0.1 μs).

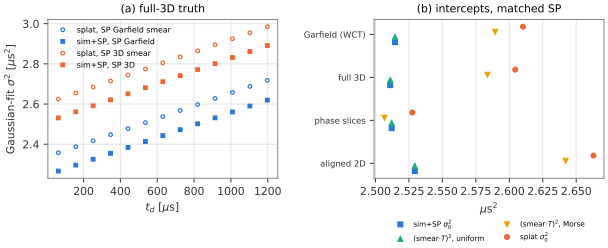
GARFIELD more different than firep than different firep models.

Pulse shape vs FR model

	centroid − Garfield [μs]		raw width [μs]			SP width [μs]	
model	raw	SP	RMS	Gauss	FWHM	Gauss	FWHM
aligned 2D	+0.34	+0.34	1.391	1.351	1.335	1.540	1.551
phase slices	+0.27	+0.27	1.349	1.303	1.294	1.499	1.511
full 3D	+0.29	+0.29	1.360	1.316	1.307	1.509	1.521
Garfield (WCT)	+0.00	+0.00	1.528	1.421	1.405	1.592	1.600

Pulses at $L = 200$ mm for each response model: centroid time relative to Garfield, and widths of the raw and the deconvolved pulse (Garfield response in the deconvolution). Averages over eight sampling phases; the crossed models are averaged over height.

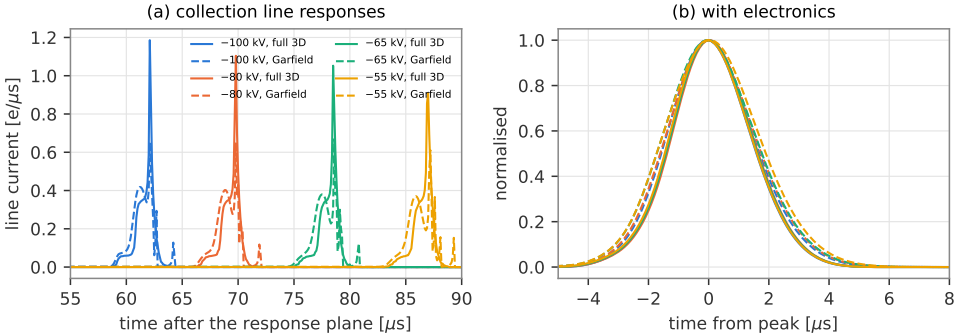
Splat - test with “true” signal



(a) Gaussian-fit squared widths of splat (open) and idealised sim+SP (filled) signals of horizontal muons, full-3D truth, with the Garfield response or the truth itself in the deconvolution (and in the Morse procedure). (b) Intercepts with matched responses: the sim+SP intercept, the squared extra smearing from the Morse procedure (six equally weighted impacts) and from all start positions weighted uniformly, and the splat intercept.

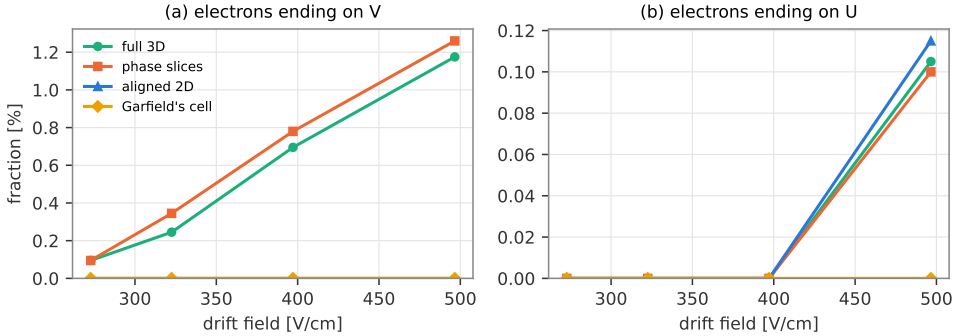
- D_L recovered exactly from splat regardless of how splat constructed.
- Intercept is splat “extra smearing” plus time bin $T/\sqrt{12}$ resolution (added in quadrature).

Cathode-HV runs: line responses



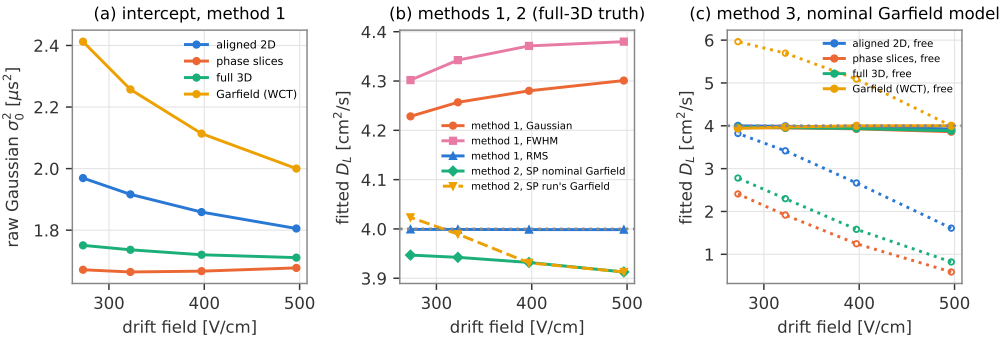
Collection line responses for the four cathode voltages: firep full 3D (solid) and Garfield (dashed). (a) Current: at lower drift fields the electrons take longer from the response plane to the wires. (b) With the electronics, normalised and aligned on the peak.

Cathode-HV runs: where the electrons end



Fraction of electrons from the response plane that end on V (a) and on U (b) against the drift field, for the firep models of SBND (mesh 3 mm behind W) and for Garfield's cell (ground plane 20 mm behind W), all traced by firep with 20000 electrons per point. No electron reaches the mesh in any case.

Cathode-HV runs: what the methods extract



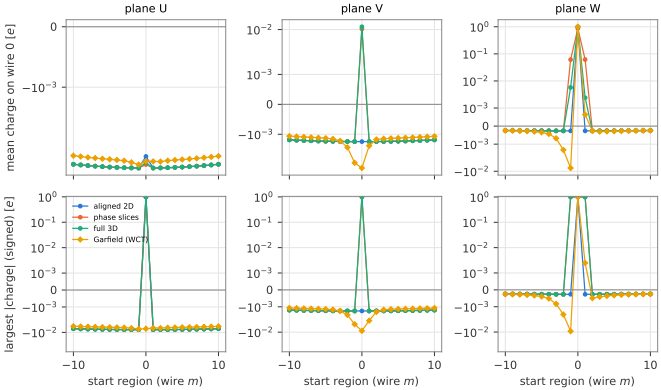
The methods against the drift field. (a) The method-1 intercept (raw Gaussian fit) for each truth response. (b) D_L from methods 1 and 2 for the full-3D truth. Method 2 deconvolves either with the nominal Garfield response, which is what SBND uses for every run, or with Garfield at the run's own field. (c) Method 3 with the nominal Garfield response as the model, with a free intercept (filled) and through the origin (open, dotted), for each truth. Dotted grey: the simulated D_L.

Cathode-HV runs: summary table

cathode	−100 kV	−80 kV	−65 kV	−55 kV
drift field [V/cm]	496.6	397.1	322.5	272.8
bulk speed [mm/μs]	1.587	1.406	1.243	1.118
D_L (Li et al.) [cm^2/s]	6.62	6.53	6.43	6.32
Garfield cell cathode [kV]	10.00	8.04	6.56	5.57
σ_θ^2 raw Gaussian, aligned 2D [μs^2]	1.805	1.859	1.916	1.969
σ_θ^2 raw Gaussian, phase slices [μs^2]	1.677	1.667	1.664	1.671
σ_θ^2 raw Gaussian, full 3D [μs^2]	1.711	1.720	1.736	1.751
σ_θ^2 raw Gaussian, Garfield (WCT) [μs^2]	2.000	2.114	2.257	2.412
M1 Gaussian, aligned 2D	4.31	4.29	4.26	4.22
M1 Gaussian, phase slices	4.30	4.30	4.27	4.24
M1 Gaussian, full 3D	4.30	4.28	4.26	4.23
M1 Gaussian, Garfield (WCT)	4.30	4.24	4.17	4.10
M1 FWHM, aligned 2D	4.52	4.54	4.50	4.45
M1 FWHM, phase slices	4.39	4.38	4.35	4.31
M1 FWHM, full 3D	4.38	4.37	4.34	4.30
M1 FWHM, Garfield (WCT)	4.46	4.38	4.24	4.13
M2 SP nominal Garfield, aligned 2D	3.91	3.92	3.92	3.92
M2 SP nominal Garfield, phase slices	3.92	3.93	3.94	3.94
M2 SP nominal Garfield, full 3D	3.91	3.93	3.94	3.95
M2 SP nominal Garfield, Garfield (WCT)	4.00	4.00	3.98	3.97
M2 SP run's Garfield, aligned 2D	3.91	3.92	3.96	3.96
M2 SP run's Garfield, phase slices	3.92	3.93	3.99	4.02
M2 SP run's Garfield, full 3D	3.91	3.93	3.99	4.02
M3 nominal Garfield, free, aligned 2D	3.93	3.97	3.99	4.00
M3 nominal Garfield, free, phase slices	3.86	3.93	3.95	3.96
M3 nominal Garfield, free, full 3D	3.88	3.94	3.96	3.97
M3 nominal Garfield, free, Garfield (WCT)	4.00	4.00	3.97	3.94
M3 nominal Garfield, origin, aligned 2D	1.61	2.66	3.42	3.82
M3 nominal Garfield, origin, phase slices	0.59	1.25	1.92	2.41
M3 nominal Garfield, origin, full 3D	0.82	1.58	2.30	2.78
M3 nominal Garfield, origin, Garfield (WCT)	4.00	5.09	5.70	5.96

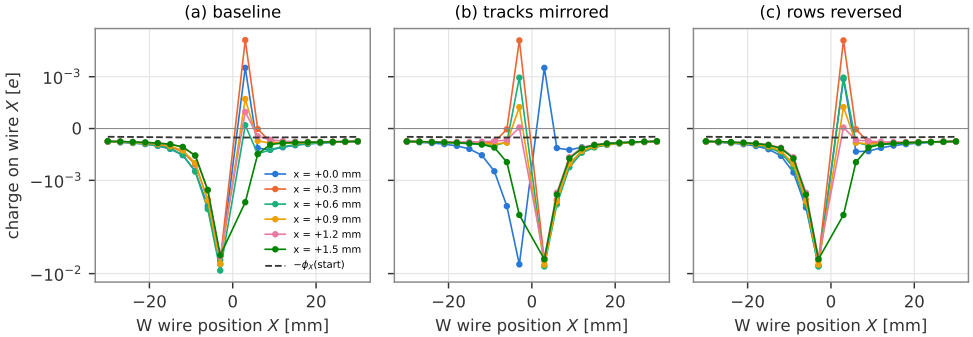
The cathode-HV runs. Drift field and bulk speed from firep's solution; D_L from Li et al. at that field; the cathode voltage of Garfield's cell that gives the same drift field. Then the method-1 intercepts and the D_L [cm^2/s] from each method, for each truth response. M2: Gaussian fit after deconvolution with the nominal Garfield response or with Garfield at the run's field. M3: raw Gaussian-fit widths with the nominal Garfield response as the model. The simulated D_L is 4.0.

Backup: charge induced by each path



Time integral of the field-response current on wire 0 of each plane, against the region in which the electron starts (the wire m it starts nearest to; $m = 0$ is wire 0's own region). Top: mean over start positions in the region. Bottom: the value of largest magnitude, with its sign. Symmetric-log axis, linear within $\pm 10^{-3}$. firep models from 3000 electrons at random start positions; Garfield from the six stored paths per region.

Backup: Garfield's asymmetry on the collection plane



Charge Garfield induces on each W wire at position X by an electron collected on wire 0, for the six tracks of the SBND cell (-10 kV), with the collecting wire left out. (a) The original runs. (b) Tracks mirrored to $x = 0 \ldots -1.5$ mm (the $x = 0$ track is nudged to $+1$ μm in both). (c) Every row of the cell deck listed right to left. Dashed: the physical bound $-\phi_X(\texttt{start})$ from the single-wire far field of Garfield's cell (note Eq. 7, $\Phi_W = 0.0182$). Symmetric-log axis, linear within $\pm 10^{-3}$.

Recommendations for measuring D_L

- Use SP signals, not raw collection ADC waveforms.
  - If must use raw, use RMS, not Gaussian fit.
- Non-Gaussian and FR mismatch hits Y-intercept more than slope.
- Apply your method to "splat" and require exact D_L recovery.
- Use bulk volume drift velocity for space/time conversions.
  - No need to consider local, near-wire velocity.
- Prefer newer FRs over GARFIELD.

A note on widths

$$\mathrm{Var}[f * g] = \mathrm{Var}[f] + \mathrm{Var}[g], \qquad \mathrm{Var}[f] = \frac{\int (t - \bar{t})^2 f \, dt}{\int f \, dt}$$

- Variances add regardless of distribution.
  - Note, variances must be over full domain for this to hold.
- Fit σ^2 or FWHM^2 add only when every convolved factor is Gaussian.
  - Raw pulses are never Gaussian
- Non-Gaussian, no-noise should use RMS or use method 3.

Becomes important with considering raw collection vs SP signal and when SP's FR mismatches sim/real data.