Rastering a fast signal

Brett Viren

November 24, 2024

Problem statement

Raster (here) means to **integrate and sample** a signal.

- Integrate over some small volume near each grid point.
  - We are free to choose any integration method → **effective charge**.
- 2D grid formed as the **impact positions** of field response path start points.

Ionization electron distribution signal can be **fast** or **sharp**

- Extreme case, no diffusion: a 0-D point depo or 1-D step line produced at anode.
- Essentially all signal may fall between grid points.

Two problems

1. The raster must not "miss" sharp signal peaking between grid points.
2. How to choose a "best" effective charge estimate.

Simple Gaussian depo method formalism

Charge density for depo d has independent Gaussian profiles in each of D dimensions.

$$Q_d(\vec{r}) = q_d \Pi_{i=1}^{D} \frac{1}{\sqrt{2\pi\sigma_{d,i}}} e^{-\frac{(r_i - c_{d,i})^2}{\sigma_{d,i}^2}}$$

Integrate $Q_d(\vec{r})$ over small elements centered on a grid point $\vec{p}_{lmn}$ in each of D dimensions.

$$q_{lmn,d} = q_d \Pi_{i=1}^{D} (erf(\rho_{lmn,i} + \Delta_i) - erf(\rho_{lmn,i}))$$

Where a coordinate transformation in each dimension i is defined:

$$\rho_{lmn,i} = ((\vec{p}_l mn)_i - (\vec{c}_d)_i)/\sigma_{d,i}$$

In order to convert Gaussian to Normal and allow $erf(\cdot)$ to be used.

Simple Gaussian depo method in practice

For each dimension:

- Find maximum range the contains all depos.
- Calculate offsets of ranges given depo centers $\vec{c}_d$.
- Define array of positions across that range **plus one more** for each depo.
- Apply Gauss $\rightarrow$ Normal transform to array $(N_{depos}, 2)$ holding $(c_{d,i}, \sigma_{d,i})$
- Form cumulative integral across positions using $erf(\cdot)$ and subtract:

```
cumu = [erf(p[i]) for i in range(N+1)]
q = cumu[1:] - cumu[:-1]
```

Note, all these are array-operations and parallelized by PyTorch.

- There are no explicit loops over depos nor positions.
- For now, we do loop over dimensions.

Results in $N + 1$ dimension array, same shape N-D array for each depo.

Integrate+sample vs sample+integrate

Integration of arbitrary distributions difficult in general.

- Gaussian + $erf(\cdot)$ allows for a special case.

May consider the reverse: sample then integrate.

- Eg, apply Newton's method to samples of **density**.
- Must sample **fine enough** to avoid bias / inefficiency.

What does **fine enough** mean?

Field response bandwidth and sampling

Field responses are continuous functions in space.

- They have some natural **band limit** in each dimension.
- But, we calculate them on a discrete grid and thus produce a **sampling**.
- Ideally, define a grid that samples at least $2\times$ faster than the band limit.

Reality

100ns for wires and 50ns for pixels time sampling is marginal.

- Some spectral power remains at these Nyquist frequencies.

In space the FR changes radically faster than sampling period/distance in some regions.

- Eg, near boundaries between wire regions or pixels.
- Oversampling these regions would require very close-spaced paths.

At the end of the day, the discrete FRs are band-limited

- Either naturally or by their (under) sampling.

Spectrum of a Gaussian signal

$$f(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}} \leftrightarrow F(\omega) = e^{-i\mu\omega} e^{-\frac{1}{2}(\sigma\omega)^2} = e^{-i\mu\omega} e^{-\frac{1}{2}(\omega/\sigma_\omega)^2}$$

Where we have identified a "frequency sigma" $\sigma_\omega \equiv 1/\sigma$ in units of `[1/distance]`.

Example

A depo with $\sigma = 100\,\mu\text{m}$ has a $3-\sigma$ band limit at $\omega = 3\sigma_\omega = \frac{3\text{samples}}{100\,\mu\text{m}}$.
Ideally, we sample at least that frequently.

But, what if we must sample less freqently?