Sample rate conversion for DUNE FD VD TPC ADC

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November 12, 2023

Context

DUNE FD VD is split into two halves

1. Top Detector Electronics (TDE)
2. Bottom Detector Electronics (BDE)

ADC sampling parameters

sampling	period (ns)	rate (MHz)	ADC (bits)
FD VD TDE	512	1.953125	14
FD VD BDE	500	2.000000	14
everyone else	500	2.000000	12

To do: official VD sampling numbers

VD CDR is somewhat unclear

- says, “12 bit ADC, 2 MHz or 14 bit ADC, 2.5 MHz or more”
- talks about downsampling from 2.5 MHz+ in FPGA.
- table 7.2: 14 bit ADC, 1.953125 MHz
  Must find a definitive and public reference!

Problems with different sample rates in DUNE FD VD TDE vs BDE

- Either **same readout time** ($\approx 4\,\mathrm{ms}$) XOR **same #samples** ($N \approx 8\mathrm{k}$), can not have both.
- Sloppy matching of 3D clusters across top/bottom boundary.
- Two separate field response calculations despite same CRP design.
- Two `tick` values floating around the configuration.

Solution: Apply **sample rate conversion** (SRC) methods

$$F = \frac{1}{T} \rightarrow F' = \frac{1}{T'},\ R \equiv \frac{F'}{F} = \frac{T}{T'}$$

We need an **upsampled interpolation**

- Sample period: $T = 512\,\text{ns} \rightarrow T' = 500\,\text{ns},\ R = \frac{512}{500} = 1.024$
- Nyquist frequency: $0.976\,562\,5\,\text{MHz} \rightarrow 1.000\,000\,\text{MHz}$
- Nyquist-Shannon sampling theorem tells us, *"You can reconstruct original signal exactly."*

Any interpolation produces **artifacts** at some level.

- Must demonstrate validity.

Historical sample rate conversion (SRC)

Schafer, R.W. and Rabiner, L.R., Proc. IEEE, 1973.

An irrational SRC ratio can be approximated by a rational ratio R to arbitrary accuracy.

Rational interpolation = integral upsample + integral downsample.

- **integral upsample** $N \to N^{iu} = LN$
- **integral downsample** $N \to N^{id} = N/M$
- **rational interpolation** $N \to N^{ri} = RN,\ R \equiv \frac{L}{M}$

Here, all natural numbers, $N, N^x, L, M \in \mathbb{N}$ and rational R.

Integral upsampling $N \rightarrow N^{iu} = LN$

Procedure

1. Apply DFT
2. **Insert** $(L-1)N$ zero-amplitude high frequency samples
3. Apply DFT^{-1}.

Insertion is at the Nyquist frequency and must preserve Hermitian symmetry.

- half of the samples inserted above “most positive” frequency, half below “most negative”.

Integral downsampling $N \rightarrow N^{id} = N/M$

Procedure

1. Select every M^{th} sample.

- Aliasing effect if signal bandwidth $> F^{id}_{nyquist}$
- No practical problem if $\frac{N}{M} \notin \mathbb{N}$.
  - Ignore that there may be $< M - 1$ original samples past the last resample.

Rational interpolation

How to assure $\frac{LN}{M}$ is an integer?
Either we get "lucky" or we must "over upsample", eg:

$$T = 512\,\text{ns}, T' = 500\,\text{ns}, N = 9600, \frac{LN}{M} = \frac{128 \cdot 384}{5}$$

Upsample by another factor of 5: $L \to L' = 5 * L = 5 \cdot 128$.
The conventional method then needs DFT^{-1} of size $n = 5 \cdot 128 \cdot 9600 = 6144000$ followed by $5\times$ larger downsampling, $M \to M' = 5 * M$.

Optimization

If we allow freedom to choose N we may be able to minimize L and M while keeping $\frac{LN}{M} \in \mathbb{N}$.
We want to shift the Nyquist frequency by:

$$\Delta F = F_{nyquist} - F'_{nyquist} = \frac{T - T'}{2TT'}$$

And we want that shift to be covered by an integral number n of bins of size $\delta F = \frac{1}{NT}$.
So, we want $n \cdot \delta F = \Delta F$ which is solved by:

$$N = \frac{2nT'}{T - T'} = \frac{n}{3} \cdot 250$$

With the numbers plugged in for the DUNE VD problem.
Thus for N any multiple of 250, we can assure the downsampling will be exactly integral.

Exact integer downsampling $N \to N^{id} = N/M \in \mathbb{N}$

When downsampling is exact, performing in frequency space is possible and advantageous.

Procedure

1. Apply DFT
2. **Remove** $(M-1)N$ high frequency samples
3. Apply DFT^{-1}

This implicitly applies a **perfect low-pass filter**.

- Though, that doesn't matter when $R > 1$.

Similar to insertion for upsampling, we must remove half the samples at the "most positive" and half at the "most negative" part of the spectrum.

Optimized procedure

1. Calcualte desired N and n.
2. Truncate or pad original signal to N
3. Apply DFT
4. Insert $2n$ $(R > 1)$ or remove $2n$ $(R < 1)$ frequency samples.
5. Apply DFT^1
6. Fix normalization

Normalization - conserve Parseval energy and ionization charge

DFT is not normalized, DFT^{-1} is normalized by $\frac{1}{N'}$

But we change $N \to N' = \frac{LN}{M}$ while ading no spectral energy. Need renormalize by $\frac{N'}{N} = R$

ADC is an integral of current over time

Normally, a sample is the value of a function at a point. But, an ADC sample is an integral of current over the sample period. Interpolating changes the integration bounds. We must also renormalize by $\frac{T'}{T} = \frac{1}{R}$.

So, it seems we do nothing!

But, do check this is correct and that Parseval energy is conserved: $\sum_i^N |x_i|^2$.

Validation

Why?

- For upsampling ($R > 1$) the method is "valid" by construction in that it is a lossless transformation (Nyquist-Shannon sampling theorem).
- However, any interpolation produces artifacts.
- And, we should illustrate the method and confirm it is bug free.

Things to check

- Generate "truth" waveforms and sample at 512 and 500ns.
- Perform signal rate conversion.
- Check Parseval energy conservation.
- Plot interval and frequency space signals at each step.
- Visual check for artifacts
- …