WCT Noise

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June 8, 2022

Intro

- formalize noise collection and generation
- understand spectral interpolation and normalization
- follow JOS formalism

Samples of a real signal $x(t)$ at discrete, regular times

Discrete sampling of signals in time

$$x(n) \triangleq x(t = n\Delta t)$$

Sampling frequency

$$f_s \triangleq \frac{1}{\Delta t}$$

Discrete Fourier Transform (DFT)

Samples of finite signal in frequency domain

$$X_k \equiv X(\omega_k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-i\frac{2\pi k}{N}}$$

$$\omega_k = 2\pi \frac{f_s}{N} k$$

- $f_n = \frac{f_s}{2}$ Nyquist frequency, largest resolved frequency
- $f_r = \frac{f_s}{N}$ Rayleigh frequency, smallest resolved frequency

Exactly recover time series from spectra

Inverse DFT

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X_k e^{i \frac{2\pi n}{N}}$$

Note DFT normalization convention: fwd is 1.0, inv is $\frac{1}{N}$.

DFT conserves "energy"

Parseval's Theorem aka Rayleigh Energy Theorem

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2$$

Autocorrelation function

Biased AC as function of "lag" l

$$(x \star x)(l) \triangleq \sum_{l} x(m)x(m+l) \leftrightarrow |X_k|^2$$

Note the simple DFT.

Unbiased "sample" AC

$$\hat{r}(l) \triangleq \frac{(x \star x)(l)}{N - |l|}$$

Defined for $|l| < N - 1$ and zero otherwise.

Periodogram

$$P_k = \frac{1}{M} |X_k|^2$$

- autocorrelation and its DFT
- white noise
- AC[0] is σ^2
- estimating and resolution vs stability