(At long last!) Lattice conquest of the Delta I=1/2 Rule and all that

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Particle Seminar,BNL

2/14/13

BASED in part on RBC-UKQCD
arXiv :1212.1474 & many more

Outline

- What the heck is it? And why care?

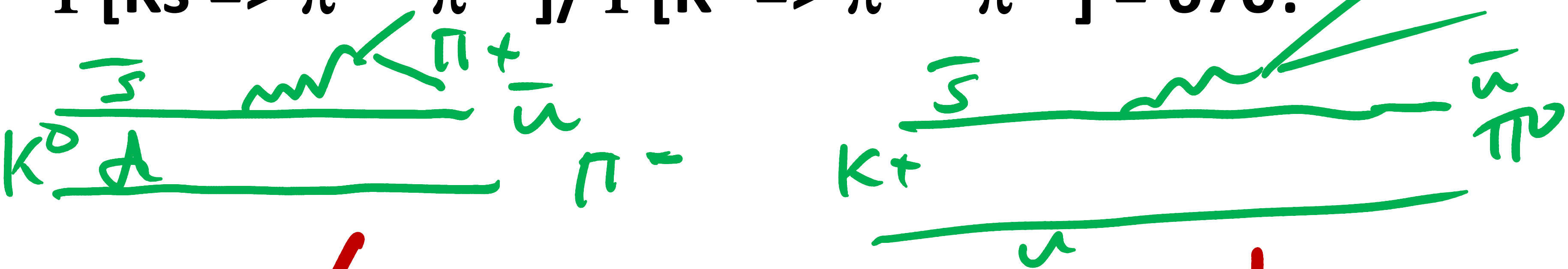
- Bit of ("textbook") History
- Bit of theory

- Lattice efforts: "Setbacks" aglore!
- Resolution: Dissecting its components
- A stupid oversight
- Some ramifications
- Outlook

HAPPY
VALENTINE'S
DAY !

What's the hooplah

- $\Gamma[K_s \Rightarrow \pi^+ \pi^-] / \Gamma[K^+ \Rightarrow \pi^+ \pi^0] = 670!$



- $\varepsilon' / \varepsilon$ ← Real Target

- **Pages & pages of K decays in PDG Br to 10^{-11} !!**

- **K-UT**
- **Haven't we tested the SM-CKM enough? ε ; m_ν**

Sad story of ε'/ε

[ACTUALLY all DIRECT CP]

- **~15% measurement obtained with heroic efforts (spanning over 20 years!) on both sides of the Atlantic at a cost very iikely well over \$200M:**

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}$$

- **Its WORTHLESS FOR NOW! It has 0 impact on theory**
- **ONLY LATTICE METHODS CAN CHANGE THIS FACT**
- **My entry into lattice methods ~1982 was motivated by wanting to reliably calculate ε'**

Reminder

$$|A_2| = 1.573(56) \times 10^{-8} GeV,$$

$$|A_0| = 3.3197(19) \times 10^{-7} GeV,$$

$$|A_0/A_2| = 21.13(77).$$

NOT $\sqrt{670}$

$\Delta I = 3/2$

$I = 2$

$I = 0, \ 1/2, 3/2$

$\Rightarrow 450$

ε' / ε: Direct CPV

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right].$$

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{\omega}{\sqrt{2}|\epsilon|}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

$$\begin{aligned} |\epsilon| &= 2.228(11) \times 10^{-3}, \\ \mathrm{Re}\,(\epsilon'/\epsilon) &= 1.65(26) \times 10^{-3}. \end{aligned}$$

Bit of history

- **Who proposed it**?

These predictions have not yet been tested because the data are too meager.

(*b*) ***Consequences of $\Delta I = \frac{1}{2}$ Rule for $K \to 2\pi$ Decays.*** The mode

$$K^+ \to \pi^+ + \pi^0 \tag{6.95}$$

constitutes $21.0 \pm 0.3\%$ (cf. Table 6.4) of the K^+ decays with a partial rate

$$\Gamma^+(K^+ \to \pi^+ + \pi^0) = (1.70 \pm 0.026) \times 10^7 \text{ sec}^{-1} \tag{6.96}$$

Two-pion decays account for essentially the whole of the K_S^0 total decay rate (cf. Table 6.4)

$$\Gamma(K_S^0 \to 2\pi) = (1.144 \pm 0.019) \times 10^{10} \text{ sec}^{-1} \tag{6.97}$$

The large difference between (6.96) and (6.97), namely

$$\frac{\Gamma^+(K^+ \to \pi^+ + \pi^0)}{\Gamma^0(K_S^0 \to 2\pi)} \simeq \frac{1}{670} \tag{6.98}$$

is one of the most interesting features of weak hadronic decays. To account for this large ratio, the $\Delta I = \frac{1}{2}$ rule was proposed [38]. The

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38. M. Gell-Mann and A. Pais, *Proc. Intern. Conf. High Energy Phys.*, Pergamon Press (1955); M. Gell-Mann and A. H. Rosenfeld, *Ann. Rev. Nucl. Sci.*, **7**, 407 (1957).

and Goldhaber, Phys. Rev. **101**, 1081 (1956).

[4] V. Fitch and R. Motley, Phys. Rev. **101**, 496 (1956); Alvarez Crawford, Good, and Stevenson, University of California Radiation Laboratory Report UCRL-3165 (unpublished).

[5] R. Dalitz, *Proceedings of the Fifth Annual Rochester Conference on High Energy Physics* (Interscience Publishers, Inc., New York, 1955), Feld, Odian, Ritson, and Wattenberg, Phys. Rev. (to be published); Orear, Harris, and Taylor (to be published).

[6] T. D. Lee and J. Orear, Phys. Rev. **100**, 932 (1955).

[7] *Note added in proof.*—T. D. Lee and C. N. Yang [Phys. Rev. (to be published)] have proposed that strong reactions be invariant with respect to a parity conjugation operator which operates only on particles of odd strangeness. In this scheme the production ratio of the τ and θ must be a constant under all conditions.

Heavy-Meson Decays and the Selection Rule $|\Delta I| \leqslant 1/2$

GREGOR WENTZEL

The Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received December 15, 1955)

THE validity of the selection rules,[1]

$$\Delta I_z = 0 \text{ for fast transitions,}$$

$$\Delta I_z = \pm\tfrac{1}{2} \text{ for slow transitions,}$$

is well established in the domain of hyperon and heavy-meson physics. Whether there exists an inde-

$$\tau^{+} \rightarrow \Big\{ 2\pi^0 + \pi^+ \quad (\text{“}\tau'\text{ decay”}).$$

The branching ratio τ/τ' has been observed to be as large as 4 (average values 4.1 and 4.6 are quoted in the literature[3]). Dalitz[4] has shown that, if the final three-pion state is an eigenstate of I, the branching ratio τ/τ' is $\frac{1}{4}$ for $I=3$, and 1 for $I=2$, whereas

$$1 \leqslant \tau/\tau' \leqslant 4 \quad \text{for } I=1. \tag{2}$$

Thus, the only single I value compatible with the observations is $I=1$. This may either mean that the τ meson has isotopic spin 1 and decays with isotopic spin conservation[4]; but this interpretation would reopen the question why the lifetime of the τ is so long. It is much more natural today to assume that the τ has isotopic spin $\frac{1}{2}$ and that the selection rule (1) is effective, leading to the same limitation for the branching ratio, *viz.* (2).

We observe further that the upper limit 4 for $\tau/\tau'[I=1]$ is obtained only if the state function of the three pions has a particularly simple symmetry property: it has the form $u(\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3)\chi(i_{1z}i_{2z}i_{3z})$ where each factor is totally symmetric in the three particles, namely u with respect to the three momentum vectors, and χ with respect to the three charge numbers. Specifically:

$$\chi = (15)^{-\frac{1}{2}}\{2[(\bar{1}11)+(1\bar{1}1)+(11\bar{1})] - [(100)+(010)+(001)]\}$$

(writing $\bar{1}$ for -1).

JUST A BIT OF THEORY

All CP violation in the Standard Model arises from a *single phase* in the (unitary) Cabibbo-Kobayashi-Maskawa mixing matrix

$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix} W^{+\mu}$$

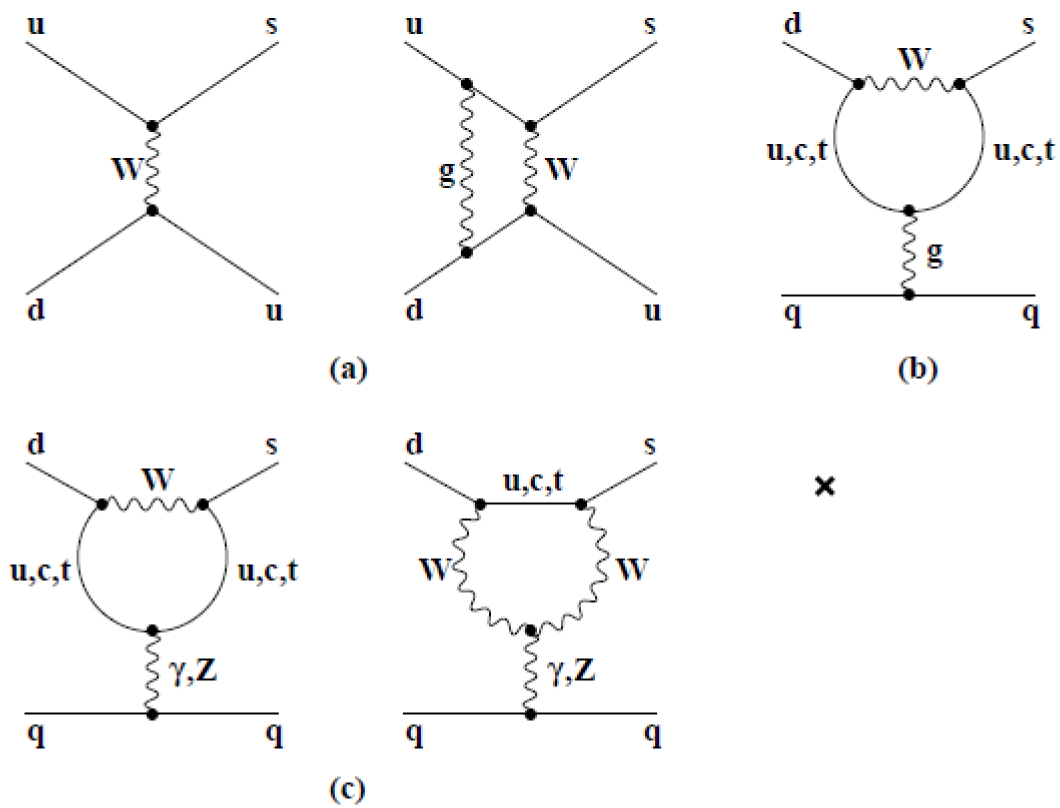
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\eta \neq 0$

$\boxed{\eta \neq 0 \leftrightarrow CP \text{ violation}}$

On the quark level, $K \to \pi\pi$ decays are mediated by strangeness changing $s \to d$ transitions ($\Delta S = 1$). In the Standard Model, typical Feynman diagrams are (a) current-current, (b) QCD penguin, and (c) Electroweak penguin:

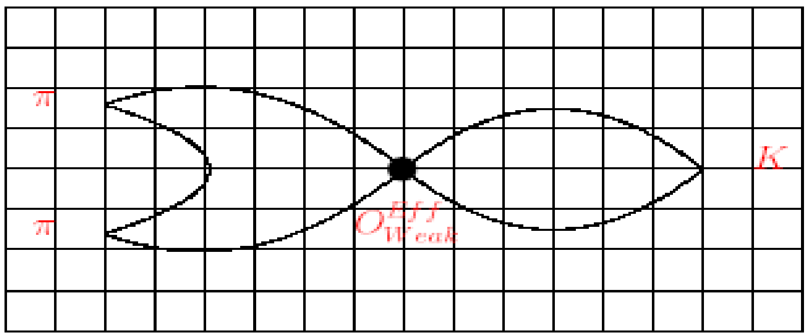


Technical problem: vastly different energy (distance) scales.

weak interactions: $\mu \sim M_W \approx 80$ GeV

strong interactions: $\mu \sim 1$ GeV.

Solution: operator product expansion (OPE) and renormalization group (RG). Integrate out short distance part perturbatively $\longrightarrow$ low energy effective theory. Effective Hamiltonian: linear combination of all local operators allowed by symmetries, coefficients given by underlying theory.



Expansion coefficients $\sim M_W^{-(d_i-4)}$, so only a few operators.

II. The $\Delta S = 1$ Effective Hamiltonian

The Standard Model Hamiltonian: a *short distance expansion* in terms of effective local **four-quark operators** $Q_i(\mu)$ (*c.f.* Fermi interaction) with **Wilson coefficients** $z_i(\mu)$ **and** $y_i(\mu)$.

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} y_i(\mu) \right] Q_i(\mu) \right\}$$

Both depend on an *arbitrary factorization scale* μ. Effective Hamiltonian is **independent** of this scale. Take μ low enough that non-perturbavtive calculations are practical ($1/a \ll M_W$), but high enough that continuum perturbation theory remains valid.

Key to the OPE is that the full amplitude is divided into low energy (hadronic matrix elements of Q_i) and high energy (Wilson coefficients) parts that can be computed separately.

$\Delta S = 1$ FOUR QUARK OPERATORS

$$
\begin{aligned}
Q_1(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\beta \\
Q_2(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\alpha \\
Q_{3,5}(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\beta \\
Q_{4,6}(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} \bar{q}_\beta \gamma_\mu (1 \mp \gamma_5) q_\alpha \\
Q_{7,9}(\mu) &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\beta \\
Q_{8,10}(\mu) &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{u,d,s,\ldots} e_q \bar{q}_\beta \gamma_\mu (1 \pm \gamma_5) q_\alpha \\
Q_{1c}(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\beta \\
Q_{2c}(\mu) &= \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \bar{c}_\beta \gamma_\mu (1-\gamma_5) c_\alpha
\end{aligned}
$$

NOTE: *active* charm and charm *integrated out* cases are being investigated; however, only charm-out is finished (at a lattice spacing of 2 GeV).

Note also: $SU(3)_L \times SU(3)_R$ very useful for classification of these

$\Delta S = 1$: 10 linearly *dependent* Q_i (12 if charm is active)

$SU_L(3) \times SU_R(3)$ chiral symmetry (L,R): (27,1), (8,1), (8,8) irr. rep's

Isospin symmetry: $\Delta I = 1/2$ and $3/2$ irr. rep's

Important examples:

$current - current$ (progenitor weak op.)

T

$$Q_2 = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \, \bar{u}_\beta \gamma_\mu (1-\gamma_5) u_\alpha$$

$$\{(8,1)\,1/2\} \, \{(27,1)\,1/2\} \, \{(27,1)\,3/2\}$$

$QCD\ Penguin$ (generated by QCD)

QCDP

$$Q_6 = \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{q=u,d,s} \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha$$

$$\{(8,1)\,1/2\}$$

$Electroweak\ Penguin$ ($\propto \alpha,\ m_t^2$)

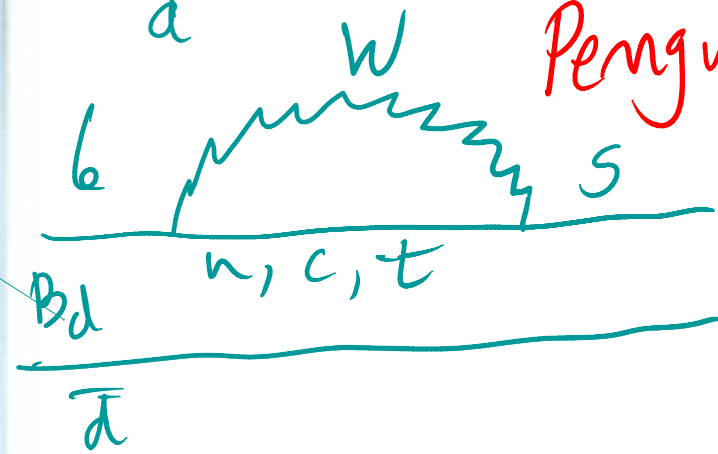
EWP

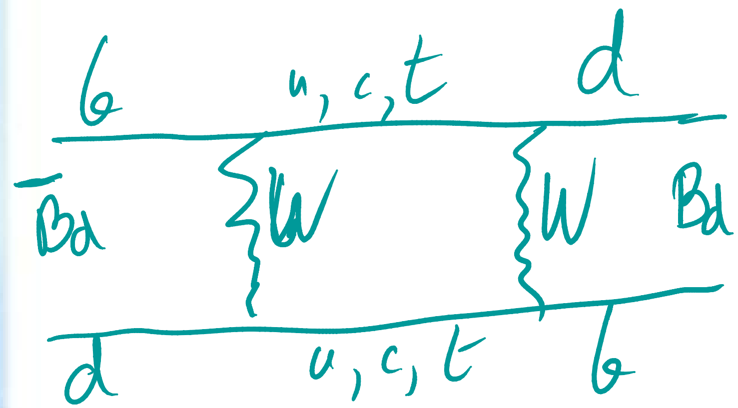
$$Q_8 = \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\beta \sum_{q=u,d,s} e_q \bar{q}_\beta \gamma_\mu (1+\gamma_5) q_\alpha$$

$$\{(8,8)\,1/2\} \, \{(8,8)\,3/2\}$$

TREE

Penguin





BOX

Operator mixing & Renormalization

LATTICE EFFORTS: "SETBACKS" AGLORE!

A sequence of many failed attempts each teaching us valuable lessons

Four stages

PhD

- **~1982 – 1994 .with Claude Bernard**

WF, ChPT, QA TD

- **~ 1995-1998 with Tom Blum**

DWF, ChPT, QA

- **1998---2008 RBC**

DWF, ChPT, QA $\rightarrow N_F=2$ C.C

J.L

- **2008 -> present RBC-UKQCD**

S.L

DWF, LL, $N_F=2+1$ (Full QCD)

QL

Computing power & accuracy

- **~1984 : 6X6X6X17, QA: $\delta(B_K)$ ~ O(100%),**

K=>$\pi\pi$ $\boldsymbol{not\ doable, because\ chial\ sym}$
$\boldsymbol{of\ WF\ found}$ to be horrible

- **~ 2012 : 32X32X32X64X16, full QCD**

i.e. more than a factor ~ 10^7 more since '84:

- **$\delta(B_K)$ < 3%; δ (Re & Im A_2) < 20%; δ (Re A_0) <30%**
- **Note: actually we could easily use another 10-100**

UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections

to Weak Matrix Elements –

The Delta-I Equals One-Half Rule

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Physics

by

Terrence Arthur James Draper

1984

PHYSICAL REVIEW D VOLUME 32, NUMBER 9 1 NOVEMBER 1985

Application of chiral perturbation theory to $K \to 2\pi$ decays

Claude Bernard, Terrence Draper,* and A. Soni
Department of Physics, University of California, Los Angeles, California 90024

H. David Politzer and Mark B. Wise
Department of Physics, California Institute of Technology, Pasadena, California 91125

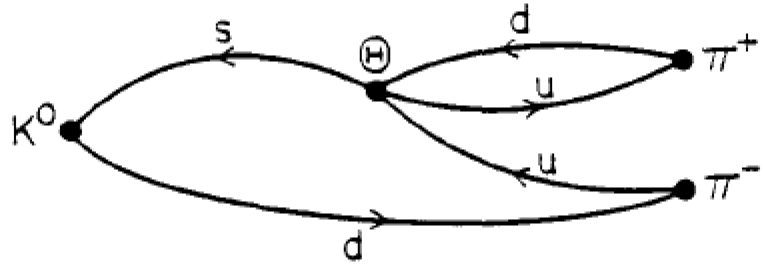
BDSPW

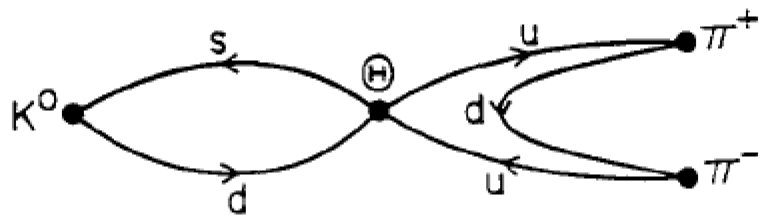
(Received 3 December 1984)

Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

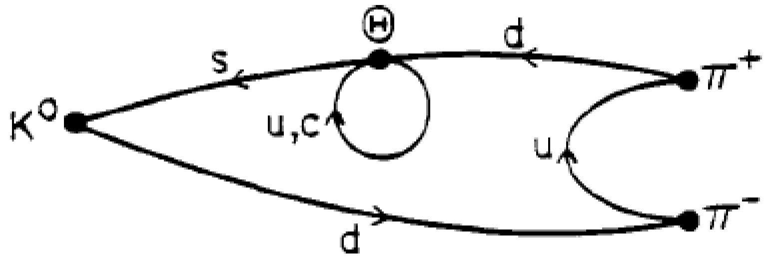
LO ChPT

J.LAIHO & AS ~2004 NLO



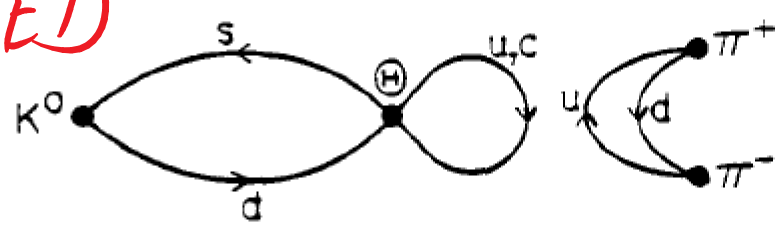


EYE-diagram



VERY HARD

DISCONNECTED DIAGRAM



SUPER HARD

Following BDSPW '84

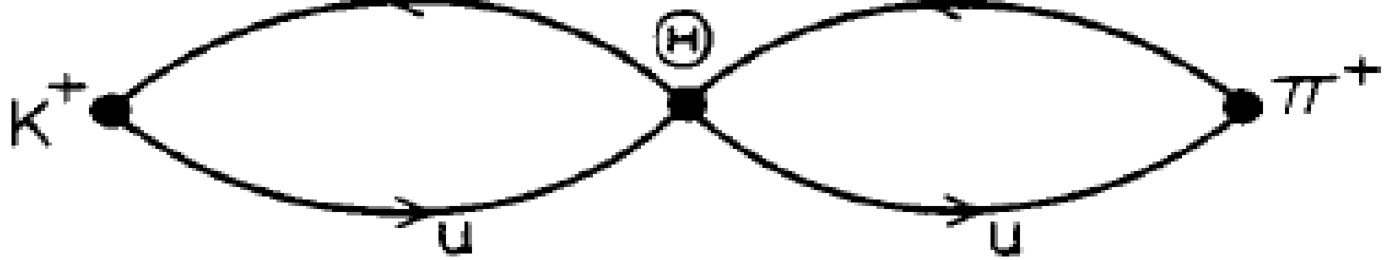
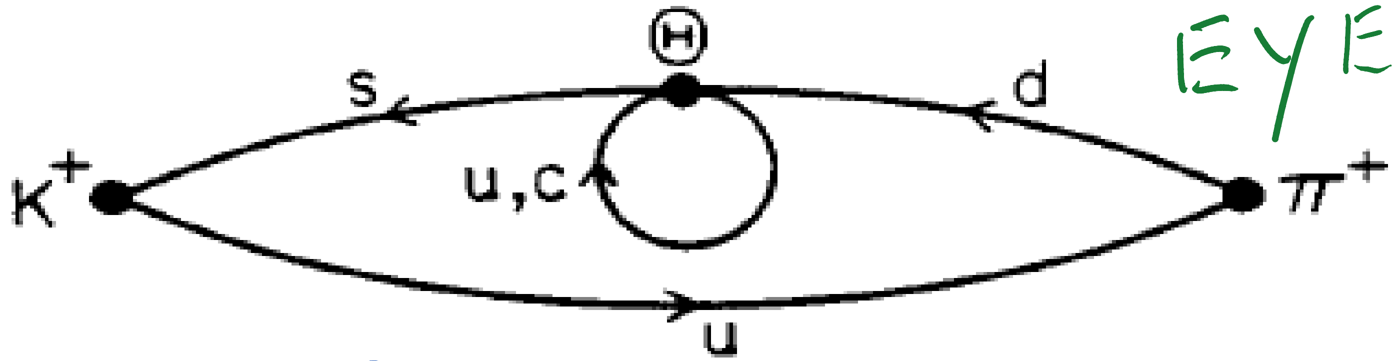
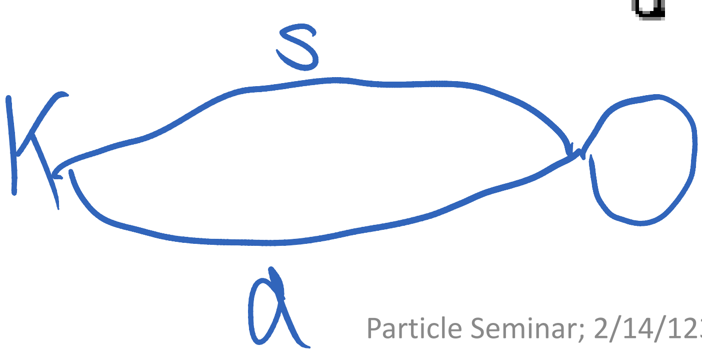


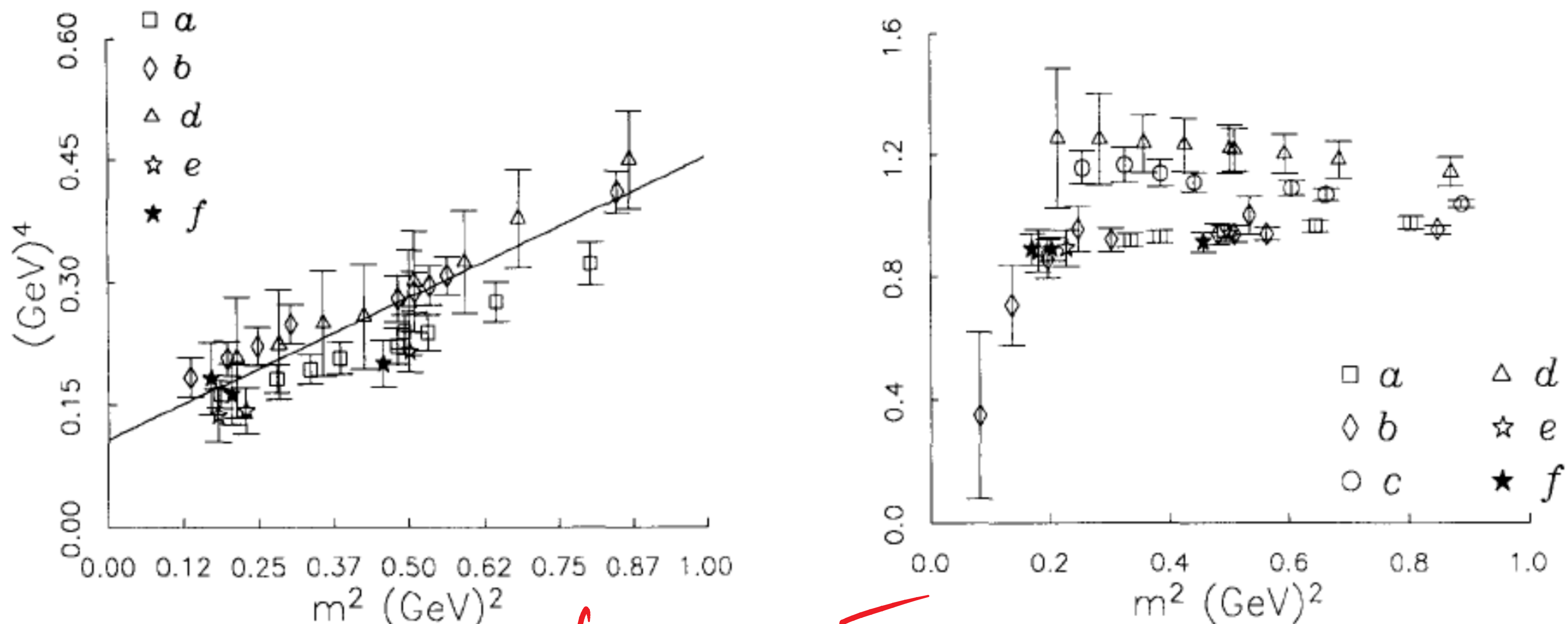
FIG 8



EYE



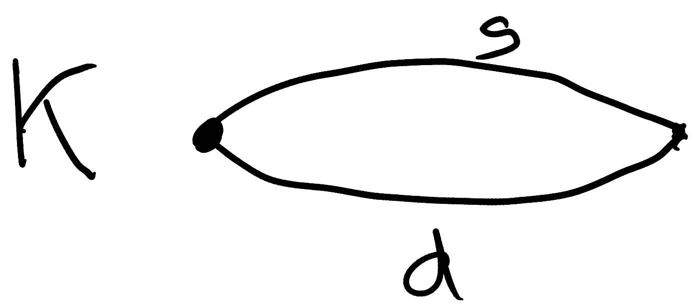
Vacuum

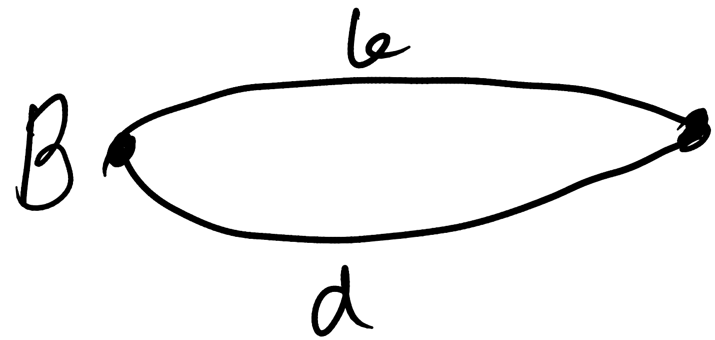


χS violation by $K-\bar{K} \Rightarrow$ FINE TUNING PROBLEM

Additional difficulties from BK to ReA0

- More diagrams
- Additonal diagrams are a lot harder
- Ops 1 => 10
- Deadily operator mixing
- Lack of chiral symmetry <=> fine-tuning problem
- 4-pt function and not 3
- Maiani-Test





Digression: Initiate Heavy-light (B) WME program-> important application to UT constraints

RATIONALE: Chiral symmetry less of a concern, utilize in a profitable way stored away light quark propagators towards exploratory application to important physical observables

Lattice calculation of weak amplitudes of D and B mesons

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(Received 27 June 1988)

A lattice calculation of the pseudoscalar decay constants and the [Δ(flavor)=2] mixing matrix elements for D and B mesons is reported. Calculations are done (in the quenched approximation) with $\beta=6.1$ on a $12^3\times 33$ lattice; results from $\beta=5.7$ on a $16^3\times 25$ lattice contribute to our estimate of the systematic errors. An extrapolation to large meson mass is required in order to treat the B meson. We find $f_{bd}=105\pm17\pm30$ MeV, $f_{bs}=155\pm31\pm48$ MeV, $f_{cd}=174\pm26\pm46$ MeV, $f_{cs}=234\pm46\pm55$ MeV (with normalization such that $f_\pi=132$ MeV). The ratios of these quantities have considerably smaller errors: $f_{bd}/f_{cd}=0.60\pm0.01\pm0.03$, $f_{bs}/f_{cs}=0.66\pm0.004\pm0.09$, $f_{bs}/f_{bd}=1.47\ \pm0.07\pm0.30$, and $f_{cs}/f_{cd}=1.35\pm0.07\pm0.21$. For the lattice "$B$ parameters" we find $B_{LL}^{\rm latt}=1.01\pm0.06\pm0.18$ and $B_{LR}^{\rm latt}=1.16\pm0.01\pm0.11$ for the *bd* system, with quite similar values for the *cu* and *bs* systems. These B parameters are defined slightly differently than in the continuum and are effectively renormalization-group invariant. The first error in each of our results is statistical; the second is an estimate of the systematic errors due to scale-breaking, finite-size, extrapolation and operator-renormalization effects.

1st application to B-physics
COARSE Lattice; Large syst. error

Lattice computation of the decay constants of *B* and *D* mesons

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(Received 1 July 1993)

Semileptonic decays on the lattice: The exclusive 0^- to 0^- case

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(Received 21 December 1990)

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SU(3) flavor breaking in hadronic matrix elements for B-$\overline{B}$ oscillations

Later Δm_s
CDF, DØ

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(Received 28 January 1998; published 5 May 1998)

Severe limitation of Wilson Fermions for application to light-light physics, e.g. Kaon mixing, ***K->$\pi\pi$*****: A serious fine tuning problem**

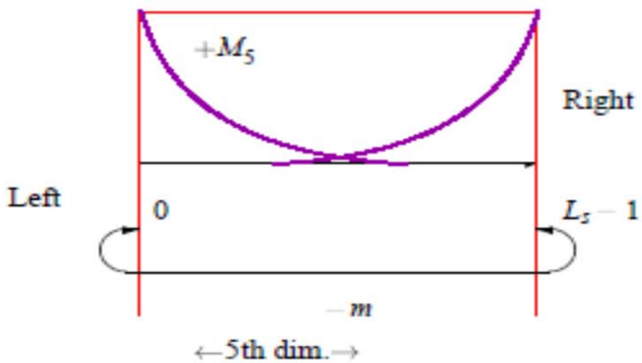
$\Rightarrow$ MOVE to DOMAIN WALL FERMION with T Blum

~95

EXACT CHIRAL SYMMETRY ON THE LATTICE

Conventional fermions do not preserve chiral-flavor symmetry on the lattice (Nielsen - Ninomiya Theorem) $\Rightarrow \Delta S = 1, \Delta I = 1/2$ case mixing with lower dim. (power-divergent) operators & or mixing of 4-quark operators with wrong chirality ones makes lattice study of $K - \pi$ physics virtually impossible.

Domain Wall Fermions (Kaplan, Shamir, Narayanan and Neuberger)



Practical viability of DWF for QCD demonstrated (96-97) Tom Blum & A. S.
Chiral symmetry on the lattice, $a \neq 0$! Huge improvement
$\Rightarrow$ Now widespread use at BNL and elsewhere

PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

QCD with domain wall quarks

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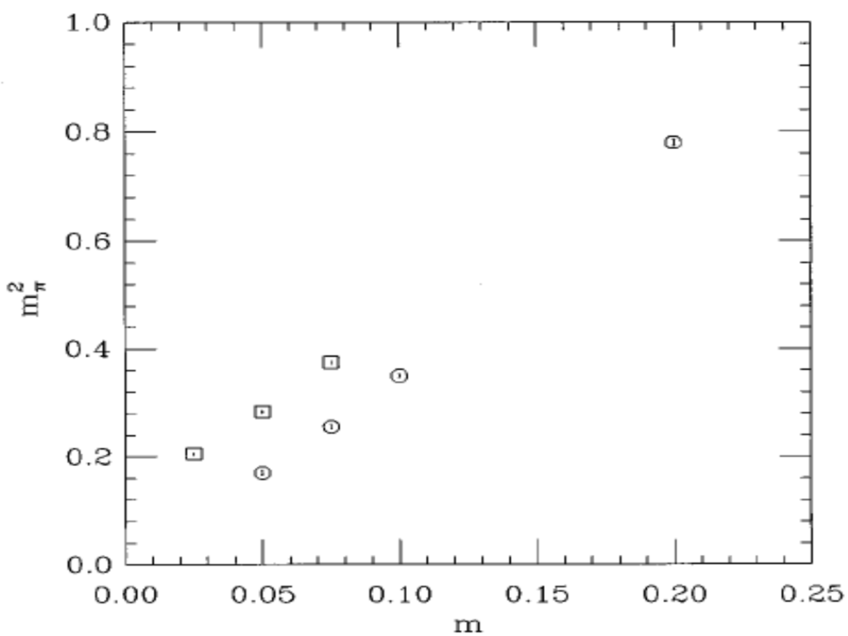
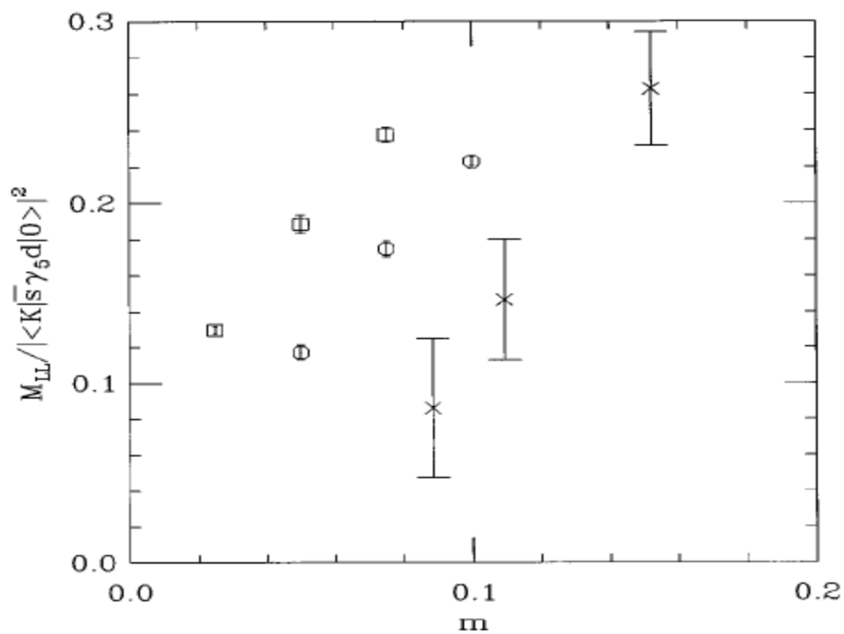
(Received 27 November 1996)

We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\overline{K_0}$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s = 10$. [S0556-2821(97)00113-6]





K→2π Via ChPT with DWQ in Quench Approx

PHYSICAL REVIEW D **68**, 114506 (2003)

Kaon matrix elements and *CP* violation from quenched lattice QCD: The 3-flavor case

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(Received 19 July 2002; published 30 December 2003)

C ALSO CP-PACS

We report the results of a calculation of the $K \to \pi\pi$ matrix elements relevant for the $\Delta I = 1/2$ rule and ϵ'/ϵ in quenched lattice QCD using domain wall fermions at a fixed lattice spacing $a^{-1} \sim 2$ GeV. Working in the three-quark effective theory, where only the u, d, and s quarks enter and which is known perturbatively to next-to-leading order, we calculate the lattice $K \to \pi$ and $K \to |0\rangle$ matrix elements of dimension six, four-fermion operators. Through lowest order chiral perturbation theory these yield $K \to \pi\pi$ matrix elements, which we then normalize to continuum values through a nonperturbative renormalization technique. For the ratio of isospin amplitudes $|A_0|/|A_2|$ we find a value of 25.3 ± 1.8 (statistical error only) compared to the experimental value of 22.2, with individual isospin amplitudes 10%–20% below the experimental values. For ϵ'/ϵ, using known central values for standard model parameters, we calculate $(-4.0 \pm 2.3) \times 10^{-4}$ (statistical error only) compared to the current experimental average of $(17.2 \pm 1.8) \times 10^{-4}$. Because we find a large cancellation between the $I=0$ and $I=2$ contributions to ϵ'/ϵ, the result may be very sensitive to the approximations employed. Among these are the use of quenched QCD, lowest order chiral perturbation theory, and continuum perturbation theory below 1.3 GeV. We also calculate the kaon B parameter B_K and find $B_{K,\overline{\rm MS}}(2\ {\rm GeV}) = 0.532(11)$. Although currently unable to give a reliable systematic error, we have control over statistical errors and more simulations will yield information about the effects of the approximations on this first-principles determination of these important quantities.

RBC Collaboration

QCDSP
~98→~'05 1TF

Kaon Matrix Elements and CP Violation from Quenched Lattice QCD

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Advisor: Professor Robert Mawhinney

Submitted in partial fulfillment of the

requirements for the degree

of Doctor of Philosophy

in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2002

TABLE XXXIX. The contribution in GeV from the renormalized continuum operator $Q_{i,\mathrm{cont}}$ to the real parts of $\langle(\pi\pi)_I|-i\mathcal{H}^{(\Delta S=1)}|K^0\rangle$ for $\mu=1.51$ GeV. The central values for the standard model parameters given in Table XXXVII have been used.

	Real A_0		Real A_2	
i	choice 1	choice 2	choice 1	choice 2
1	$3.02(68)\times10^{-8}$	$4.28(97)\times10^{-8}$	$-4.11(18)\times10^{-9}$	$-4.82(22)\times10^{-9}$
2	$2.00(18)\times10^{-7}$	$2.83(25)\times10^{-7}$	$1.392(62)\times10^{-8}$	$1.635(73)\times10^{-8}$
3	$1.4(29)\times10^{-10}$	$2.0(41)\times10^{-10}$	0.0	0.0
4	$-3.80(84)\times10^{-9}$	$-5.4(12)\times10^{-9}$	0.0	0.0
5	$-6.9(29)\times10^{-10}$	$-9.8(41)\times10^{-10}$	0.0	0.0
6	$4.99(77)\times10^{-9}$	$7.1(11)\times10^{-9}$	0.0	0.0
7	$4.04(21)\times10^{-11}$	$8.00(42)\times10^{-11}$	$2.86(15)\times10^{-11}$	$3.63(19)\times10^{-11}$
8	$-5.74(32)\times10^{-11}$	$-1.137(63)\times10^{-10}$	$-4.06(22)\times10^{-11}$	$-5.15(28)\times10^{-11}$
9	$-3.91(39)\times10^{-12}$	$-5.54(56)\times10^{-12}$	$4.69(21)\times10^{-13}$	$5.51(25)\times10^{-13}$
10	$2.27(41)\times10^{-12}$	$3.23(59)\times10^{-12}$	$3.70(17)\times10^{-13}$	$4.35(20)\times10^{-13}$

QA ; ChPT

TABLE XLIX. Our final values for physical quantities using one-loop full QCD extrapolations to the physical kaon mass (choice 2) and a value of $\mu = 2.13$ GeV for the matching between the lattice and continuum. The errors for our calculation are statistical only.

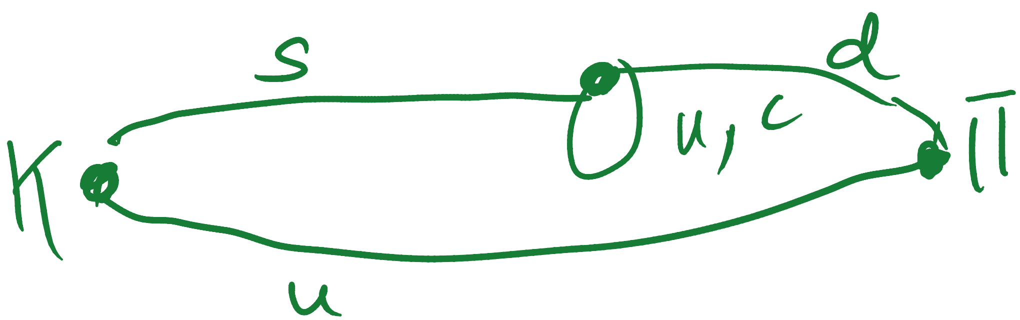
Quantity	Experiment	This calculation (statistical errors only)
Re A_0(GeV)	3.33×10^{-7}	$(2.96 \pm 0.17) \times 10^{-7}$
Re A_2(GeV)	1.50×10^{-8}	$(1.172 \pm 0.053) \times 10^{-8}$
ω^{-1}	22.2	(25.3 ± 1.8)
Re(ϵ'/ϵ)	$(15.3 \pm 2.6) \times 10^{-4}$(NA 48)	$(-4.0 \pm 2.3) \times 10^{-4}$
	$(20.7 \pm 2.8) \times 10^{-4}$(KTEV)	

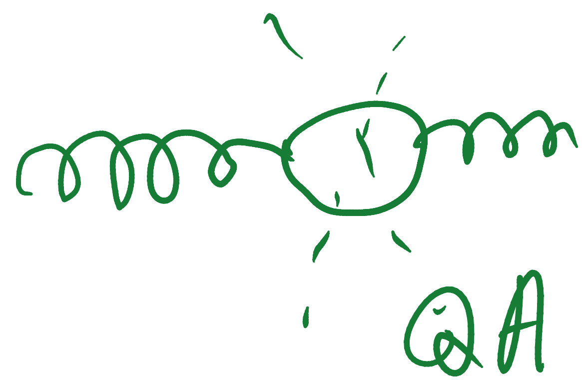
$$Q_c = (\bar{s}_\alpha d_\beta)_{V-A} \sum (\bar{q}_\beta q_\alpha)_{V+A} \qquad (8,1)$$

$$Q_8 = (\bar{s}_\alpha d_\beta)_{V-A} \sum e_q (\bar{q}_\beta q_\alpha) \qquad (8,8)$$

Golterman & Pallante; JHEP'01

Laiho et al [RBC] PRD'06





VERY SERIOUS QUENCH PATHOLOGY

Framework for improved lattice calculations of epsilon'/epsilon

by *Laiho, Jack*, Ph.D., Princeton University, 2004, 188 pages; AAT 3142342

Abstract (Summary)

In this thesis we show that it is possible to construct ε'/ε to NLO using both full and partially quenched chiral perturbation theory (PQChPT) from amplitudes that are computable using numerical lattice gauge theory. We find that the electro-weak penguin ($\Delta I = 3/2$ and $1/2$) contributions to ε'/ε in PQChPT can be determined to NLO using only degenerate ($m_K = m_\pi$) K [arrow right] π computations without momentum insertion. All one-loop formulas needed to extract the necessary NLO constants from the lattice are presented in this work. Issues pertaining to power divergent contributions, originating from mixing with lower dimensional operators in a lattice regularization, are addressed.

In embedding the QCD penguin left-right operator onto PQChPT an ambiguity arises when the number of light sea quarks is not the physical value of three, as first emphasized by Golterman and Pallante. In the quenched theory they have pointed out that there are additional effective operators that appear in the quenched chiral perturbation theory needed to make contact with K [arrow right] $\pi\pi$ amplitudes at physical kinematics. They have also proposed a method for determining the leading order low-energy constant, [Special characters omitted.] , associated with the new operators. We show that their method has difficulties due to power divergent contributions and propose a new method to obtain this constant from the lattice which does not suffer from this problem. Using this alternative method, we obtain [Special characters omitted.] , and show that our value implies a large ambiguity in the quenched contribution of Q_6 to ε'/ε.

Indexing (document details)

Advisor:	Marlow, Dan, Soni, Amarjit
School:	Princeton University
School Location:	United States -- New Jersey
Keyword(s):	Lattice calculations, Epsilon', Chiral perturbation theory
Source:	DAI-B 65/07, p. 3515, Jan 2005
Source type:	Dissertation
Subjects:	Particle physics

$K \to \pi\pi$

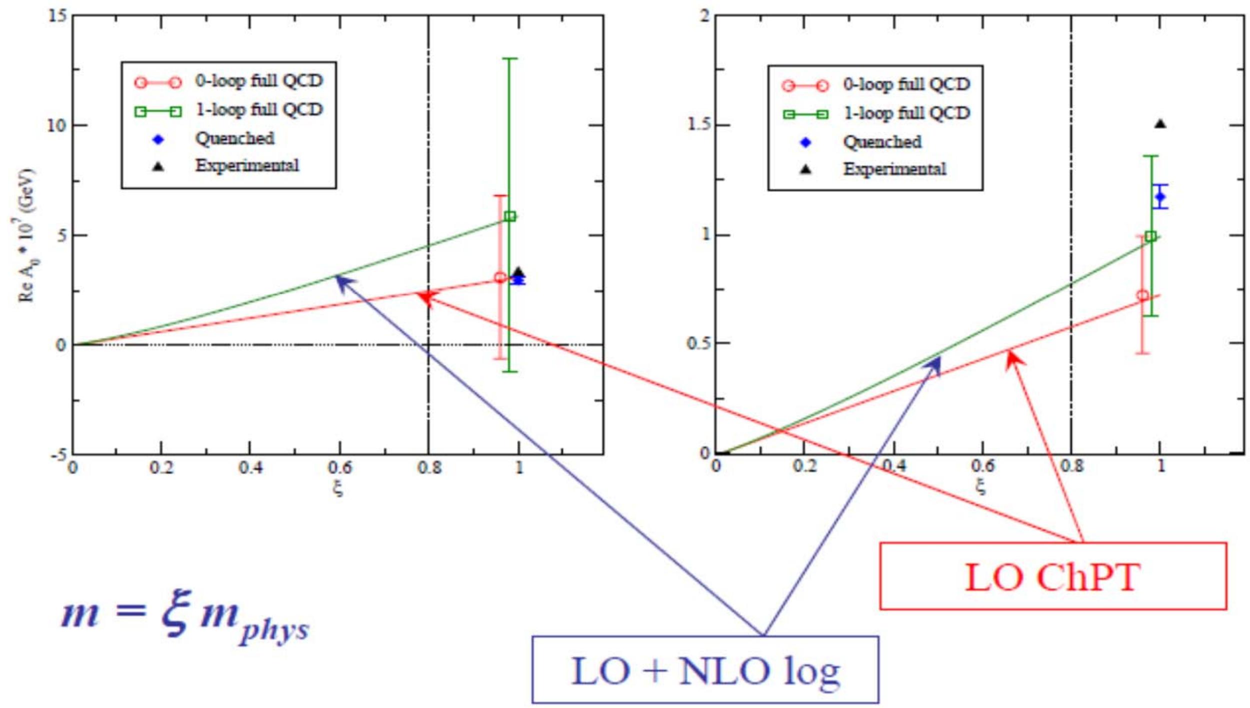
ChPT

CNLO

Estimate $K \to \pi\pi$ amplitudes (con't)

ChPT BUT WITH FULL QD

Re A_0 Re A_2



$$m = \xi\, m_{phys}$$

Lattice 2008, July 14, 2008 (21)

Lattice 2008, July 14, 2

Sam Li (CU thesis)
N Christ, LAT'08

QCDOC 10 Tf N05 — 11

Kaon Matrix Elements and CP Violation from Lattice QCD with 2+1 Flavors of Domain Wall Fermions

Shu Li

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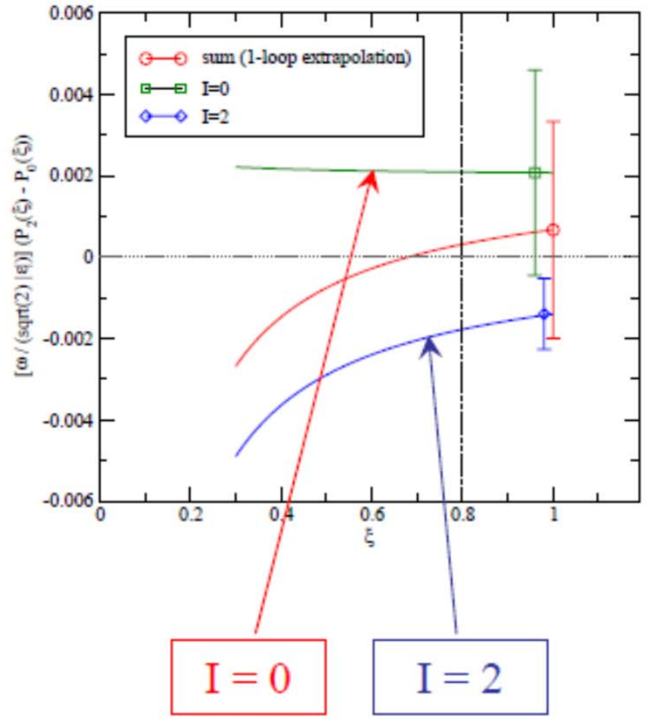
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

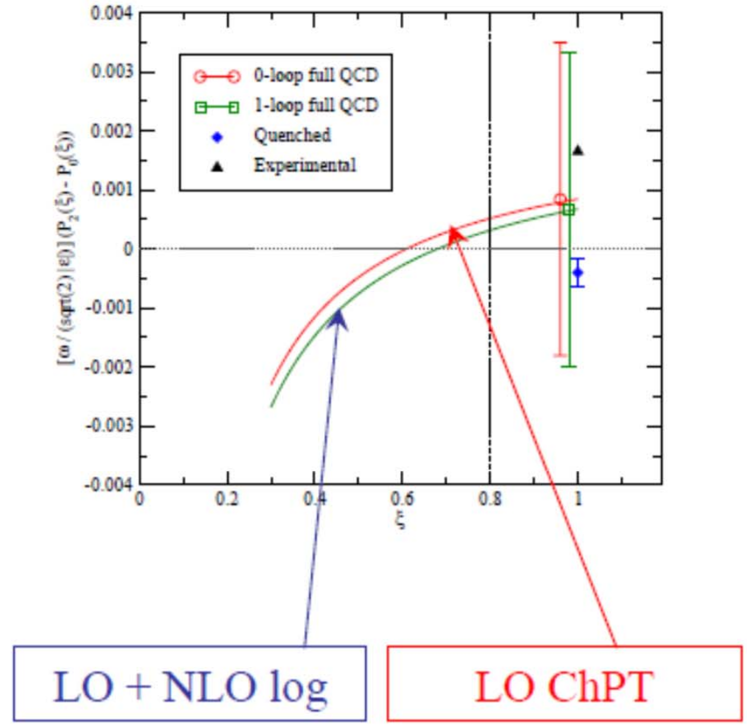
COLUMBIA UNIVERSITY
2008

Estimate $K \rightarrow \pi\pi$ amplitudes (con't)

Re ε′/ε



Re ε′/ε



$m = \xi\, m_{phys}$

Lattice 2008, July 14, 2008 (22)

N.Christ @ LAT'08

Table 49: The contribution of each operator to the physical quantity ReA_0 and ReA_2 at the energy scale $\mu = 1.92$ GeV. The unit of each number in this table is GeV. The values in the columns with the superscript "(1)" are extrapolated with zeroth order ChPT formula, and those in the columns with superscript "(2)" have the contribution of the chiral logarithms included. The errors are statistical only.

i	$\mathrm{Re}(A_0)^{(1)}$	$\mathrm{Re}(A_0)^{(2)}$	$\mathrm{Re}(A_2)^{(1)}$	$\mathrm{Re}(A_2)^{(2)}$
1	$4.19(96) \times 10^{-8}$	$7.9(1.8) \times 10^{-8}$	$-2.20(22) \times 10^{-9}$	$-3.02(30) \times 10^{-9}$
2	$2.77(56) \times 10^{-7}$	$5.3(1.1) \times 10^{-7}$	$9.74(97) \times 10^{-9}$	$1.34(13) \times 10^{-8}$
3	$-1.8(6.1) \times 10^{-12}$	$-0.3(1.2) \times 10^{-11}$	0.0	0.0
4	$1.24(34) \times 10^{-9}$	$2.36(64) \times 10^{-9}$	0.0	0.0
5	$-3.12(61) \times 10^{-10}$	$-5.9(1.2) \times 10^{-10}$	0.0	0.0
6	$-1.56(28) \times 10^{-9}$	$-2.96(53) \times 10^{-9}$	0.0	0.0
7	$2.47(38) \times 10^{-11}$	$7.7(1.2) \times 10^{-11}$	$1.75(27) \times 10^{-11}$	$2.75(43) \times 10^{-11}$
8	$-1.60(24) \times 10^{-10}$	$-4.96(74) \times 10^{-10}$	$-1.13(17) \times 10^{-10}$	$-1.78(26) \times 10^{-10}$
9	$-7.8(1.6) \times 10^{-13}$	$-1.47(30) \times 10^{-12}$	$4.15(41) \times 10^{-14}$	$5.71(57) \times 10^{-14}$
10	$1.13(25) \times 10^{-11}$	$2.15(48) \times 10^{-11}$	$8.66(86) \times 10^{-13}$	$1.19(12) \times 10^{-12}$

Conclusion

Quantity	This analysis	Quenched	Experiment
$\mathrm{Re}A_0$ (GeV)	$4.5(11)(53) \times 10^{-7}$	$2.96(17) \times 10^{-7}$	3.33×10^{-7}
$\mathrm{Re}A_2$ (GeV)	$8.57(99)(300) \times 10^{-9}$	$1.172(53) \times 10^{-8}$	1.50×10^{-8}
$\mathrm{Im}A_0$ (GeV)	$-6.5(18)(77) \times 10^{-11}$	$-2.35(40) \times 10^{-11}$	
$\mathrm{Im}A_2$ (GeV)	$-7.9(16)(39) \times 10^{-13}$	$-1.264(72) \times 10^{-12}$	
$1/\omega$	$50(13)(62)$	$25.3(1.8)$	**22.2**
$\mathrm{Re}(\epsilon'/\epsilon)$	$7.6(68)(256) \times 10^{-4}$	$-4.0(2.3) \times 10^{-4}$	1.65×10^{-3}

$\Rightarrow$

- ChPT approach to $K \to \pi\pi$ faces severe difficulties.
- RBC/UKQCD studying physical $\pi\pi$ final states.
- DWF on coarse lattices and large volumes: $4 \to 5$ fm?
- Vranas auxiliary determinant (Renfrew talk on Wed.)

LARGE SYSTEMATIC
errors DUE ChPT

Lattice

N. Christ @LAT08

RBC-UKQCD mK too heavy for ChPT

$\Rightarrow$ K SU(2) ChPT

Direct K-> $\pi\pi$ (a la Lellouch-Luscher), using finite volume correlation functions, [i.e. w/o ChPT] RBC initiates around 2006

PRL 108, 141601 (2012) PHYSICAL REVIEW LETTERS week ending 6 APRIL 2012

$K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD

T. Blum,[1] P. A. Boyle,[2] N.H. Christ,[3] N. Garron,[2] E. Goode,[4] T. Izubuchi,[5,6] C. Jung,[5] C. Kelly,[3] C. Lehner,[6] M. Lightman,[3,7] Q. Liu,[3] A. T. Lytle,[4] R. D. Mawhinney,[3] C. T. Sachrajda,[4] A. Soni,[5] and C. Sturm[8]

(RBC and UKQCD Collaborations)

DWQ + Full QCD + Physical Kinematics!

$m_K \sim 511$ MeV, $m_\pi \sim 142$ MeV ✓

$\Delta I = 3/2$ $K \longrightarrow \pi\pi$ Decays Using Lattice QCD with Domain Wall Fermions

Matthew Lightman

Advisor: Professor Norman Christ

Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY
2011

ESSENTIALLY Physical K, π and Physical Kinematics for I = 2

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

	m_{K^+}	m_{π^+}	$E_{\pi\pi}$	$m_K - E_{\pi\pi}$
Simulated	511.3(3.9)	142.9(1.1)	492.6(5.5)	18.7(4.8)
Physical	493.677(0.016)	139.57018(0.00035)	m_K	0

Table 1: Kaon and pion masses and the two-pion energy $E_{\pi\pi}$ in the simulation together with the corresponding physical values. The results are given in MeV.

RBC-UKQCD, arXiv:1111.1699, PRL 2012

$$\mathrm{Re}\,A_2 = (1.436 \pm 0.062_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}}) \times 10^{-8}\,\mathrm{GeV}$$

$$\mathrm{Im}\,A_2 = -(6.83 \pm 0.51_{\mathrm{stat}} \pm 1.30_{\mathrm{syst}}) \times 10^{-13}\,\mathrm{GeV}.$$

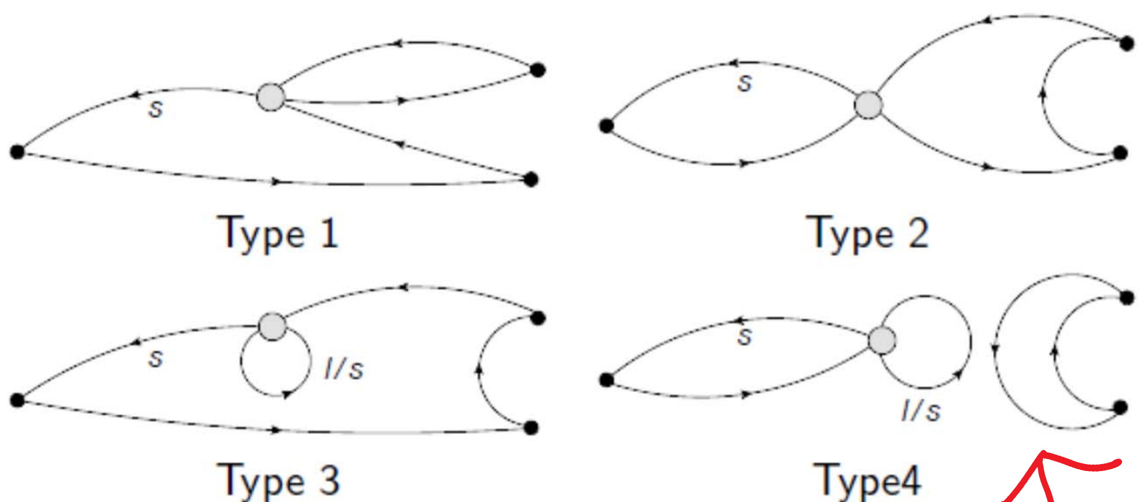
EXPT Re A_2 $\left\{\begin{array}{l} 1.479(4)\,K^+ \\ 1.573(57)\,K_S \end{array}\right.$

ERROR Re, Im $A_2 \sim 20\%$

I=0 Channel

Difficulties of a direct Lattice calculation

Disconnected (Vacuum) Graphs!



$$Type4 = \langle K(0)O(t)\pi\pi(t_\pi)\rangle - \langle K(0)O(t)\rangle\,\langle\pi\pi(t_\pi)\rangle$$

extremely difficult

Qi Liu, LAT'11

K^0 to $\pi\pi$ decay Results

Qi Liu @ LAT'11

Preliminary Results [2]:

We take the kinematics point $m_k \approx 2m_\pi$, therefore, the decay is a little bit off shell because of $\pi - \pi$ interaction. Real part ($\times 10^{-8}$ GeV), Imaginary part ($\times 10^{-12}$GeV) .

m_π	$\mathrm{Re}(A_0)$	$\mathrm{Re}(A_0')$	$\mathrm{Im}(A_0)$	$\mathrm{Im}(A_0')$	$\mathrm{Re}(A_2)$	$\mathrm{Im}(A_2)$
329.3	36.8(6.6)	27.7(1.1)	-31(23)	-34.9(21)	2.663(19)	-0.6527(43)
421.4	45(10)	48.8(24)	-41(26)	-74.6(47)	4.911(31)	-0.5502(40)

16^3

800 gc

24^3

789 c

$\mathrm{Re}\, A_0^{expt} = 33 \times 10^{-8}$

78 gc -> 140 around 9/15/11 END of QCDOCawaiting QCDCQ to accumulate more data

Kaon to Two Pions decays from Lattice QCD: $\Delta I = 1/2$ rule and CP violation

Qi Liu

Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2012

Organization

RBC-UKQCD

- <u>BNL HEP Theory</u>
  M. Creutz, TI, C. Jung*,
  A. Soni, R. Van de Water,
  O. Witzel*,
  R. Arthur, T. Kawanai¥, T. Misumi¥
  (* SciDAC, ¥ JSPS)
- RIKEN BNL Columbia (RBC) Collaboration (1998-)
  - RIKEN-BNL Research Center
    1.5 fellows, 2 PostDocs,
    3 long-term visiting scientists
  - Columbia University
    University of Connecticut
    2 faculties, 2 PostDoc,
    8 Students
  - University of Connecticut
    1 faculties, 2 PostDoc, 2 Students

  Harvard, Yale,
  Virginia (Google), Regensburg

16 current students,
~20 PhD theses since 2005

- + UKQCD Collaboration (2005-)
  - Univ. of Edinburgh
    5 faculties, 1 fellows, 1 staff,
    2 PostDocs, 3 students
  - Univ. of Southampton
    2 faculties, 1 Postdoc, 2 students

  CERN, Julich
- + JLQCD (planned since 2010)
  - KEK, Tsukuba & Osaka Univ

(# of personnel: accumulation of last 3 years
# of PhD thesis: accumulation of last 5 years)

T Izubuchi

Emerging understanding of the $\Delta I = 1/2$ Rule from Lattice QCD

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C. Lehner,[5] Q. Liu,[2] A.T. Lytle,[4] C.T. Sachrajda,[4] A. Soni,[6] and D. Zhang[2]
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There has been much speculation as to the origin of the $\Delta I = 1/2$ rule ($\mathrm{Re}A_0/\mathrm{Re}A_2 \simeq 22.5$). We find that the two dominant contributions to the $\Delta I = 3/2$, $K \to \pi\pi$ correlation functions have opposite signs leading to a significant cancellation. This partial cancellation occurs in our computation of $\mathrm{Re}A_2$ with physical quark masses and kinematics (where we reproduce the experimental value of A_2) and also for heavier pions at threshold. For $\mathrm{Re}A_0$, although we do not have results at physical kinematics, we do have results for pions at zero-momentum with $m_\pi \simeq 420\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 9.1(2.1)$) and $m_\pi \simeq 330\,\mathrm{MeV}$ ($\mathrm{Re}A_0/\mathrm{Re}A_2 = 12.0(1.7)$). The contributions which partially cancel in $\mathrm{Re}A_2$ are also the largest ones in $\mathrm{Re}A_0$, but now they have the same sign and so enhance this amplitude. The emerging explanation of the $\Delta I = 1/2$ rule is a combination of the perturbative running to scales of $O(2\,\mathrm{GeV})$, a relative suppression of $\mathrm{Re}A_2$ through the cancellation of the two dominant contributions and the corresponding enhancement of $\mathrm{Re}A_0$. QCD and EWP penguin operators make only very small contributions at such scales.

A SURPRISE FINDING:
SIGNFICANT SUPPRESSED Re A2!

Vacuum saturation

The discussion of direct calculations of the nonleptonic amplitudes is beyond the scope of this book. Suffice it to say that no treatment is presently adequate. Let us give the simplest estimate, called *vacuum saturation*, as a convenient benchmark with which to compare the theory. For simplicity we consider only O_1 (the largest $\Delta I = 1/2$ operator) and O_4 (the $\Delta I = 3/2$ operator),

$$\mathcal{H}_W \simeq \frac{G_F}{2\sqrt{2}} V_{ud}^* V_{us} (c_1 O_1 + c_4 O_4) \quad , \tag{4.16}$$

with $c_1 \simeq 1.9$ and $c_4 \simeq 0.5$. The vacuum saturation approximation consists of inserting the vacuum intermediate state between the two currents in any way possible, *e.g.*

$$\begin{aligned} &\langle \pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)\,|\bar{d}\gamma^\mu(1+\gamma_5)\,u\bar{u}\gamma^\mu(1+\gamma_5)\,s|\,\bar{K}^0(\mathbf{k})\rangle \\ &\quad = \langle \pi^-(\mathbf{p}_-)\,|\bar{d}\gamma^\mu\gamma_5 u|\,0\rangle\,\langle \pi^+(\mathbf{p}_+)\,|\bar{u}\gamma^\mu s|\,\bar{K}^0(\mathbf{k})\rangle \\ &\qquad + \langle \pi^+(\mathbf{p}_+)\pi^-(\mathbf{p}_-)\,|\bar{u}_\beta\gamma^\mu u_\alpha|\,0\rangle\,\langle 0\,|\bar{d}_\alpha\gamma^\mu\gamma_5 s_\beta|\,\bar{K}^0(\mathbf{k})\rangle \\ &\quad = -i\sqrt{2}\,F_\pi f_+ p_-^\mu\,(k+p_+)_\mu - \frac{i}{3}\sqrt{2}\,F_K f_+ k_\mu\,(p_- - p_+)^\mu \quad . \end{aligned} \tag{4.17}$$

In obtaining this result the Fierz rearrangement property

$$\bar{d}_\alpha\gamma^\mu(1+\gamma_5)\,u_\alpha\bar{u}_\beta\gamma^\mu(1+\gamma_5)\,s_\beta = \bar{d}_\alpha\gamma^\mu(1+\gamma_5)\,s_\beta\bar{u}_\beta\gamma^\mu(1+\gamma_5)\,u_\alpha$$

has been used, where α, β are color indices which are summed over. In addition, the color singlet property of currents is employed,

$$\langle 0\,|\bar{d}_\alpha\gamma_\mu\gamma_5 s_\beta|\,\bar{K}^0(\mathbf{k})\rangle = i\sqrt{2}\,F_K k_\mu \frac{\delta_{\alpha\beta}}{3} \quad . \tag{4.18}$$

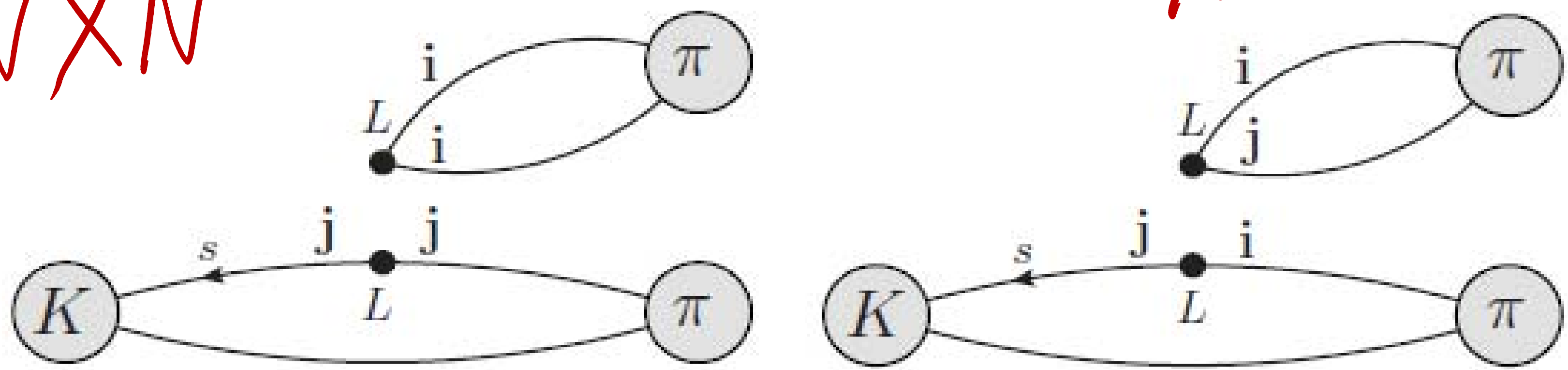
Within the vacuum saturation approximation, we see that the amplitudes are given completely by known semileptonic decay matrix elements. Putting in all of the constants, we find that

$$\begin{aligned} A_0 &= \frac{G_F}{3} V_{ud}^* V_{us} F_\pi\,(m_K^2 - m_\pi^2)\,c_1 = 0.84 \times 10^{-7} m_K \ , \\ A_2 &= \frac{2\sqrt{2}\,G_F}{3} V_{ud}^* V_{us} F_\pi\,(m_K^2 - m_\pi^2)\,c_4 = 0.42 \times 10^{-7} m_K \quad . \end{aligned} \tag{4.19}$$

We see that the above estimate of A_2 works reasonably well, but that A_0 falls considerably short of the observed $\Delta I = 1/2$ amplitude. This demonstrates that vacuum saturation is not a realistic approximation. However, it does serve to indicate how much additional $\Delta I = 1/2$ en-

Donoghue,,G,H
“Dynamics of
The SM” ‘92

NXN N



Contraction ①. Contraction ②.

FIG. 1: The two contractions contributing to ReA_2. They are distinguished by the color summation (**i**, **j** denote color). s denotes the strange quark and L that the currents are left-handed.

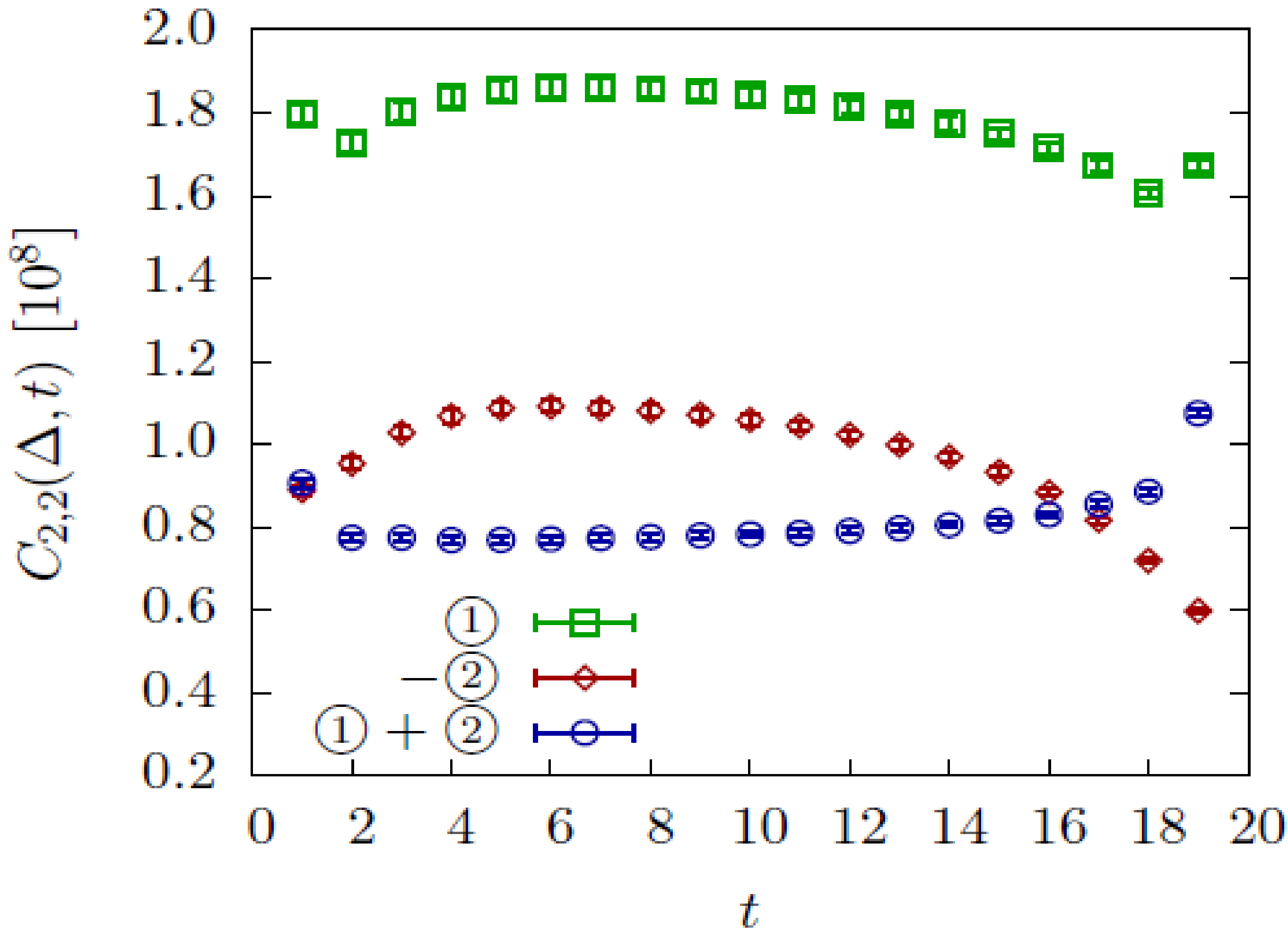


FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\text{MeV}$ and $\Delta = 20$.

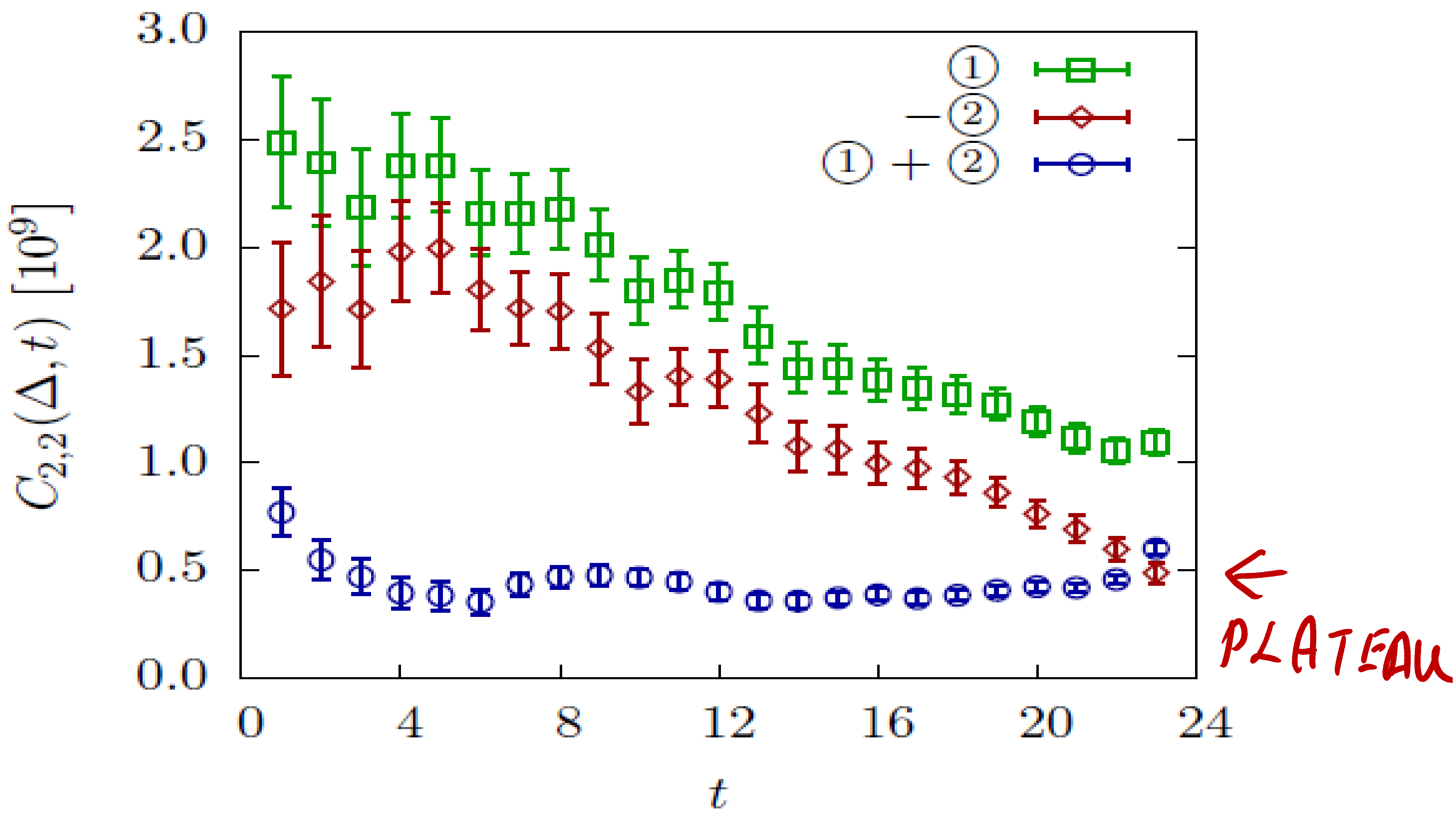


FIG. 2: Contractions ①, -② and ① + ② as functions of t from the simulation at physical kinematics and with $\Delta = 24$.

Mass depends of ReA2, A0

	a^{-1} [GeV]	m_π [MeV]	m_K [MeV]	ReA_2 [10^{-8} GeV]	ReA_0 [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	notes
16^3 Iwasaki	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold calculation
24^3 Iwasaki	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold calculation
IDSDR	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical kinematics
Experiment	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

TABLE I: Summary of simulation parameters and results obtained on three DWF ensembles.

i	Q_i^{lat} [GeV]	$Q_i^{\overline{\text{MS}}\text{-NDR}}$ [GeV]
1	8.1(4.6) 10^{-8}	6.6(3.1) 10^{-8}
2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
3	-0.6(1.0) 10^{-8}	5.4(6.7) 10^{-10}
4	—	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	—	2.0(0.6) 10^{-14}
10	—	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

TABLE II: Contributions from each operator to ReA_0 for $m_K = 662$ MeV and $m_\pi = 329$ MeV. The second column contains the contributions from the 7 linearly independent lattice operators with $1/a = 1.73(3)$ GeV and the third column those in the 10-operator basis in the $\overline{\text{MS}}$-NDR scheme at $\mu = 2.15$ GeV. Numbers in parentheses represent the statistical errors.

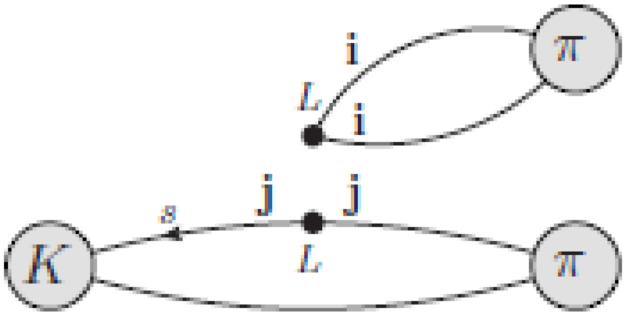
Sources of ReA0/ReA2 enhancement

- **Factor of 3 to 4 suppression of ReA2 due to (unanticipated) cancellation between the 2 contractions**

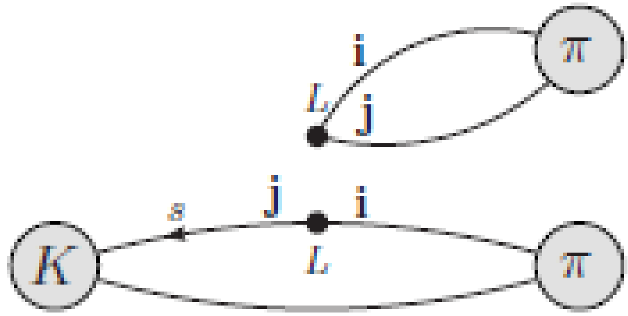
- **Factor of 2 + some perturbative running for ReA0 vs ReA2**

Factor of around 2 to 3 in the matrix elements for I=1/2 versus 3/2

REPERCUSSIONS FOR B_K



Contraction ①.



Contraction ②.

THE $\Delta S = 2$ MATRIX ELEMENT FOR K^0–$\bar{K}^0$ MIXING☆

John F. DONOGHUE, Eugene GOLOWICH and Barry R. HOLSTEIN

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Received 29 July 1982

We use SU(3) and PCAC to relate the $\Delta S = 2$ matrix element $\mathcal{M} = \langle K^0 | \bar{d}\gamma_\mu(1+\gamma_5)s\,\bar{d}\gamma^\mu(1+\gamma_5)s | \bar{K}^0 \rangle$ to experimental information on the $\Delta I = 3/2$ contribution to $K \to 2\pi$. Our result is $\mathcal{M} = 0.10\, m_K^4$ which is about 33% of the vacuum insertion value.

Following the discovery that ① and ② have opposite signs we examined separately the two contributions to the matrix element $\langle \bar{K}^0 | (\bar{s}d)_L (\bar{s}d)_L | K^0 \rangle$ which contains the non-perturbative QCD effects in neutral kaon mixing [11]. The two contributions correspond to Wick contractions in which the two quark fields in the K^0 interpolating operator are contracted i) with fields from the same current in $(\bar{s}d)_L(\bar{s}d)_L$ and ii) with one field from each of the two currents. Color counting and the vacuum insertion hypothesis suggest that the two contributions come in the ratio 1:1/3, whereas we find that in QCD they have the opposite sign.

RBC-UKQCD arXiv 1212.1474

CONTIUUM EXPECTATIONS FOR M_{LL}

K-K Mixing

To get an order of magnitude of the size of $\langle K|\mathcal{O}_{JJ}|\bar{K}\rangle$, we make the 'vacuum saturation' approximation

$2[1+1/3]$

$$\langle K|[\bar{d}\gamma^{\mu}(1-\gamma_5)s][\bar{d}\gamma_{\mu}(1-\gamma_5)s]|\bar{K}\rangle = \frac{8}{3}\langle K|\bar{d}\gamma^{\mu}\gamma_5 s|0\rangle\langle 0|\bar{d}\gamma_{\mu}\gamma_5 s|\bar{K}\rangle$$

$$= \frac{8}{3}\frac{f_K^2 m_K^2}{2m_K} \qquad (12.93)$$

Cheng & Li, Gauge Field Theory

where $f_K \simeq 1.23\, f_\pi$ is the kaon decay constant; the factor $(2m_K)^{-1}$ arises from the normalization of the state. The factor 8/3 corresponds to the four ways of Wick contraction times a colour factor 2/3. The hope in making such

To make contact with phenomenology, one must evaluate the matrix element of $O^{\Delta S=2}$ between K^0 and $\bar{K}^0$ states. It is conventional to express the results in terms of the so-called *B-parameter*,

DGH

$$\left\langle K^0 \left| O^{\Delta S=2} \right| \bar{K}^0 \right\rangle \equiv \frac{16}{3} F_K^2 m_K^2 B \quad , \qquad (1.19)$$

where $B = 1$ corresponds to the simple vacuum saturation approximation would be too large. Such a statement requires some estimate of the matrix element of the $\Delta S = 2$ operator. Gaillard and Lee used a version of the vacuum insertion approximation [see 9.4.7 and 9.4.8]. If we insert a vacuum state between all possible pairs of quark fields in $O_+^{\bar{d}s}$, we get

Weak Interactions & Modern Particle Theory
Howard Georgi

$$\langle K^0 | O_+^{\bar{d}s} | \overline{K^0} \rangle \simeq \frac{8}{3} f_K^2 m_K^2 \tag{10.2.7}$$

WHAT A STUPID OVERSIGHT!

79, NUMBER 19 PHYSICAL REVIEW LETTERS 10 NOVEMBER 1997

Domain Wall Quarks and Kaon Weak Matrix Elements

±15%

T. Blum and A. Soni

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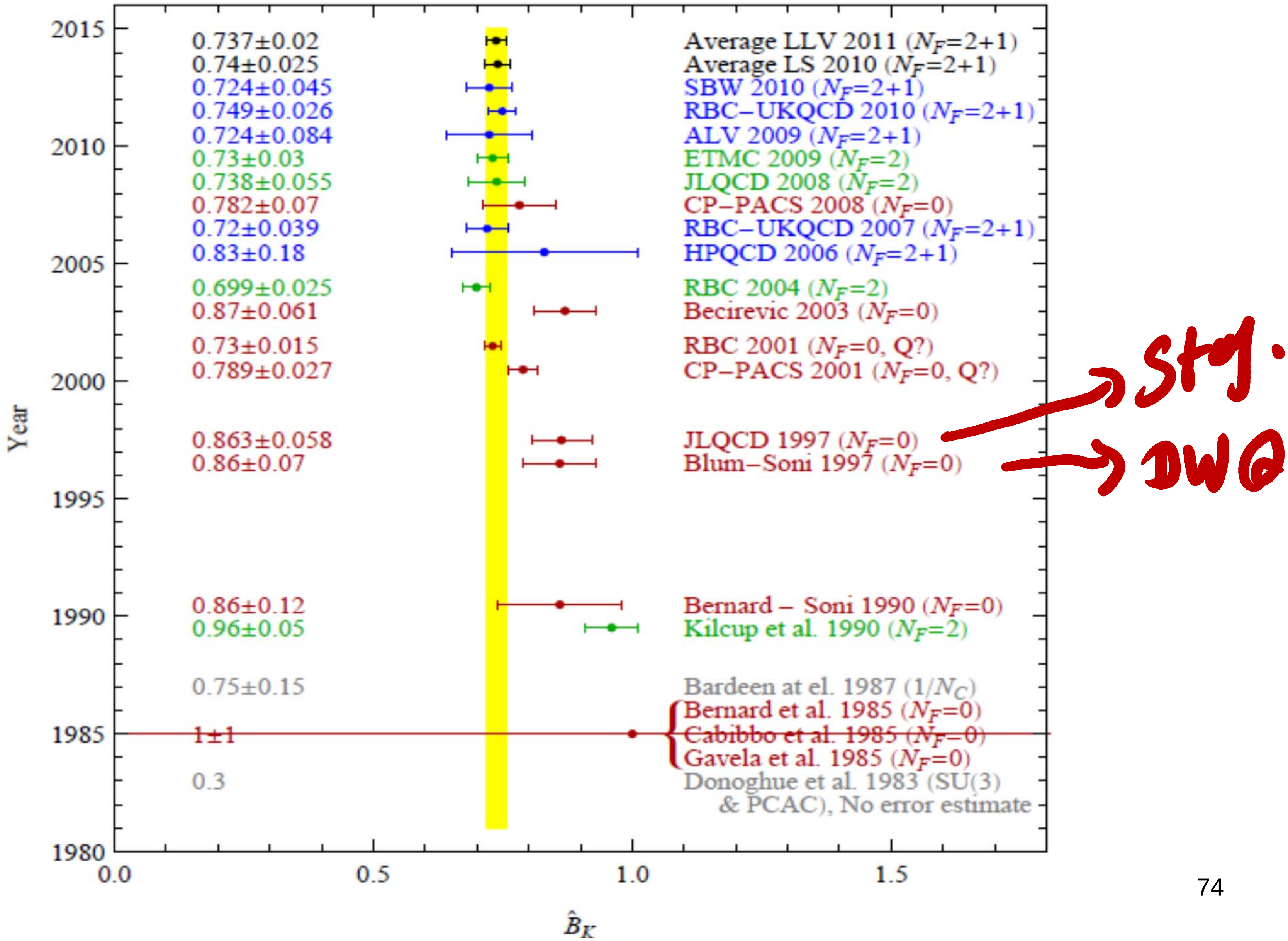
(Received 30 June 1997)

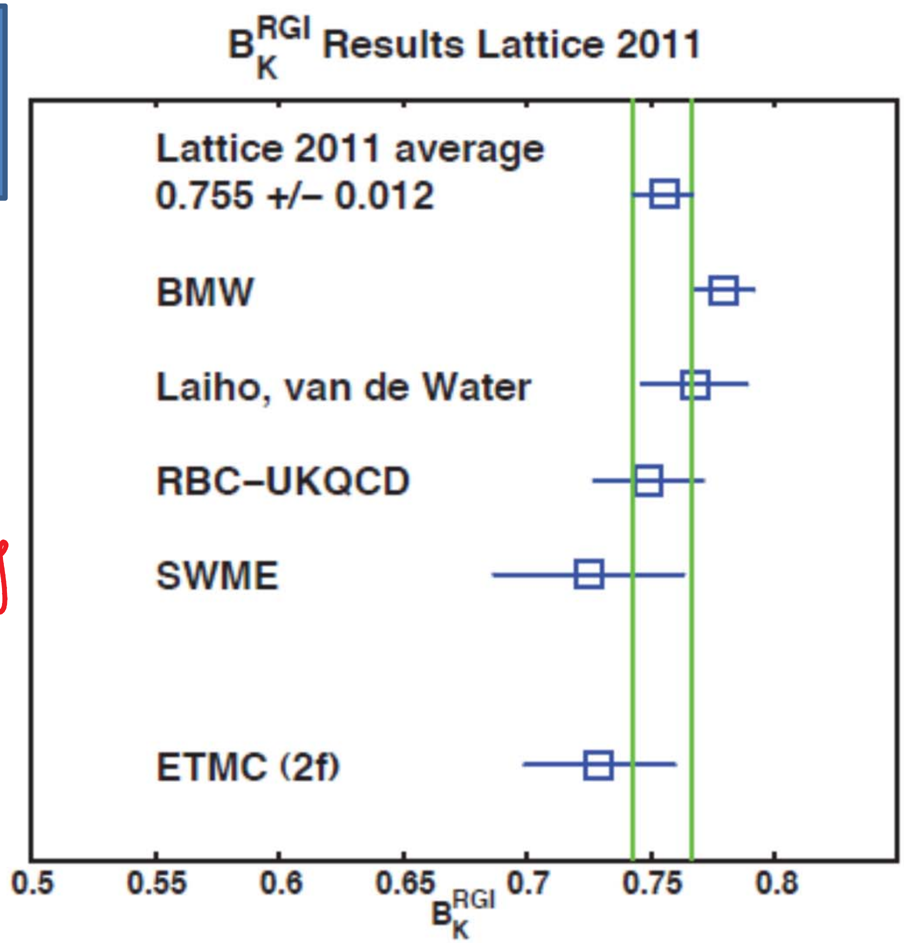
We present lattice calculations of kaon matrix elements with domain wall fermions. Using lattices with $6/g^2 = 5.85$, 6.0, and 6.3, we estimate $B_K(\mu \approx 2\ \text{GeV}) = 0.628(47)$ in quenched QCD, which is consistent with previous calculations. At $6/g^2 = 6.0$ and 5.85 we find the ratio f_K/m_ρ, in agreement with the experimental value, within errors. These results support expectations that $O(a)$ errors are exponentially suppressed in low energy ($E \ll a^{-1}$) observables, and indicate that domain wall fermions have good scaling behavior at relatively strong couplings. We also demonstrate that the axial current numerically satisfies the lattice analog of the usual continuum axial Ward identity. [S0031-9007(97)04682-6]

Because we did not look separately at the contribution from the two contractions,
the discovery of this remarkable suppression in Δ I=2 channel got delayed
By 15 years!!

A BRIEF ~25 years history of B_K



Mawhinney, plenary LAT'11



Several Lattice groups using completely diff methods reporting B_K with total error $\leq 3\%$!

HUGE STRIDES IN LATTICE CALCULATION OF BK!

- Average the four 2+1 flavor calculations presented
- Except for BMW, all are preliminary, although all groups have recently published B_K results from earlier datasets, so preliminary work should be fairly reliable.

See also recent summary by FLAG working group of FLAVIANET (arXiv:1011.4408)
They quote $\hat{B}_K = 0.738(20)$ for $N_f = 2+1$

EXPECTED PROGRESS NEAR FUTURE[RBC – UKQCD]

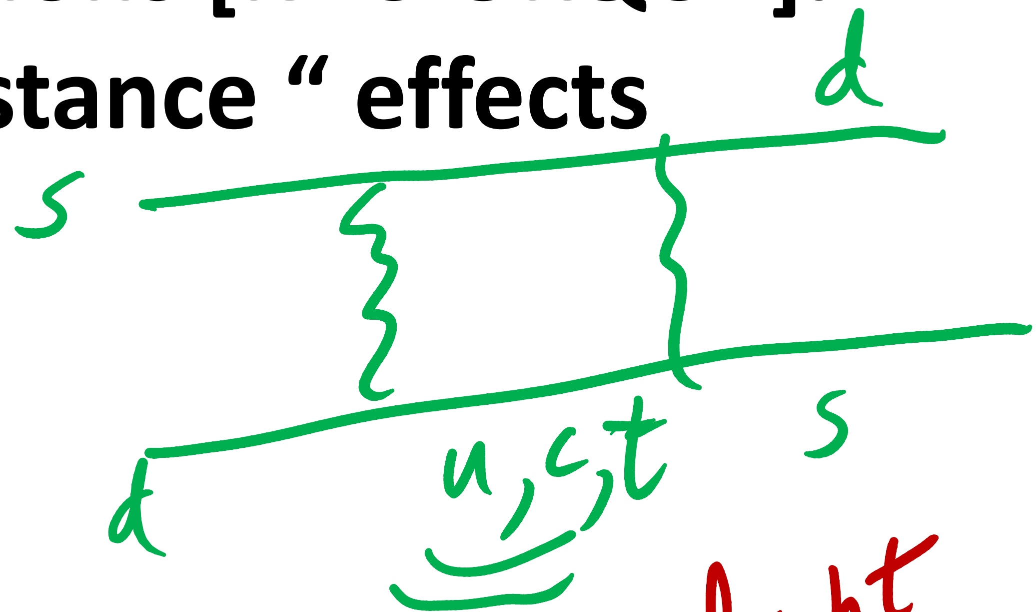
ARRIVED: NEW BGQs ⇒ 20-50 X COMPUTING POWER!

Expected progress near future[RBC –UKQCD]

NOW ~1y~($m_\pi \cong 140$ MeV)

- Kl3 ~ 1/2% ~ 1/4%
- B_K ~ 3% ~ 1-2%
- Re A_2 ~ 20% ~ 10%
- Im A_2
- Re A_0 ~ 25% ~ 15%
- Im A_0 ~ 100% ~ 30%

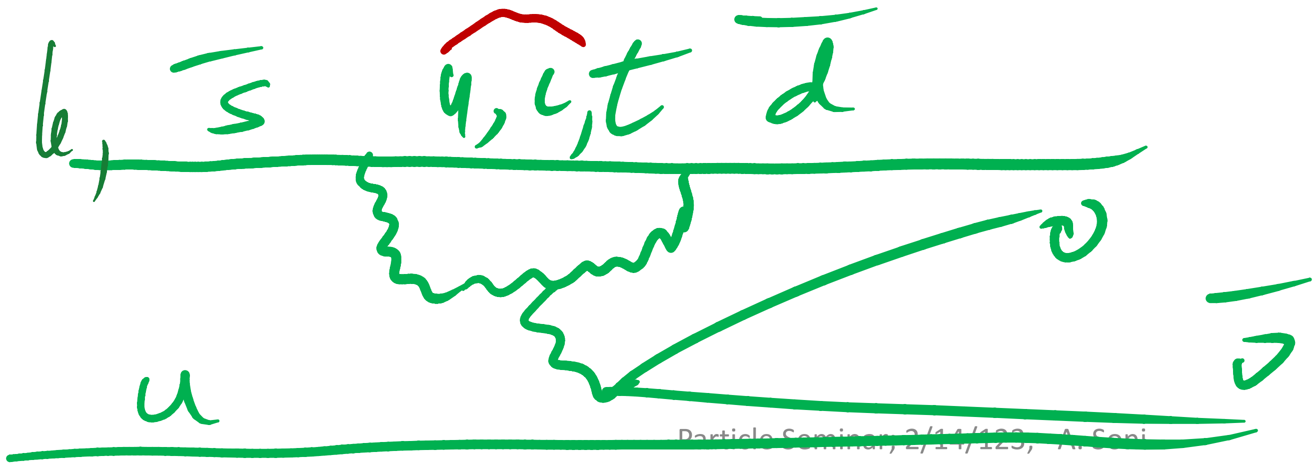
New applications [RBC-UKQCD]: "Long-distance " effects



- **KL-Ks mass diff**

too light for OPE

- **K$^+$ => $\pi^+ \nu\nu$ & other rare decays**



Also Relevant to Expected Progress

- **Super-KEKB/BELLEII**

- **S(LHCb)**

Some implications

- 1. Improved determination of the B-UT
- 2. significant progress towards a separate K-UT
- Crucial 1st step in factorization fails completely

pervasive use in K, D, B decays, so many phenomenological expectations will need to be modified.

HAVEN'T WE TESTED THE SM-CKM ENOUGH?

Haven't we tested the SM-CKM enough?

- Recall current tests around 15-10%

- Recall also $\varepsilon \sim 2 \times 10^{-3}$; if BNL had stopped experimental searches at the level of even 1%, history of Particle Physics would have been completely different

- ν mass & osc is another example.

We are looking for small effects

Adapted from Browder

A lesson from history (I)

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single $K_L \rightarrow \pi^+ \pi^-$ event among 600 decays into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

-Lev Okun, "The Vacuum as Seen from Moscow"

1964: BF= 2×10^{-3}

A failure of imagination ? Lack of patience ?

Christenson, Cronin, FITCH + TURLAY BNL '64

Lesson learnt from ν's

~ Circa 1983, after long and arduous efforts, Δm^2 upper bound used to be around a few ev^2 but efforts to Search oscillations continued basically because there was no good theoretical reason for m_ν to be zero.

- *Recall it took more than a decade beyond '83* and Δm^2 had to be lowered by almost 4 orders of magnitude (!) before osc were discovered.
- Moral: Physical "principles" shouldn't be abandoned easily

SUMMARY & Outlook

- **With the use of DWF significant progress has been made in tackling outstanding non-perturbative effects in weak decays**
- **Improved precision in Kl3, BK, Re&IM A2, ReA0**
- **Sources of Delta I=1/2 enhancement have been quantitatively identified**
- **$\varepsilon' / \varepsilon$ (Im A0) still needs more work but is underway**
- **Hitherto inaccessible LD effects are for the first time becoming computable**
- **Since QCD is intrinsically a non-perturbative theory,**

it is extremely important that we put greater efforts

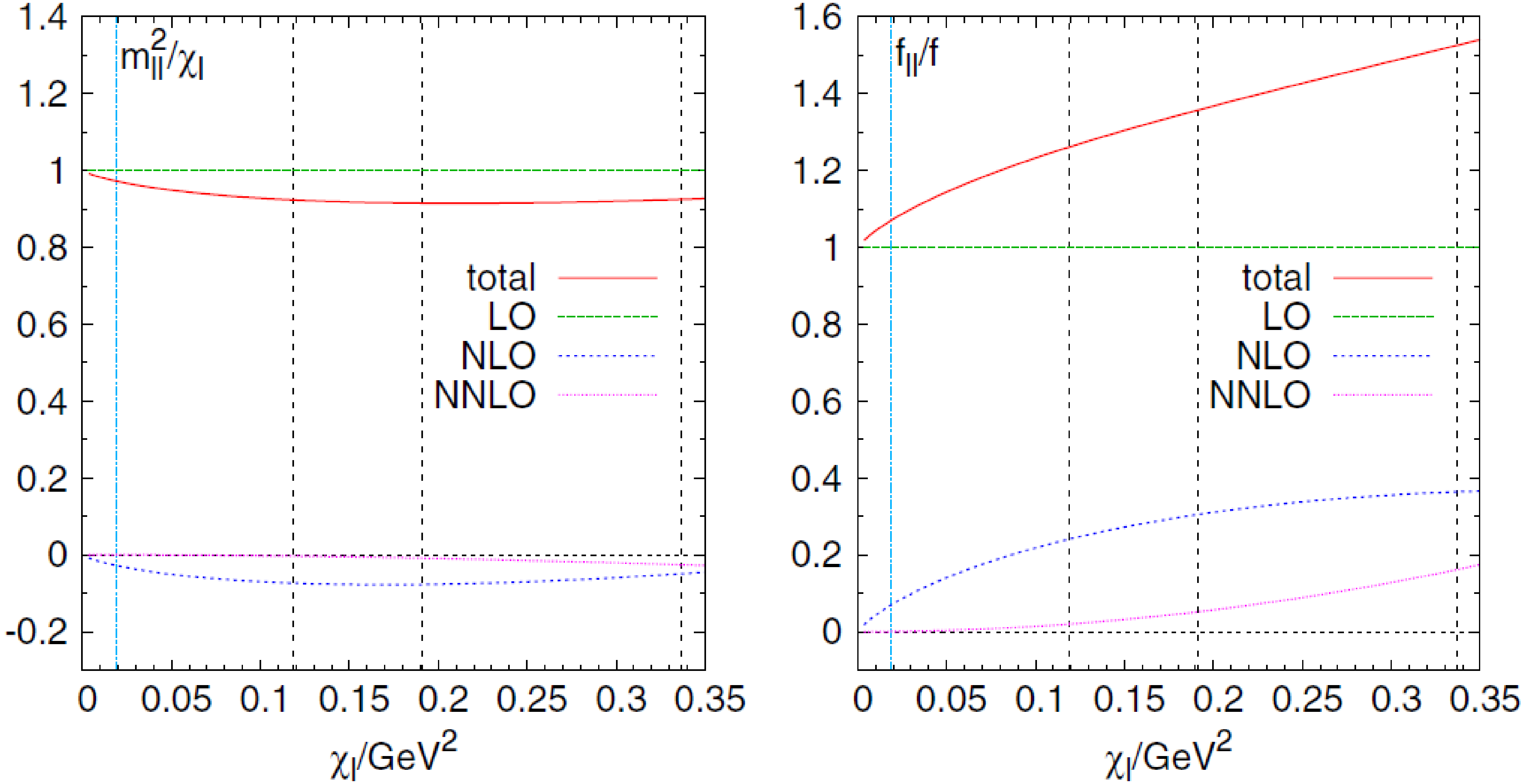
in sharpening our computational capabilities on & off the lattice [Higgs & Strong Dynamics] to enable us to interpret experiments in a more cost effective manner

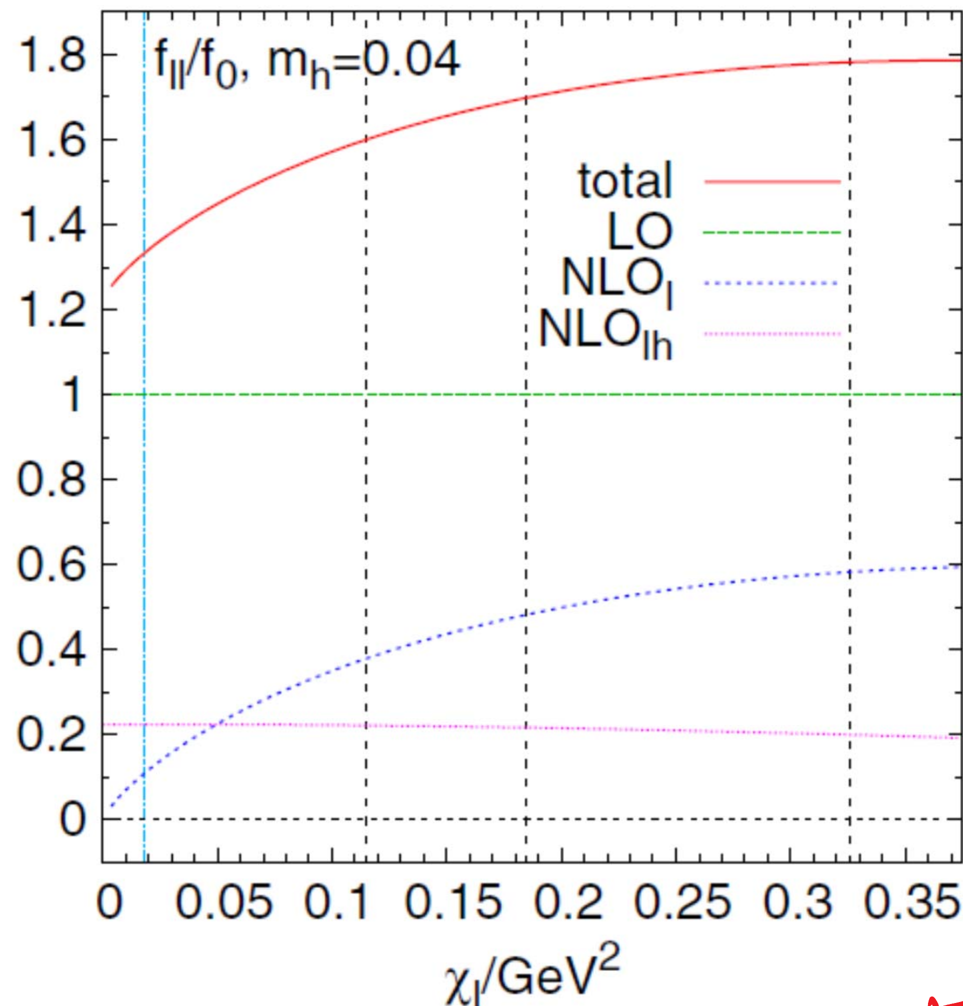
XTRAS

	$\mathrm{Re}A_2$	$\mathrm{Im}A_2$
lattice artefacts	15%	15%
finite-volume corrections	6.2%	6.8%
partial quenching	3.5%	1.7%
renormalization	1.7%	4.7%
unphysical kinematics	3.0%	0.22%
derivative of the phase shift	0.32%	0.32%
Wilson coefficients	7.1%	8.1%
Total	18%	19%

Table 2: Systematic error budget for Re A_2 and Im A_2.

RBC-UKQCD PRD'08





SU3 ChPT VERY Large corrections:
m_K too heavy!

RBC-UKQCD (-07) Initiate use of SU(2)XSU(2) ChPT for chiral extrapolations-> significant improvement in BK and many other light-light entities {07->}

Why hadronic matrix elements from the lattice are needed: Illustration BK

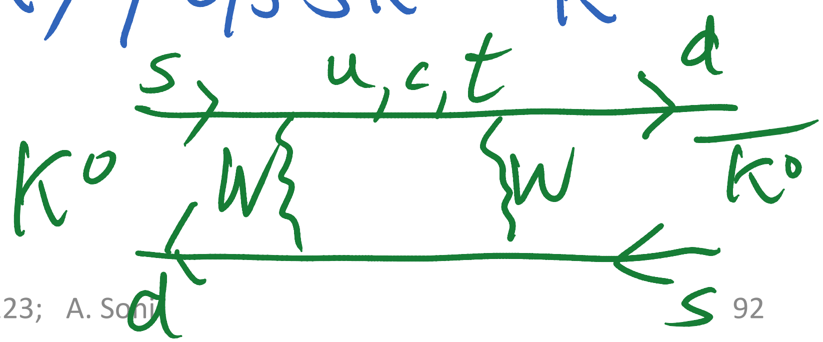
2.23×10^{-3} (BNL '64)

$$|\varepsilon_K| = \frac{G_F^2 m_W^2 f_K^2 m_K}{12\sqrt{2}\pi^2 \Delta m_K^{\rm exp}} \hat{B}_K \kappa_\varepsilon \, \mathrm{Im}\Big(\eta_1 S_0(x_c)\,(V_{cs}V_{cd}^*)^2 + 2\eta_3 S_0(x_c,x_t) V_{cs}V_{cd}^* V_{ts}V_{td}^* + \eta_2 S_0(x_t)\,(V_{ts}V_{td}^*)^2\Big)\,. \qquad (2.3)$$

BROWN MUCK

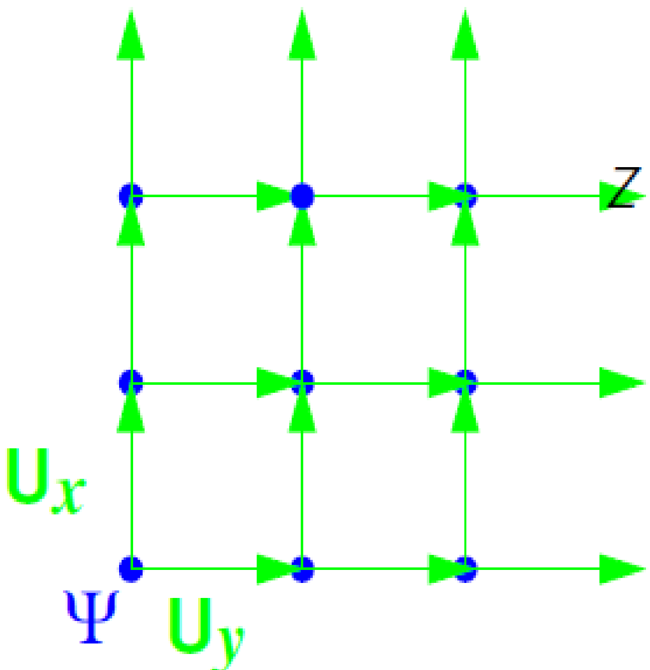
$\epsilon_K \sim$ Known Const $* \, B_K * \eta$ ← gem

$B_K \equiv \langle K|(\bar{s}\gamma_\mu L\, d)^2|\bar{K}\rangle \,/\, 8/3\, f_K^2\, m_K^2$



VeryVery Brief Introduction to Lattice methods

Quantum ChromoDynamics (QCD): Theory of strong interaction which governs interaction between quarks and gluons.
In contrast to Quantum Electrodynamics (QED), The effective coupling of QCD decreases in high energy, hence is calculable by hand, but not in low energy. $\rightarrow$ Nonperturbative techniques such as lattice QCD is needed for *ab initio* calculations. $(\psi(x), A_\mu(x)) \rightarrow (\psi(n), U_\mu(n) = \exp(-iA_\mu))$



$$Z = \int [dU] \det(\not{D} + m) \exp(-(S_g))$$

$$= \int [dU][d\bar{\psi}][d\psi] \ \exp(-(S_g + S_f))$$

$$S_f = \bar{\psi}(M^\dagger M)^{-1}\psi, \qquad S_{eff} = S_g + S_f$$

$$S_g = \beta \sum \left[(U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x) \right]$$

Courtesy Chulwoo Jung

SYMBOLICALLY

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int d[A, \bar{\psi}, \psi] \mathcal{O} e^{-S}$$

$$Z = \int d[A, \bar{\psi}, \psi] e^{-S}.$$

Monte Carlo methods and importance sampling ideas are used to numerically compute these starting from path integrals over discretized space-time boxes