On Neutrino Oscillometry—New Neutrino Oscillation Features with Low Energy Mono-energetic Neutrinos

(Low Energy Neutrinos in a box)

J. D. Vergados

University of Ioannina+

+ in collaboration with:

Y. Giomataris* and Yu.N. Novikov**

** for the LAGUNA collaboration (LENA);

*for the NOSTOS Collaboration: Saclay, APC-Paris, Saragoza, Ioannina, Thessaloniki, Dimokritos, Dortmund, Sheffield

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NOSTOS: SPHERICAL TPC’s (STPC) for detecting Earth or sky neutrinos

• A) Neutrino Oscillometry-Low energy neutrinos in a spherical box (electron recoils from low energy neutrinos)

• B) Neutral Current Spherical TPC’s (nuclear recoils)
  • B1: For Dedicated SUPERNOVA NEUTRINO DETECTION
  • B2: For exotic neutrino Oscillometry (Reactor Antineutrino Anomaly)

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NEUTRINO OSCILLATIONS

Neutrino mass terms: Beyond the standard model
1. Dirac +(heavy neutrino) Majorana type or
2. Light neutrino Majorana type

Result in all cases: Neutrino mixing

One then distinguishes between the weak interaction states $\nu^0_L$ and the mass eigenstates $\nu_L$. 

$$\nu^0_L = U \nu_L$$
Standard Parameterization of Mixing Matrix (2 Majorana phases not shown)

\[ U = R_{23} W_{13} R_{12} \]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta_{13}} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[ \theta_{23} \simeq \theta_{\text{ATM}} \]
\[ \theta_{13} = \theta_{\text{CHOOZ}} \]
\[ \theta_{12} = \theta_{\text{SUN}} \]

\[
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}
\end{pmatrix}
\]
The mixing matrix is called PNMS (Pontecorvo–Maki–Nakagawa–Sakata matrix).

It has not yet been derived from a basic theory. From neutrino oscillations we know that, unlike the C-M matrix for quarks, it has large off diagonal elements. Some models yield "bi-tri maximal" form consistent with v-oscillations, i.e.

\[
\begin{pmatrix}
\frac{\sqrt{2}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0(?) \\
\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\]
Massive Neutrinos Oscillate!

- **Flavor states**: $\nu_{\alpha}$, $\alpha = e, \mu, \tau$.
- **Mass eigenstates**: $\nu_i$, $i = 1, 2, 3$
- **Flavor $\alpha$ at time $t=0$**, $\nu_{\alpha} = \sum_i U_{\alpha i} \nu_i$
- **Flavor $\alpha$ at a later time $t \neq 0$**, $\nu_{\alpha}(t) = \sum_i U_{\alpha i} \nu_i \exp(iE_i t)$
- $<\nu_{\alpha}(0) | \nu_\beta(t)> \# \delta_{\alpha \beta}$ -->
- $P(\nu_\alpha \rightarrow \nu_\beta) = \sum_j (U_{\beta j})^* U_{\alpha j} \exp(iE_j t) \# \delta_{\alpha \beta}$
Neutrino Oscillations (two \(\nu\) types)

\[ L = ct, \ L_0 = \text{oscillation length} \leftrightarrow \text{period} \]

Mixing matrix

\[
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]

Q.M. Evolution Equation

Then for \(\nu_\alpha, \nu_\beta\) two neutrino flavors,

\[
\nu_\alpha(0) = \cos \theta \nu_1 + \sin \theta \nu_2, \quad \nu_\beta(L) = -\sin \theta \nu_1 + \cos \theta \nu_2 e^{-2i\Delta_{12}L}
\]

\[
\Delta_{12} = \frac{E_2 - E_1}{2} \approx \frac{m_2^2 - m_1^2}{4p} \approx \frac{m_2^2 - m_1^2}{4E_\nu}
\]

\[
P(\alpha \rightarrow \beta) = \sin^2 2\theta \sin^2 \pi \frac{L}{L_0}, \quad \alpha \neq \beta
\]

\[
P(\alpha \rightarrow \alpha) = 1 - P(\alpha \rightarrow \beta) = 1 - \sin^2 2\theta \sin^2 \pi \frac{L}{L_0}
\]

\[
L_0 \equiv \ell_{12} = \frac{4\pi E_\nu}{|m_2^2 - m_1^2|}
\]

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Neutrino Oscillation Experiments
Effectively analyzed as two generations

- **Appearance**
  \[ P(\nu_\alpha \rightarrow \nu_\beta, \alpha \neq \beta) = \sin^2 2\theta \sin^2 \pi(L/L_0) \]

- **Disappearance**
  \[ P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \pi(L/L_0) \]

- **\( \theta \)** the effective mixing angle

- **\( L_0 \)** the oscillation Length = \( \frac{4\pi E_\nu}{\Delta m^2} \) or
  \[ L_0 = 2.476 \text{ km } \left\{ \frac{E_\nu}{1 \text{ MeV}} \right\} / \left\{ \frac{\Delta m^2}{10^{-3} \text{ eV}^2} \right\} = 2.476 \text{ m } \left\{ \frac{E_\nu}{1 \text{ keV}} \right\} / \left\{ \frac{\Delta m^2}{10^{-3} \text{ eV}^2} \right\} \]

- **\( L \)** is the source detector distance
Two generation Oscillations
$\theta = \pi/4$ (atmospheric), $\theta = \pi/5$ (solar)
Table I: Best fit values from global data (solar, atmospheric, reactor (KamLand and CHOOZE) and K2K experiments)

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit</th>
<th>2σ</th>
<th>3σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_{21}^2 , [10^{-5} \text{eV}^2]$</td>
<td>$7.59^{+0.23}_{-0.18}$</td>
<td>$7.22-8.03$</td>
<td>$7.03-8.27$</td>
</tr>
<tr>
<td>$</td>
<td>\Delta m_{31}^2</td>
<td>, [10^{-3} \text{eV}^2]$</td>
<td>$2.40^{+0.12}_{-0.11}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.318^{+0.019}_{-0.016}$</td>
<td>$0.29-0.36$</td>
<td>$0.27-0.38$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.50^{+0.07}_{-0.06}$</td>
<td>$0.39-0.63$</td>
<td>$0.36-0.67$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.013^{+0.013}_{-0.009}$</td>
<td>$\leq 0.039$</td>
<td>$\leq 0.053$</td>
</tr>
</tbody>
</table>
Neutrino energy regions for various detectors

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The standard \((\nu, e)\) cross section

\[
\frac{d\sigma}{dT} = \left(\frac{d\sigma}{dT}\right)_{weak} + \left(\frac{d\sigma}{dT}\right)_{EM}
\]  

(2.1)

\[
\left(\frac{d\sigma}{dT}\right)_{weak} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left[ 1 - \frac{T}{E_\nu} \right]^2 + \left( g_A^2 - g_V^2 \right) \frac{m_e T}{E_\nu^2} \right]
\]  

(2.2)

\[g_V = 2 \sin^2 \theta_W + 1/2 \quad (\nu_e), \quad g_V = 2 \sin^2 \theta_W - 1/2 \quad (\nu_\mu, \nu_\tau)\]

\[g_A = 1/2 \quad (\nu_e), \quad g_A = -1/2 \quad (\nu_\mu, \nu_\tau)\]

For antineutrinos \(g_A \rightarrow -g_A\).

The scale is set by the weak interaction:

\[
\frac{G_F^2 m_e}{2\pi} = 4.45 \times 10^{-48} \frac{cm^2}{keV}
\]  

(2.3)
In \((\nu_e,e)\) reaction all flavors contribute
\[
\sigma_e(E_v,T_e,L) = \sigma(E_v,T_e,0) \ P(\nu_e\rightarrow\nu_e) + \\
\sigma'(E_v,T_e,0) \ \sum_{a\neq e} \ P(\nu_e\rightarrow\nu_a)
\]

- \(\sigma_e(E_v,T_e,0) (\sigma'_a(E_v,T_e,0))\) are the standard \(\nu_e\) 
  \((\nu_a \neq \nu_e)\) -electron cross sections in the absence of 
  oscillation.

- The 3-generation oscillation probability (after 
  integration over the electron energies) will appear as:

\[
P(\nu_e\rightarrow\nu_e) \approx 1 - \chi(E_v)
\]

\[
\{ \sin^2 (2\theta_{12}) \ \sin^2 [\pi(L/L_{12})] + \\
\sin^2 (2\theta_{13}) \ \sin^2 [\pi(L/L_{13})] \}, \ L_{13} \approx L_{23}
\]

\[
\chi(E_v) = 1 - \ \sigma'_d(E_v,0) / \sigma_e(E_v,0) \approx 1
\]
The neutrino disappearance probability $E_{\nu} = 13$ keV, $\theta_{12} = \pi/5$, $\sin^2 2\theta_{13} = 0.175, 0.085, 0.045$

Detector close to the source  Detector far from the source
Standard Long baseline (L->km)

Short baseline (L->m) - Oscillometry

Long baseline ($E_\nu >> 1$ MeV) $\rightarrow$ L in [km]

Short baseline ($E_\nu \ll 1$ MeV) $\rightarrow$ L in [m] - Oscillometry
More Exotic Neutrino Oscillation Experiments to extract more precise Neutrino Oscillation Parameters

- Very low energy neutrinos \(10^5\) small oscillation lengths
- The full oscillation takes place inside the detector (many standard experiments simultaneously)
- Due to thresholds available are only:
  - neutrino electron and neutral current scattering are open
Experimental Issues:
The main idea of NOSTOS Set Up (the position is determined via a radial Electric field)

The detector

The neutrino source

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A novel large-volume spherical detector with proportional amplification read-out

I. Giomataris,\textsuperscript{a} I. Irastorza,\textsuperscript{b} I. Savvidis,\textsuperscript{c} S. Andriamonje,\textsuperscript{a} S. Aune,\textsuperscript{a} M. Chapellier,\textsuperscript{d} Ph. Charvin,\textsuperscript{e} P. Colas,\textsuperscript{f} J. Derre,\textsuperscript{g} E. Ferrer,\textsuperscript{a} M. Gros,\textsuperscript{a} X.F. Navick,\textsuperscript{a} P. Salin\textsuperscript{a} and J.D. Vergados\textsuperscript{d}  
\textsuperscript{a}IRFU, Centre d'\textacute{e}tudes de Saclay, 91191 Gif-sur-Yvette CEDEX, France
\textsuperscript{b}University of Saragoza, Spain
\textsuperscript{c}Aristotle University of Thessaloniki, Greece
\textsuperscript{d}APC, Université Paris 7 Denis Diderot, Paris, France
\textsuperscript{e}University of Ioannina, Greece
E-mail: ioanis.giomataris@cern.ch

ABSTRACT: A new type of radiation detector based on a spherical geometry is presented. The detector consists of a large spherical gas volume with a central electrode forming a radial electric field. Charges deposited in the conversion volume drift to the central sensor where they are amplified and collected. We introduce a small spherical sensor located at the center acting as a proportional amplification structure. It allows high gas gains to be reached and operates in a wide range of gas pressures. Signal development and the absolute amplitude of the response are consistent with predictions. Sub-keV energy threshold with good energy resolution is achieved. This new concept has been proven to operate in a simple and robust way and allows reading large volumes with a single read-out channel. The detector performance presently achieved is already close to fulfill the demands of many challenging projects from low energy neutrino physics to dark matter detection with applications in neutron, alpha and gamma spectroscopy.

KEYWORDS: Gaseous detectors; Very low-energy charged particle detectors; Large detector systems for particle and astroparticle physics; Neutron detectors (cold, thermal, fast neutrons).
The prototype operating at LSM (Laboratoire Souterrain de Modane)

- $D=1.3 \text{ m}$
- $V=1 \text{ m}^3$
- Spherical vessel made of Cu (6 mm thick)
- $P$ up to 5 bar possible (up to 1.5 tested up to now)
- Vacuum tight: $\sim 10^{-6} \text{ mbar}$
  (outgassing: $\sim 10^{-9} \text{ mbar/s}$)
Experimental Requirements for Oscillometry by detecting electrons

- $10^4$ Events per year are adequate against $10^6$
- bgd events per year (feasible)
- The source can be shielded employing 15-20cm of Pb (total absorption intensity $10^{17}s^{-1}$) for a source like $^{55}$Fe. Precise simulations are under way. Perhaps we can manage with 10cm
- The source can be replaced many times. $^{37}$Ar, $^{51}$Cr can be produced in intensities higher than those of GALLEX and SAGE
- Detector: Spherical Gaseous TPC with Micromegas (under development using KET, Kapton Etching Technology).
- The detector will be cooled and placed underground
- Good energy resolution and low threshold, 0.1 keV
- Position resolution better than 0.1m

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The number of events for a spherical gaseous detector (source at the origin)

The number of events between \( L \) and \( L + dL \) is given by:

\[
dN = N_\nu n_e \frac{4\pi L^2 dL}{4\pi L^2} \sigma(L, x, y_{th}) = N_\nu n_e dL \sigma(L, x, y_{th}) \quad (5.22)
\]

or

\[
\frac{dN}{dL} = N_\nu n_e \sigma(L, x, y_{th}), \quad x = \frac{E_\nu}{m_e}, \quad y_{th} = \frac{(T_e)_{th}}{m_e} \quad (5.23)
\]

To compare with other geometries we rewrite this as follows:

\[
R_0 \frac{dN}{dL} = R_0 N_\nu n_e \sigma(L, x, y_{th}) \quad (5.24)
\]

or

\[
R_0 \frac{dN}{dL} = \Lambda g_s(L/R_0) \tilde{\sigma}(L, x, y_{th}), \quad g_s(L/R_0) = 1 \quad (5.25)
\]

where

\[
\Lambda = \frac{G_F^2 m_e^2}{2\pi} R_0 N_\nu n_e \quad (5.26)
\]
Part I \((\nu_e, e)\) scattering

Extract

- \(\sin^2 (2\theta_{13})\) from the total number of events
  \[ R = A + B \sin^2 (2\theta_{13}) \]

- \(\delta m_{13}^2\) from the oscillation pattern
  \[ \frac{dR}{dL} \sim \{1 - \sin^2 (2\theta_{13}) \sin^2 [\pi (L/L_{13})]\} \]

- Or both from the oscillation pattern

- Compare with T2K experiment: \(\nu_\mu \rightarrow \nu_e\)
  \(0.03 (0.04) \leq \sin^2 (2\theta_{13}) \leq 0.28 (0.34)\) Normal (Inv)

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Some sources of low energy Monoenergetic Neutrinos for STPC measuring $\sin^2 (2\theta_{13})$ and $\delta m^2_{13}$

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$T_{1/2}$</th>
<th>$Q_e$</th>
<th>$E_\nu$</th>
<th>$L_{23}/2$</th>
<th>$E_{e,max}$</th>
<th>weight</th>
<th>$\nu$-intensity (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{139}$Ce</td>
<td>138y</td>
<td>113*</td>
<td>74</td>
<td>37</td>
<td>20</td>
<td>1.5</td>
<td>$2 \times 10^{14}$</td>
</tr>
<tr>
<td>$^{157}$Tb</td>
<td>70y</td>
<td>60.0(3)</td>
<td>9.8</td>
<td>5</td>
<td>0.4</td>
<td>5</td>
<td>$2 \times 10^{14}$</td>
</tr>
<tr>
<td>$^{163}$Ho</td>
<td>4500y</td>
<td>$\approx 2.6$</td>
<td>$\approx 0.5; \approx 0.8$</td>
<td>0.2-1.3</td>
<td>$\leq 0.03$</td>
<td>250</td>
<td>$5 \times 10^{12}$</td>
</tr>
<tr>
<td>$^{193}$Pt</td>
<td>50y</td>
<td>568.0(3)</td>
<td>44(70%)</td>
<td>22</td>
<td>6.5</td>
<td>300</td>
<td>$5 \times 10^{14}$</td>
</tr>
</tbody>
</table>
Event rate \( \frac{dN}{dL} \) (per m), \( P=10 \text{Atm} \), Ar target for \( m=0.2 \) and 0.3 kg of source

\[
\sin^2 \theta_{13} = 0.175, 0.085, 0.045 \quad \text{T}_{th} = 0.1 \text{keV}
\]

\( L=10 \text{m}, \ E_\nu = 9.8 \text{ keV} \) (\(^{157}\text{Tb}\))

\( L=50 \text{m}, \ E_\nu = 50 \text{ keV} \) (\(^{193}\text{Pt}\))
Oscillometry with Larger Detectors, e.g.
LENA, \( R_0 = 11 \text{m}, \ h = 90 \text{m}, \ E_{\text{th}} = 5 \text{keV} \)

Courtesy of A. Popov

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Candidate sources for oscillometry for $\theta_{13}$ using the LENA detector

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$T_{1/2}$ (d)</th>
<th>$m_t$ (kg)</th>
<th>$t_{ir}$ (d)</th>
<th>$E_{e,\text{max}}$ (keV)</th>
<th>$m_s$ (g)</th>
<th>$N_\nu$ ($s^{-1}$)</th>
<th>$N_{ir}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}\text{Ar}$</td>
<td>35</td>
<td>0.36 ($^{36}\text{Ar}$)</td>
<td>30</td>
<td>617</td>
<td>2.2</td>
<td>$10^{16}$</td>
<td>5</td>
</tr>
<tr>
<td>$^{51}\text{Cr}$</td>
<td>27.7</td>
<td>15 ($^{50}\text{Cr}$)</td>
<td>30</td>
<td>560</td>
<td>209</td>
<td>$7 \times 10^{17}$</td>
<td>5</td>
</tr>
<tr>
<td>$^{75}\text{Se}$</td>
<td>120</td>
<td>1000</td>
<td>100</td>
<td>287</td>
<td>1475</td>
<td>$8 \times 10^{17}$</td>
<td>3</td>
</tr>
<tr>
<td>$^{85}\text{Sr}$</td>
<td>64.9</td>
<td>1000</td>
<td>60</td>
<td>363</td>
<td>8.64</td>
<td>$7.5 \times 10^{15}$</td>
<td>5</td>
</tr>
<tr>
<td>$^{103}\text{Pd}$</td>
<td>17</td>
<td>1000</td>
<td>10</td>
<td>315</td>
<td>11.5</td>
<td>$3 \times 10^{16}$</td>
<td>5</td>
</tr>
<tr>
<td>$^{113}\text{Sn}$</td>
<td>115</td>
<td>1000</td>
<td>100</td>
<td>436</td>
<td>17.3</td>
<td>$6.4 \times 10^{15}$</td>
<td>3</td>
</tr>
<tr>
<td>$^{121}\text{Te}$</td>
<td>16.8</td>
<td>1000</td>
<td>10</td>
<td>280</td>
<td>1.6</td>
<td>$3.8 \times 10^{15}$</td>
<td>5</td>
</tr>
<tr>
<td>$^{145}\text{Sm}$</td>
<td>340</td>
<td>1000</td>
<td>300</td>
<td>340</td>
<td>480</td>
<td>$4.7 \times 10^{16}$</td>
<td>1</td>
</tr>
<tr>
<td>$^{169}\text{Yb}$</td>
<td>32</td>
<td>1000</td>
<td>30</td>
<td>304</td>
<td>3000</td>
<td>$2.8 \times 10^{18}$</td>
<td>5</td>
</tr>
</tbody>
</table>
Cylindrical geometry (source at the origin of one of its bases) radius $R_0$)

$$R_0 \frac{dN}{dL} = N_\nu n_e R_0 \frac{1}{2} g_{av}(u, L/R_0) \sigma(L, x, y_{yh})$$

$$= \Lambda \frac{1}{2} g_{av}(u, L/R_0) \tilde{\sigma}(L, x, y_{yh})$$

(7.36)

where $g_{av}(u, L/R_0)$ is a geometric factor that takes care of the variation of the neutrino flux in the various positions described by $L$. It can be cast in the form:

$$g_{av}(u, \nu) = \begin{cases} 
1, & 0 < \nu < 1 \\
1 - \sqrt{\nu^2 - 1/\nu}, & 1 < \nu < 1/u \\
1/(uv) - \sqrt{\nu^2 - 1/\nu}, & 1/u < \nu < \sqrt{1 + 1/u^2}
\end{cases}$$

(7.37)
Oscillometry with the LENA detector (Liquid Ar) events/m divided by the geometric factor $g_{av}$, $u = R_0/h = 11/91$

Events/meter; $^{51}$Cr; Width=$N^{1/2}$
The Experimentalist’s width: $^{51}\text{Cr}$ source in LENA detector
Part II: $\left(\nu_e, e\right)$ scattering for oscillations to a Sterile Neutrino measuring $\sin^2(2\theta_{14})$ and $\delta m^2_{14}$

- Motivated by
  The reactor neutrino anomaly and LSND:
  $\sin^2(2\theta_{14}) = 0.17 \pm 0.1 (95\%)$, $\delta m^2_{14} > 1.5 \text{ eV}^2$

*Now $\delta m^2$ is larger -> The optimal $\nu$-energy can be larger -> The cross sections are higher
In $(\nu_e,e)$ reaction all flavors contribute

$$\sigma_e(E_v,T_e,L) = \sigma(E_v,T_e,0) \ P(\nu_e-->\nu_e) + \sigma'(E_v,T_e,0) \ \sum_a \sigma_a \ P(\nu_e-->\nu_a)$$

- $\sigma_e(E_v,T_e,0)$ ($\sigma'_a(E_v,T_e,0)$) are the standard $\nu_e$ ($\nu_a \neq \nu_e$) -electron cross sections in the absence of oscillation. The sterile does not interact!

- The 3-generation oscillation probability (after integration over the electron energies) will appear as:

$$P(\nu_e-->\nu_e) \approx 1 - \left\{ \sin^2(2\theta_{12}) \sin^2 \left[ \pi \frac{L}{L_{12}} \right] + \sin^2(2\theta_{13}) \sin^2 \left[ \pi \frac{L}{L_{13}} \right] + \sin^2(2\theta_{14}) \sin^2 \left[ \pi \frac{L}{L_{14}} \right] \right\},$$

$$L_{14} \approx L_{24} \ll L_{13} \approx L_{23} \ll L_{12}$$
Some sources (0.1 kg) of low energy Monoenergetic Neutrinos for measuring \( \sin^2 (2\theta_{14}) \) and \( \delta m^2_{14} \) (electron recoils) To check the Reactor neutrino anomaly \( \sin^2 (2\theta_{14}) = 0.17 \pm 0.01, \delta m^2_{14} \approx 1.5 \text{ eV}^2 \)
Sterile neutrino oscillations: $R_0=4\text{m}, P=10\text{ Atm}$

$E_\nu := 747\text{ keV (90\% to gs)}; = 530\text{ keV (10\% to excited)}$ small effect

On the left full, dotted, dashed curve

$\sin^2(2\theta_{14}) = 0.27, 0.17, 0.07$

Oscillation Pattern (10d)

Expected Spectra (55d)

Statistical corridor $1\sigma$
Determination of $\theta_{14}$ by $^{40}$Ar ($v_e,e$) detector: $\sin^2(2\theta_{14})=0.05$ (99%)

- The total number of events:
  $N_0 = A + B \sin^2 (2\theta_{14})$

- For $^{51}$Cr (measuring for 55 days):
  $A=1.59 \times 10^4$, $B=-7.56 \times 10^4$
Part III: Neutral Current detectors* for oscillations to a Sterile Neutrino measuring $\sin^2 (2\theta_{14})$ and $\delta m^2_{14}$

- Motivated by
  The reactor neutrino anomaly and LSND:
  $\sin^2 (2\theta_{14}) = 0.17 \pm 0.1 (95\%)$, $\delta m^2_{14} > 1.5 \text{ eV}^2$
  Now $\delta m^2$ is larger ➔ The optimal $\nu$-energy can be larger

- *Expect large cross sections due to the $N^2$ dependence instead of $Z$ for ($\nu_e$, $e$)
- * Benefit from the experience of dark matter searches
Neutrino oscillations with NC interactions?

- All four neutrinos are active.
  Then
  \[ \sigma_{\text{tot}} = (P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) + P(\nu_e \rightarrow \nu_4)) \sigma, \quad (3) \]
  but
  \[ P(\nu_e \rightarrow \nu_e) = 1 - (P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) + P(\nu_e \rightarrow \nu_4)) , \quad (4) \]
  i.e.
  \[ \sigma_{\text{tot}} = \sigma, \quad (5) \]
  no oscillation is observed.

- The fourth neutrino is sterile.
  Then
  \[ \sigma_{\text{tot}} = (P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau)) \sigma, \quad (6) \]
  i.e. the sterile neutrino does not contribute. Eq. 4, however, is still valid (neutinos are lost from the flux). Thus
  \[ \sigma_{\text{tot}} = (1 - P(\nu_e \rightarrow \nu_4)) \sigma. \quad (7) \]
  If, in addition, the new oscillation length is much smaller than the other two, one finds:
  \[ \sigma_{\text{tot}} = \left(1 - \sin^2 2\theta_{14} \sin^2 \pi \frac{L}{L_{14}} \right) \sigma. \quad (8) \]
Neutrino – Nucleus elastic scattering

\[
\left( \frac{d\sigma}{dT_A} \right) (T_A, E_\nu) = \frac{G_F^2 A m_N}{2\pi} \left( \frac{N^2}{4} \right) F_{coh}(T_A, E_\nu),
\]

with

\[
F_{coh}(T_A, E_\nu) = F^2(q^2) \left( 1 + \left( 1 - \frac{T_A}{E_\nu} \right) - \frac{A m_N T_A}{E_\nu^2} \right)
\]

where \( N \) is the neutron number and \( F(q^2) = F(T_A^2 + 2A m_N T_A) \).
Some sources (0.1 kg) of low energy Monoenergetic Neutrinos for measuring $\sin^2(2\theta_{14})$ and $\delta m^2_{14}$ (nuclear recoils) To check the Reactor neutrino anomaly $\sin^2(2\theta_{14}) = 0.17 \pm 0.01$, $\delta m^2_{14} \approx 1.5$ eV$^2$

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>$T_{1/2}$ (d)</th>
<th>$E_{\nu}$ (keV)</th>
<th>$L_{14}$ (m)</th>
<th>$N_{\nu}$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}$Ar</td>
<td>35</td>
<td>811</td>
<td>1.4</td>
<td>$1.8 \times 10^{16}$</td>
</tr>
<tr>
<td>$^{51}$Cr</td>
<td>27.7</td>
<td>747</td>
<td>1.2</td>
<td>$4.1 \times 10^{17}$</td>
</tr>
<tr>
<td>$^{65}$Zn</td>
<td>244</td>
<td>1343</td>
<td>2.2</td>
<td>$3.0 \times 10^{16}$</td>
</tr>
<tr>
<td>$^{59}$Ni</td>
<td>$2.8 \times 10^7$</td>
<td>1065</td>
<td>1.8</td>
<td>$1.1 \times 10^{14}$</td>
</tr>
<tr>
<td>$^{113}$Sn</td>
<td>116</td>
<td>617</td>
<td>1.0</td>
<td>$3.7 \times 10^{16}$</td>
</tr>
<tr>
<td>$^{32}$P</td>
<td>14.3</td>
<td>continuum</td>
<td>$\approx 2.5$</td>
<td>$5.0 \times 10^{16}$</td>
</tr>
</tbody>
</table>
An empirical quenching factor
(a fit based on a $^3$He gas: Santos et al arXiv:0810.1137(astro-ph))

Figure 1: The quenching factor as a function of the recoil energy of interest in the present work (a). Due to quenching the threshold energy for nuclear recoils is shifted upwards from $T_{th}$ to $T'_{th}$, e.g. from 0.10 to 0.18 keV (b).
Unexpected snug: Threshold effect kills the benefit of large $N^2(\text{large } \sigma)$. Large mass \[\square\] Small recoil energy

Figure 2: We show the minimum neutrino energy required as a function of threshold without quenching (a) and with quenching in (b). From top to bottom for the targets of $^{131}\text{Xe}$, $^{40}\text{Ar}$, $^{20}\text{Ne}$ and $^{4}\text{He}$. The threshold value is very crucial, especially for heavy targets.
Sterile neutrino oscillations: $R_0 = 4\text{m}, P = 10\text{ Atm}$ He target; (NC); full, dotted, dashed curve $\square \sin^2(2\theta_{14}) = 0.27, 0.17, 0.07$

$^{37}\text{Ar} (E_\nu = 811\text{ keV})$ $^{51}\text{Cr} (E_\nu = 747\text{ keV})$
Sterile neutrino oscillations: $R_0 = 4m, P = 10$ Atm, $E_\nu = 1343$ keV; (NC) full, dotted, dashed curve, $\sin^2(2\theta_{14}) = 0.27, 0.17, 0.07$

source: $^{65}$Zn; target: $^{20}$Ne

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Sterile neutrino oscillations: $R_0=4\text{m}, P=10\text{ Atm}$

Antineutrino (continuous) source; (NC)

NC cross section (no oscillation)

Source spectrum

Figure 7: The non oscillating part of the antineutrino cross section in units of $(G_F m_e)^2/(2\pi) = 2.29 \times 10^{-49}\text{m}^2$ as a function of the energy in MeV for a target $^{40}\text{Ar}$ (a) and $^{40}\text{Ne}$ (b), assuming a threshold of 0.1 keV.

Figure 8: The normalized antineutrino spectrum following the beta decay of $^{38}\text{P}$. The vertical line indicates the space on its right allowed for $^{39}\text{Ar}$ (dotted line) and $^{20}\text{Ne}$ (dashed line) targets, assuming a threshold of 0.1 keV.
Sterile neutrino oscillations: $R_0=4\text{m}, P=10\text{ Atm}$

Antineutrino (continuous) source; (NC) full, dotted, dashed curve \(\sin^2(2\theta_{14})=0.27, 0.17, 0.07\)

- Source: $^{32}\text{P}$; Target: $^{40}\text{Ar}$
- Source: $^{32}\text{P}$; Target: $^{20}\text{Ne}$

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The total number of NC events:

\[ N_0 = A + B \sin^2(2\theta_{14}) \]

<table>
<thead>
<tr>
<th>target-source</th>
<th>( A ) (no quenching)</th>
<th>( B ) (no quenching)</th>
<th>( A ) (quenching)</th>
<th>( B ) (quenching)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{40}\text{Ar-}^{32}\text{P})</td>
<td>(2.4 \times 10^2)</td>
<td>(-1.2 \times 10^2)</td>
<td>(4.2 \times 10^2)</td>
<td>(-1.8 \times 10^2)</td>
</tr>
<tr>
<td>(^{40}\text{Ar-}^{205}\text{Bi})</td>
<td>(1.4 \times 10^4)</td>
<td>(-6.6 \times 10^3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{20}\text{Ne-}^{32}\text{P})</td>
<td>(8.8 \times 10^2)</td>
<td>(-4.6 \times 10^2)</td>
<td>(1.0 \times 10^2)</td>
<td>(-5.4 \times 10)</td>
</tr>
<tr>
<td>(^{20}\text{Ne-}^{65}\text{Zn})</td>
<td>(2.9 \times 10^4)</td>
<td>(-1.6 \times 10^4)</td>
<td>(5.3 \times 10^2)</td>
<td>(-2.8 \times 10^2)</td>
</tr>
<tr>
<td>(^{20}\text{Ne-}^{205}\text{Bi})</td>
<td>(7.2 \times 10^3)</td>
<td>(-3.3 \times 10^3)</td>
<td>(3.8 \times 10^3)</td>
<td>(-1.7 \times 10^3)</td>
</tr>
<tr>
<td>(^4\text{He-}^{37}\text{Ar})</td>
<td>(7.8 \times 10)</td>
<td>(-3.9 \times 10)</td>
<td>(3.6 \times 10)</td>
<td>(-1.8 \times 10)</td>
</tr>
<tr>
<td>(^4\text{He-}^{51}\text{Cr})</td>
<td>(8.7 \times 10^2)</td>
<td>(-4.1 \times 10^2)</td>
<td>(3.1 \times 10^2)</td>
<td>(-1.5 \times 10^2)</td>
</tr>
<tr>
<td>(^4\text{He-}^{65}\text{Zn})</td>
<td>(4.0 \times 10^3)</td>
<td>(-2.1 \times 10^3)</td>
<td>(3.3 \times 10^3)</td>
<td>(-1.8 \times 10^3)</td>
</tr>
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<td>(4.6 \times 10^2)</td>
<td>(-2.0 \times 10^2)</td>
<td>(4.3 \times 10^2)</td>
<td>(-1.9 \times 10^2)</td>
</tr>
</tbody>
</table>
Determination of $\theta_{14}$ by NC $^{20}$Ne detector: $\sin^2(2\theta_{14}) = 0.1$ (99%)

- The total number of events:
  
  $N_0 = A + B \sin^2 (2\theta_{14})$

- For $^{65}$Zn (measuring for 50 days):
  
  $A = 5.3 \times 10^2$, $B = -2.8 \times 10^2$
Conclusions A (neutrino oscillations):

- The discovery of neutrino oscillations gave neutrino physics and astrophysics a new momentum.
- The two mass square differences, except for a sign, are known
- The mixing angles $\theta_{12}$ and $\theta_{23}$ are understood.
- The angle $\theta_{13}$ and the phase $\delta_{13}$ are unknown. This is crucial for CP violation in the leptonic sector.
- Neutrino Oscillations like double CHOOZE and NOSTOS may help in determining the neutrino oscillation parameters, including $\theta_{13}$, more precisely.
- The Reactor Neutrino Anomaly implies a fourth (sterile?) neutrino. Neutrino oscillometry with the gaseous STPC detector (NOSTOS) employing relatively intense monochromatic neutrino sources are ideally suited to resolve this issue.
- There remain some technical problems, but they seem to be under control.
Questions that cannot be answered by neutrino oscillations: The mass scale and the sign of $\Delta m^2_{31}$ (normal vs inverted hierarchy or almost degenerate scenario)
Conclusions B (involving neutrinos)

The absolute scale of neutrino mass is still elusive. The combination neutrinoless double beta decay, triton decay, astrophysics may provide the answer.

- We do not know whether the neutrinos are Dirac or Majorana type particles (only neutrinoless double beta decay can settle this issue).
- Neutrinos may be the best probes for studying the deep sky and the interior of dense objects, like supernovae. A network of cheap easily maintainable and robust STPC detectors may be a useful in supernova neutrino detection.
- Shall we ever see the neutrino background radiation? Will we see it before the gravitational background radiation?

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A powerful money problem solver

Once all mighty God gave 5 golden cows, made of pure gold, to be divided among the three neutrino groups as follows:

- 1/3 goes to the neutrino factories.
- 1/3 goes to the long baseline experiments
- 1/6 goes to the short baseline and small detector size experiments

Condition: No cow should be carved.

Obviously the three groups did not know how to do this.
So they went, where else?, to Mulah Nasrudin.
He thought for a while and ordered his wife to bring into the pack their own cow, which he valued more than golden.
Now there are 6 cows in the pack. It was easy for the first two groups to get 2 each and the small group to get 1.
His cow was left and his wife took it back to the stable.
Was this a fair deal?
Was it a fair deal?

- Mullah Nasrudin did not know how to sum infinite series. However:

\[
\frac{5}{3} + \frac{1}{3} \left( \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} \cdots \right) = \frac{5}{3} + \frac{1}{3} = 2
\]

\[
\frac{5}{6} + \frac{1}{5} \left( \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} \cdots \right) = \frac{5}{6} + \frac{1}{6} = 1
\]
THE END
Shielding problem (Preliminary)

\[ z \text{ distance from source, } y \text{ source dimension} \] (by Novikov)

300 keV, photons on lead

J.D. Vergados  BNL 08/04/11
Electron capture—a source of mono-energetic neutrinos

\[ Q_v = Q_\xi - B_e \]
Nuclear process

Atomic process

Time range

0 \(10^{-18}\)s

10^{-10}s

Z-1

N+1

Z

N

Electron vacancy

KX

LX

Auger electron

e\nu

courtesy of J. Khuyagbaatar

J.D. Vergados    BNL 08/04/11
The standard \((v,e)\) cross section

(In the absence of neutrino oscillations)

For low energy neutrinos the historic process neutrino-electron scattering [16] [12] is very useful. The differential cross section [17] takes the form

\[
\frac{d\sigma}{dT} = \left( \frac{d\sigma}{dT} \right)_{\text{weak}} + \left( \frac{d\sigma}{dT} \right)_{\text{EM}}
\]  

(2.1)

\[
\left( \frac{d\sigma}{dT} \right)_{\text{weak}} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left[ 1 - \frac{T}{E_V} \right]^2 + \left( g_A^2 - g_V^2 \right) \frac{m_e T}{E_V^2} \right]
\]  

(2.2)

\[ g_V = 2 \sin^2 \theta_W + 1/2 \quad (\nu_e) \quad, \quad g_V = 2 \sin^2 \theta_W - 1/2 \quad (\nu_\mu, \nu_\tau) \]

\[ g_A = 1/2 \quad (\nu_e) \quad, \quad g_A = -1/2 \quad (\nu_\mu, \nu_\tau) \]

For antineutrinos \(g_A \rightarrow -g_A\).

The scale is set by the weak interaction:

\[
\frac{G_F^2 m_e}{2\pi} = 4.45 \times 10^{-48} \quad \text{cm}^2 \quad \text{keV}
\]  

(2.3)
II: Measure the Weinberg angle at very low momentum transfers

\[ \left( \frac{d\sigma}{dT} \right)_{\text{weak}} = \frac{G_F^2 m_e}{2\pi} \left[ (2\sin^2 \theta_W)^2 + (1 + 2\sin^2 \theta_W)^2 (1 - T/E_\nu)^2 - 2\sin^2 \theta_W (1 + 2\sin^2 \theta_W) (m_e T/E_\nu^2) \right] \] (1.12)
πI : At low neutrino energies: The EM interaction competes with the weak

\[
\left( \frac{d\sigma}{dT} \right)_{EM} = \xi_1^2 \left( \frac{d\sigma}{dT} \right)_{Weak} \left( \frac{\mu_\nu}{10^{-12} \mu_B} \right)^2 \frac{0.1 \text{KeV}}{T} \left( 1 - \frac{T}{E_\nu} \right)
\]

(1.15)

- With \( \mu_\nu \) the neutrino magnetic moment and \( \xi_1 \approx 0.25 \)
- Thus we can obtain the limit: \( \mu_\nu \leq 10^{-12} \mu_B \)
- (present limit: \( \mu_\nu \leq 10^{-10} \mu_B \)
Simulations: $\sin^2(2\theta_{13}) = 0.170$ (left), $\sin^2(2\theta_{13}) = 0.085$ (right)
Current Limits

NEUTRINO OSCILLATIONS 2009

From Schwetz et al, NJP 10 (2008) 113011

Homestake, SAGE+ GALEX/GNO, Super-K, SNO Borexino
KamLAND (180 Km)

... Super-K
K2K (250 Km) MINOS (735 Km)
Neutrino mass terms -
Dirac mass term $M_D$

1. Dirac mass terms like in the charged fermions:

$$\nu_L^0 (M_D) \nu_R^0 + H.C.$$

- It is absent in the SM (the right handed neutrino
does not exist).
- If this is the only mass term, the neutrinos are
  Dirac particles.
- It cannot occur by itself (in GUT’s the neutrino
  should be as heavy as the up-quarks).
- In extra dimensions one can have a small such
  matrix, but one also has Majorana mass terms.
Neutrino mass terms - Majorana mass terms $\mathcal{M}_\nu$ & $\mathcal{M}_N$

2. Majorana mass terms:

$$\bar{\nu}_L^0 (\mathcal{M}_\nu) \nu_R^{0C} + \bar{\nu}_L^{0C} (\mathcal{M}_N) \nu_R^0 + H.C.$$  

- These presuppose lepton violating interactions.
- If any of them occurs the neutrinos are Majorana particles.
- The term $\bar{\nu}_L^{0C} (\mathcal{M}_N) \nu_R^0$ can occur in any theory, since the right handed neutrino carries no standard model quantum numbers.
- The term $\bar{\nu}_L^0 (\mathcal{M}_\nu) \nu_R^{0C}$ is much harder to get.
1. No light Majorana mass term, $\bar{\nu}_L^0 (\mathcal{M}_\nu) \nu_R^{0C} = 0$

- one can get an effective light majorana mass term of the form

$$\bar{\nu}_L^0 (-) (\mathcal{M}_D) (\mathcal{M}_N)^{-1} (\mathcal{M}_D)^T \nu_R^{0C}$$

- This is the celebrated ”see-saw mechanism”
- The neutrinos are light, so long as the right handed Majorana mass is superheavy.
Majorana neutrino mass

2. $\bar{\nu}_L^0 (M_\nu) \nu_R^{0C} \neq 0 \implies$

No need of right handed neutrino. Such matrix is obtained:

- Via isotriplet of Higgs scalars (not without tears).
- Radiatively at one loop level or higher (two Higgs isodoublets).
- Via SUSY R-parity (and hence lepton number) violating interactions
The Mass Hierarchies
- Flavor Content

"Normal" hierarchy

\[ \Delta m_{23}^2 \] (atm.)

\[ \{ \mu, \tau \} \]

\[ \Delta m_{12}^2 \] (solar)

\[ \{ e, \mu, \tau \} \]

or

"Inverted" hierarchy

\[ \Delta m_{12}^2 \]

\[ \{ e, \mu, \tau \} \]

\[ \Delta m_{23}^2 \]

\[ \{ \mu, \tau \} \]
(1): Astrophysics Mass Limit

\[ \sum_k m_k = m_{\text{astro}} = 0.71 \text{eV} \]

- Normal Hierarchy:

\[ \Delta m^2_{SUN} = m_2^2 - m_1^2, \quad \Delta m^2_{ATM} = m_3^2 - m_1^2 \]

\[ m_1 + \sqrt{\Delta m^2_{SUN} + m_1^2} + \sqrt{\Delta m^2_{ATM} + m_1^2} \leq m_{\text{astro}} \]

- Inverted Hierarchy:

\[ \Delta m^2_{SUN} = m_2^2 - m_1^2, \quad \Delta m^2_{ATM} = m_2^2 - m_3^2 \]

\[ m_3 + \sqrt{\Delta m^2_{ATM} + m_3^2} + \sqrt{\Delta m^2_{ATM} - \Delta m^2_{SUN} + m_3^2} \leq m_{\text{astro}} \]
Astrophysics bound: 0.71 eV, \( \text{Log}(0.71) = -0.15 \)

black \( \sum m_k \),
green \( m_3 \),
dotted \( m_1, m_2 \)
green \( m_1 \),
dotted \( m_3, m_2 \)
(2): Triton decay mass limit

\[ m_{\text{decay}} = 2.2 \text{eV} \]

- Normal Hierarchy:

\[ \Delta m_{SUN}^2 = m_2^2 - m_1^2, \quad \Delta m_{ATM}^2 = m_3^2 - m_1^2 \]

The condition is:

\[ c_{12}^2 c_{13}^2 m_1^2 + s_{12}^2 c_{13}^2 (\Delta m_{SUN}^2 + m_1^2) + s_{13}^2 (\Delta m_{ATM}^2 + m_1^2) \leq m_{\text{decay}}^2 \]

- Inverted Hierarchy:

\[ \Delta m_{SUN}^2 = m_2^2 - m_1^2, \quad \Delta m_{ATM}^2 = m_2^2 - m_3^2 \]

The condition is:

\[ s_{13}^2 m_3^2 + s_{12}^2 c_{13}^2 (\Delta m_{ATM}^2 + m_3^2) + c_{12}^2 c_{13}^2 (\Delta m_{ATM}^2 - \Delta m_{SUN}^2 + m_3^2) \leq m_{\text{decay}}^2 \]
Triton decay limit: \( m_{\text{decay}} = 2.2 \text{eV}, \quad \log(2.2) = 0.34 \)

KATRIN \( 0.2 \text{ eV}, \quad \log(0.2) = -0.7 \);

Black \( m_{\text{decay}}(m_1) \),

green \( m_3 \),

dotted \( m_1, \) red \( m_2 \),

\[ m_1 \approx m_2 \approx m_{\text{decay}} \]

dotted \( m_3 \),
Majorana Mass Mechanism

\((\nu)^c = e^{i\phi} \nu, \phi = \alpha_k\) (Majorana condition)
Effective neutrino mass $<m_\nu>$ encountered in $0\nu\beta\beta$-decay

$[\alpha = \alpha_2 - \alpha_1, \beta = \alpha_3 - \alpha_1 + 2\delta_{13}, \alpha_1, \alpha_2, \alpha_3$ Majorana phases $]$

Mass scale: $m_1$ (normal); $m_3$ (inverted)

$$<m_\nu> = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3$$
lower $m_{ee}$ bound from 0ν $\beta\beta$-decay

(From J Valle)

Normal hierarchy                    Inverted

NME from Rodin, Faessler, Simkovic, Vogel

SPECTRUM + ABSOLUTE SCALE + MAJ. PHASE

J.D. Vergados      BNL 08/04/11
The $(\nu, e)$ scattering cross section

The total neutrino-electron scattering cross section as a function of $x$ and $L$ can be cast in the form:

$$\sigma(L, x) = \sigma(0, x) \left(1 - \chi(x)p(L, x)\right)$$  \hspace{1cm} (4.12)

with $x = \frac{E_x}{m_e}$ and

$$\sigma(0, x) = \frac{G_F^2 m_e^2}{2\pi} \frac{x^2 \left(17.7464x^2 + 15.3098x + 3.36245\right)}{(2x + 1)^3}$$  \hspace{1cm} (4.13)

is the total cross section in the absence of oscillations. Furthermore

$$p(L, x) = \sin^2 \left(\frac{0.122959L}{330x}\right) \sin^2(2\theta_{solar}) + \sin^2 \left(\frac{0.122959L}{10x}\right) \sin^2(2\theta_{13})$$  \hspace{1cm} (4.14)

with $L$ the source detector distance in meters and

$$\chi(x) = \frac{2.8664x^2 + 4.1498x + 1.50245}{17.7464x^2 + 15.3098x + 3.36245}$$  \hspace{1cm} (4.15)
Minimal set of Neutrino Parameters

- 3 masses
- 3 angles $\theta_{ij}$
- 3 phases
  - 1 KM-like phase
  - 2 Majorana phases

simplest form of 3-f lepton mixing $K = \omega_{23}\omega_{13}\omega_{12}$

with each factor

\[
\begin{pmatrix}
    c_{12} & e^{i\phi_{12}}s_{12} \\
    -e^{-i\phi_{12}}s_{12} & c_{12}
\end{pmatrix}
\]

for $\Delta L = 0$ oscillations we can drop Maj phases & take KM-like form

\[
\begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
    -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\
    s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23}
\end{pmatrix}
\]
CAST: Another “Greek” Collaboration

- **Probing eV-scale axions with CAST**

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