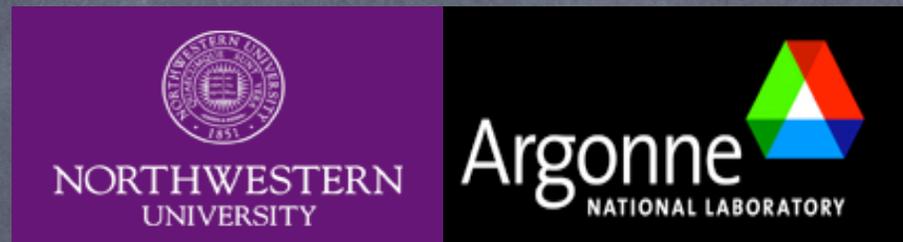


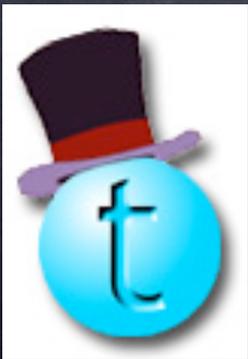
# Truth

(Top Theory)

Tim M.P. Tait



Northwestern University / Argonne National Laboratory



Brookhaven  
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# Outline of the Lectures

- Lecture I: Introduction and Motivation to study top
- Lecture II: Production and Decay
- Lecture III: Special topic: Could top be composite?

Please feel free to stop me with questions at any time!

# Lecture I : Motivation

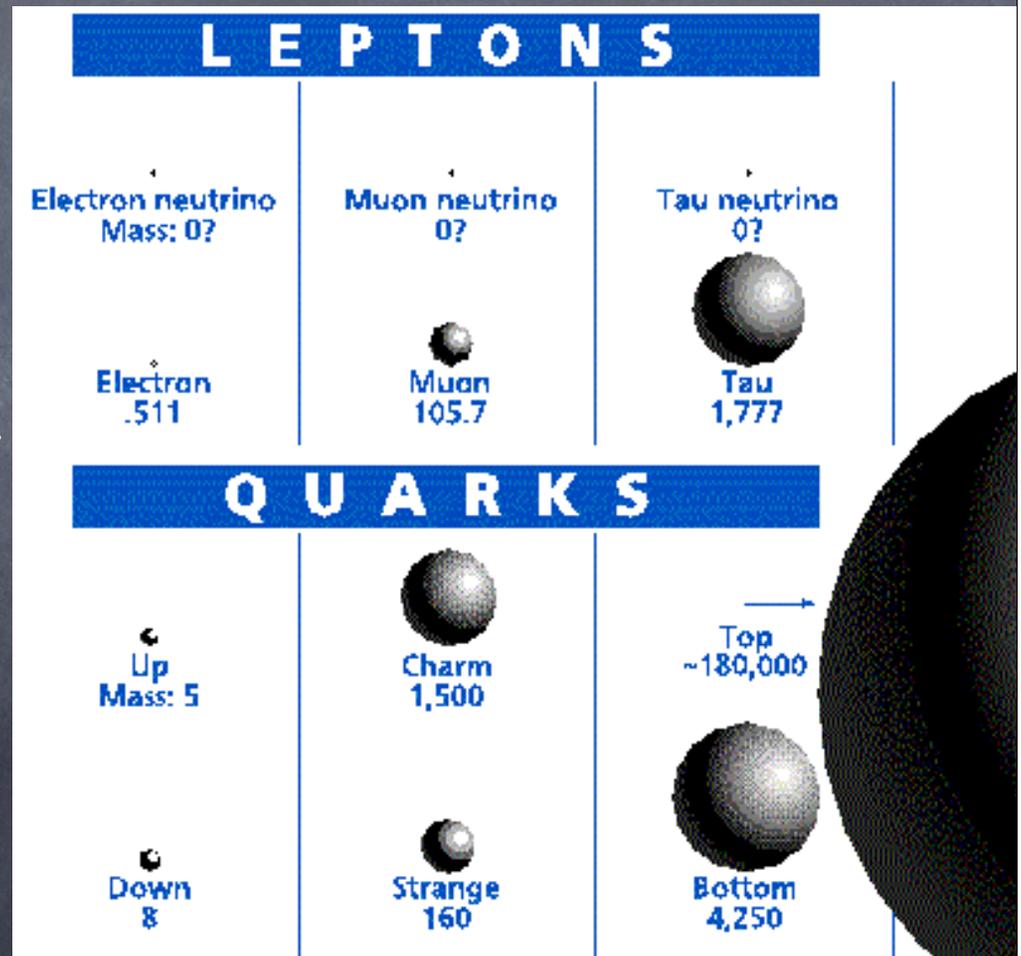
- Importance of Top in the Standard Model
  - Interactions and Parameters
  - SM Phenomena
- Importance of Top Beyond-the-SM
  - Top as a Motivation for new physics.
  - A special role for top in new physics.
  - Top as a "Portal" to new physics.

# Top in the Standard Model

# The King of Fermions!

- In the SM, top is superficially much like other fermions.
- What really distinguishes it is the huge mass, roughly 40x larger than the next lighter quark, bottom.
- This may be a strong clue that top is special in some way.
- It also implies a special role for top within the Standard model itself.
- Top is only fermion for which the coupling to the Higgs is important: it is a laboratory in which we can study EWSB.

## SM Fermions



# Top in the SM

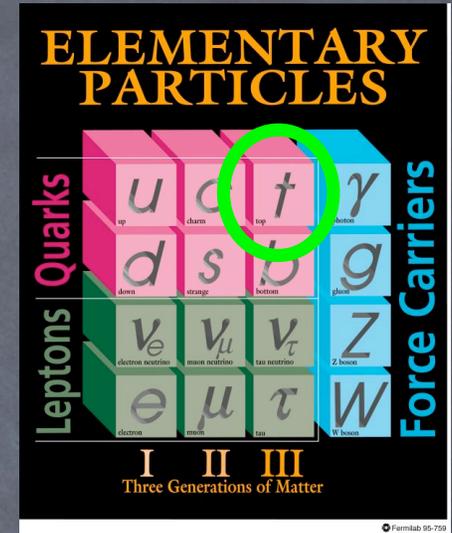
$$Q_{i\alpha} = \begin{bmatrix} t_L^i \\ b_L^i \end{bmatrix}$$

$\alpha=1$ : top  
 $\alpha=2$ : bottom

$i$ : color index  
 $t_R^i$

Top is two separate objects:  
A left-handed quark doublet.  
A right-handed quark singlet.

In four component notation:  $t = \begin{bmatrix} t_R \\ t_L \end{bmatrix}$



$$L = i\bar{Q}_{i\alpha}\gamma^\mu (D_\mu Q)_{i\alpha} + i\bar{t}_i\gamma^\mu (D_\mu t)_i + y_t\tilde{H}\bar{Q}t + H.c.$$

$$(D_\mu Q)_{i\alpha} \equiv \partial_\mu Q_{i\alpha} + ig_S T_{ij}^a G_\mu^a Q_{j\alpha} + ig_W \tau_{\alpha\beta}^n W_\mu^n Q_{i\beta} + i\frac{1}{6}g_Y B_\mu Q_{i\alpha}$$

$$(D_\mu t)_i \equiv \partial_\mu t_i + ig_S T_{ij}^a G_\mu^a t_j + i\frac{2}{3}g_Y B_\mu t_i$$

Dictated by gauge invariance  
and renormalizability!

H: Higgs (doublet)

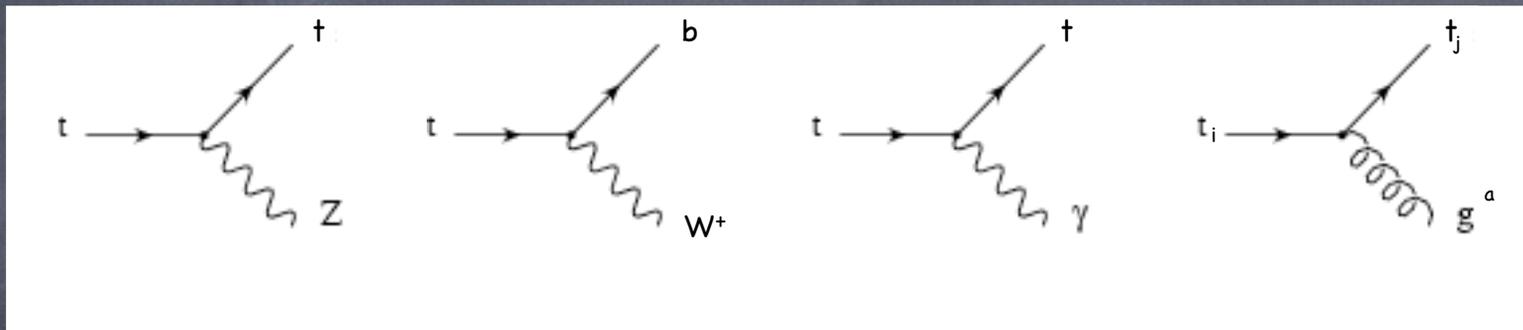
G: gluons

B, W<sup>n</sup>: W<sup>±</sup>, Z, γ

# Feynman Rules

- From the Lagrangian we can read off the SM Feynman rules involving top.

- Gauge bosons:



$$-i \frac{e}{\sin\theta_W \cos\theta_W} \gamma^\mu \times$$

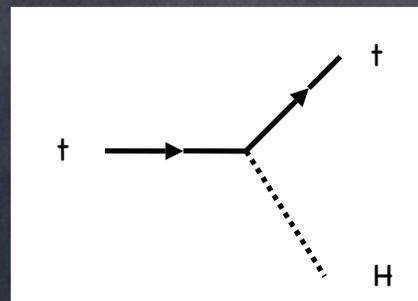
$$\left[ \left( \frac{1}{2} - \frac{2}{3} \sin^2\theta_W \right) P_L + \left( -\frac{2}{3} \sin^2\theta_W \right) P_R \right]$$

$$-i \frac{e}{\sin\theta_W} \gamma^\mu P_L$$

$$+i \frac{2e}{3} \gamma^\mu$$

$$-ig_S T_{ij}^a \gamma^\mu$$

- Higgs:



$$-i \frac{m_t}{v}$$

# Top in the SM

- In the SM, top is the marriage between a left-handed quark doublet and a right-handed quark singlet.
- This marriage is consummated by EWSB, with the mass ( $m_t$ ) determined by the coupling to the Higgs ( $y_t$ ). At tree level,  $m_t = y_t v$ .
- This structure fixes all of the renormalizable interactions of top, and determines what is needed for a complete description of top in the SM.
- Couplings:  $g_s$  and  $e$  are fixed by gauge invariance. The weak interaction has NC couplings, fixed in addition by  $s_w^2$ . CC couplings are described by  $V_{tb}$ ,  $V_{ts}$ , and  $V_{td}$ .

# Current Measurements

- $g_s$ ,  $e$ , and  $s_w^2$  are well known ( $g_s$  at per cent level, EW couplings at per mil level) from other sectors.
- $m_t$  is reconstructed kinematically at the Tevatron:
  - World average:  $m_t = 173.1 \pm 0.6 \pm 1.1 \text{ GeV}$
- $V_{td}$  and  $V_{ts}$  ( $V_{tb}$ ) are determined indirectly from flavor physics:
  - $V_{td}$ :  $0.00874 \pm 0.00003$
  - $V_{ts}$ :  $0.0407 \pm 0.001$
  - $V_{tb}$ :  $0.9999133 \pm 0.00004$

PDG: <http://pdg.lbl.gov/pdg.html>

  - These limits assume the 3 generation SM, reconstructing the values using the unitarity of the CKM matrix.
- $V_{tb}$  is also measured directly from single top production:  $V_{tb} > 0.74$

# Top's Role in the SM

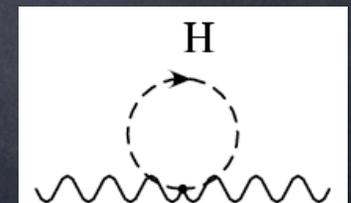
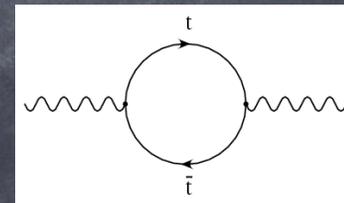
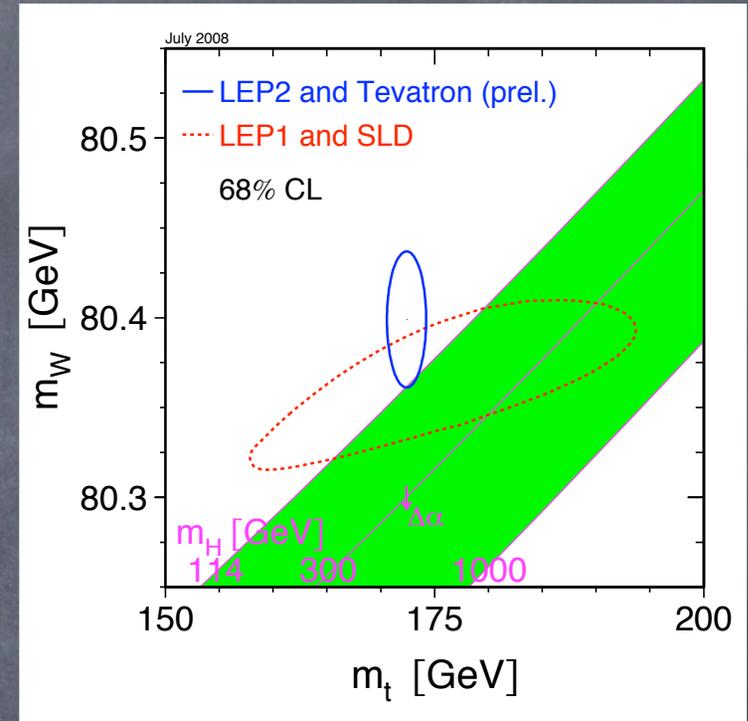
## Precision EW Physics:

- The large top-bottom mass splitting is a strong violation of a custodial SU(2) symmetry (interchanging  $t_R$  and  $b_R$ )
- This results in large corrections to  $\Delta\rho$  ( $\Delta T$ ):

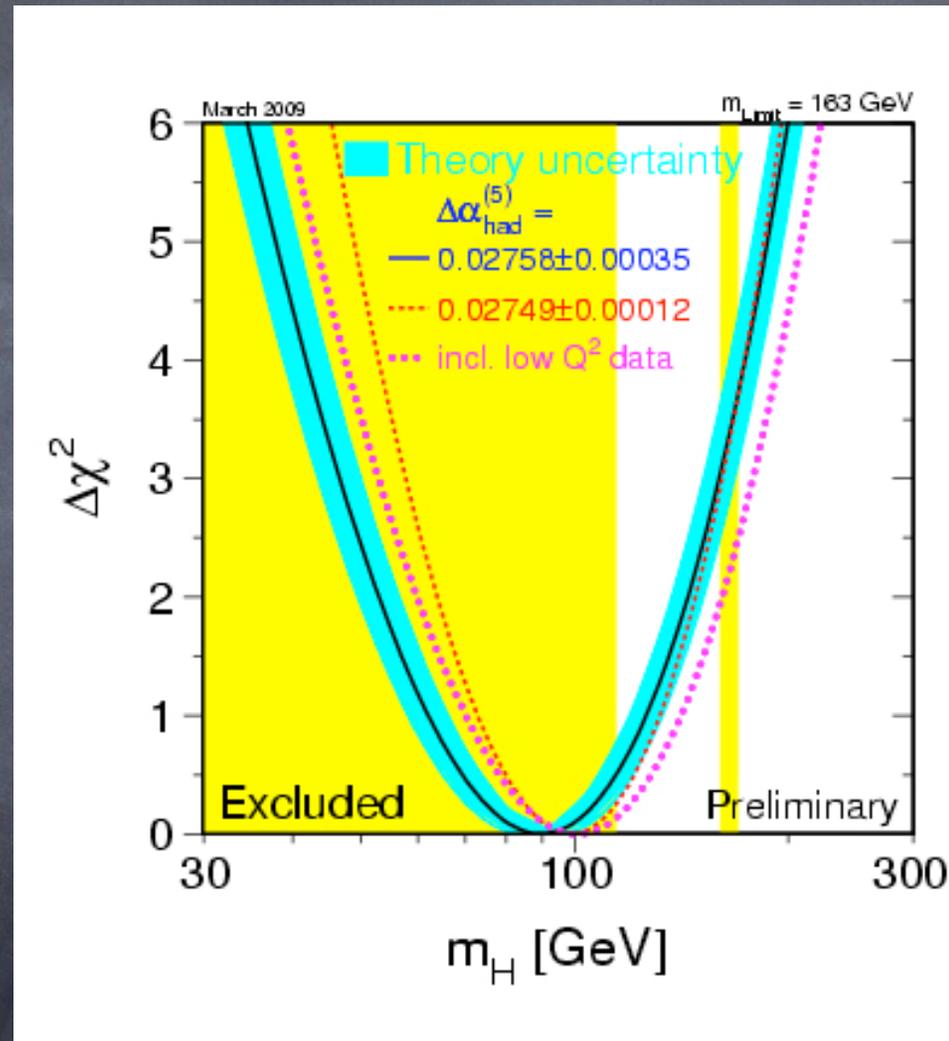
$$= \frac{N_c}{16\pi \sin^2 \theta_W \cos^2 \theta_W} \frac{m_t^2}{m_Z^2}$$

- The one loop corrections are so sensitive to the top mass that precision measurements at LEP/SLD could predict  $m_t$  before top was observed at Tevatron.
- Once  $m_t$  was directly measured, could look for subdominant effects like from the Higgs.

As we move into the era of precision  $m_t$ , it is important to know what exactly we are measuring!



# The SM Higgs Mass

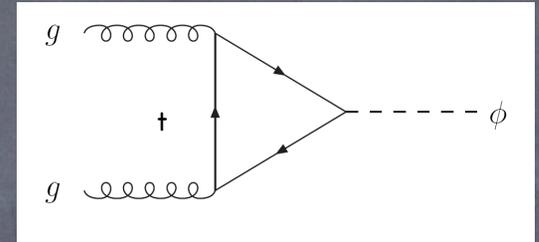


The errors on the top mass control the width of the  $\Delta\chi^2$  distribution.

# Higgs Production

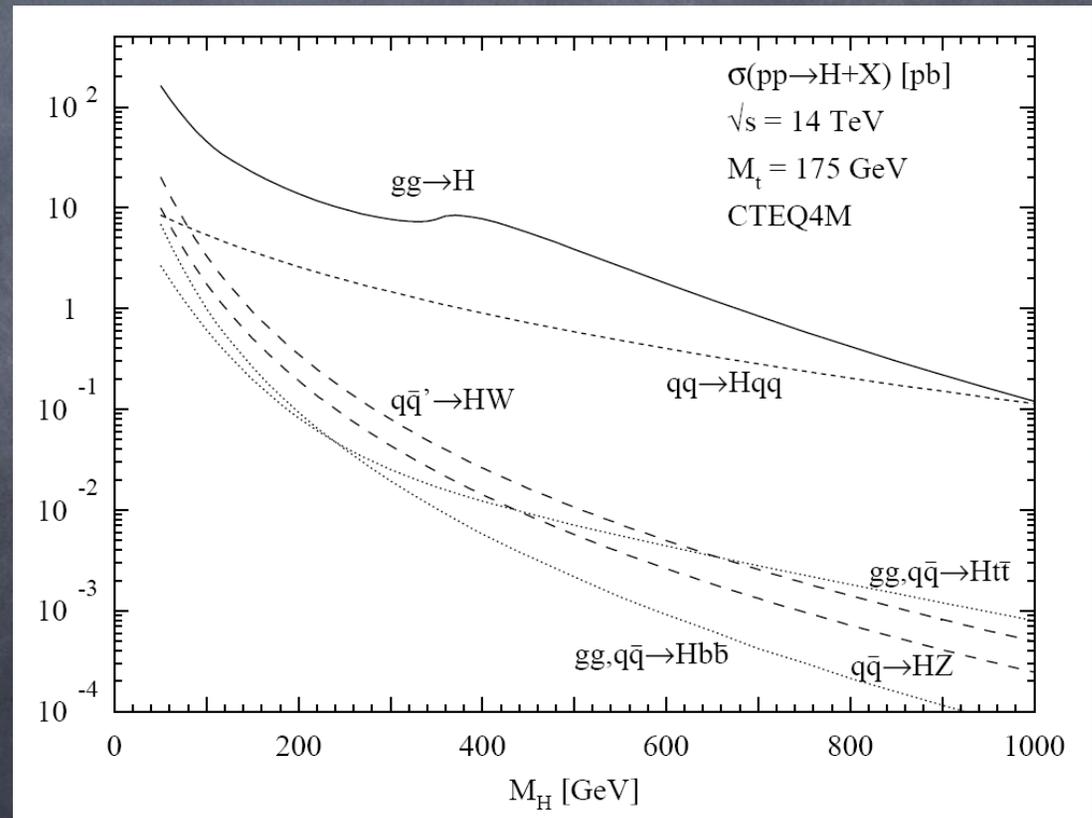
The large top mass means a strong coupling to the Higgs. Thus, several mechanisms of Higgs production rely on Top.

$$\sigma(gg \rightarrow h) \sim m_t^4$$



One in particular takes advantage of the fact that top is colored. Loops of top quarks mediate an interaction between Higgs and gluons. Despite being loop suppressed, this process *dominates* Higgs production at the LHC!

Top also contributes to the Higgs coupling to two photons.



# Top and Flavor Physics

Flavor Physics:

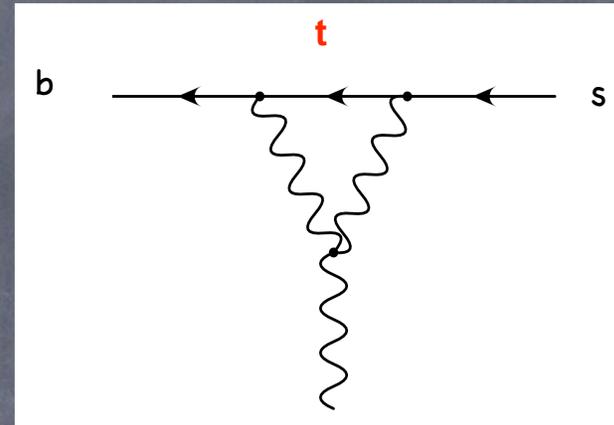
i.e.:

$$B \rightarrow X_s \nu \bar{\nu}$$

$$\text{BR}(B \rightarrow X_s \nu \bar{\nu}) = 4.1 \times 10^{-5} \frac{|V_{ts}|^2}{|V_{cb}|^2} \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{2.30}$$

$$B_s \rightarrow \mu^+ \mu^-$$

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = 4.18 \times 10^{-9} \left[ \frac{|V_{ts}|}{0.04} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{3.12}$$



Buchalla, Buras, Lautenbacher RMP68,1125 (1996)

Precision inputs  
from the top sector  
for precision SM  
predictions.

Top's large mass disrupts the GIM mechanism!

# Perspective: Top in the SM

- In the Standard Model, top is described by gauge couplings we already know, CKM elements we can infer from unitarity, b physics, and single top production.
- So within the context of the SM, the important measurements involving top per se are the **top mass**, and perhaps  $V_{tb}$ .
- These inputs are important in terms of understanding the Standard model, and Higgs physics.
- My interest in the top quark arises from the possibility that top motivates/plays a role in/acts as a portal to physics beyond the Standard Model.

# Top Beyond the Standard Model

# Motivation:

## The Hierarchy Problem

- One of the major dissatisfactions of the SM picture of EWSB is the hierarchy problem.
- The Higgs mass is quadratically sensitive to new physics.
- Since we believe there IS new physics at the GUT or Planck scales, this raises the question: how did the weak scale turn out so low compared to those energies?
- The top, because of its large coupling to the Higgs, provides us with a generic way to “define” this problem...

# Hierarchy Problem

- Usually at this point theorists start talking about “quadratic divergences” and “cut-off dependence”.
  - I’m going to try to explain this in a more physical way.
- The Higgs potential has a dimensionful (“mass”) term and dimensionless (“quartic”) term:

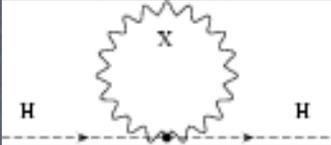
$$V_{\Phi} = \lambda \left( |\Phi|^2 - v^2 \right)^2$$

$$M_W = g v$$

- The Higgs VEV, and thus the  $W/Z$  masses are linearly related to  $v$ .
  - So  $\lambda v$  can’t be much bigger than  $M_W$  and  $M_Z$ .
- Imagine there are new heavy particles that couple to the Higgs.
  - Heavy gauge bosons left-over from a GUT theory.
  - Right-handed neutrino needed in the seesaw theory of neutrino masses.
  - These examples couple to the Higgs directly.
  - All particles couple to it through gravity.

# Naturalness

- So what does this hypothetical heavy particle do to  $v^2$ ?
- It corrects it through loops. At one loop, in the specific case of a GUT gauge boson, the correction looks like:



A Feynman diagram showing a loop of a heavy particle X (represented by a wavy line) between two Higgs bosons H (represented by dashed lines). A blue arrow points to the right, indicating the resulting correction term.

$$\frac{g^2}{16\pi^2} M_{GUT}^2$$

- The GUT scale has appeared as the mass of the vector boson.
- In perturbation theory, the  $v^2$  measured in experiments (for example, when we measure  $M_W$  at LEP or CDF) is the sum of the tree level piece plus all of the higher order corrections:

$$v^2 = v_0^2 + \frac{g^2}{16\pi^2} M_{GUT}^2 + \dots$$

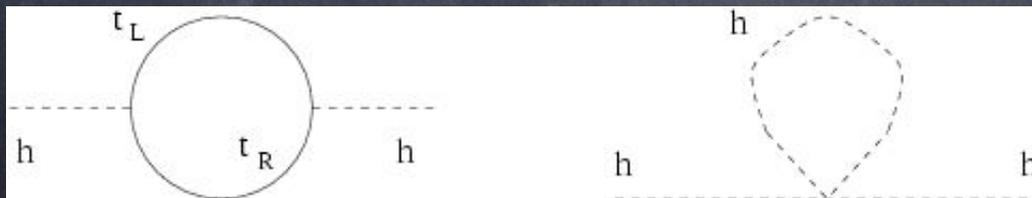
- Here we see the issue: the loops should be small, but if the masses that go into the loops are large, then they are huge.
- But we don't know what the tree-level piece ( $v_0^2$ ) was...

# Fine-Tuning

- Our GUT doesn't have any **technical** problem.
  - We can always choose  $v_0^2$  such that it compensates for the big corrections, and get  $v^2$  to turn out the way we need it to.
  - But this is a (drastic!) "fine-tuning" of parameters.
  - Since we know  $M_{\text{GUT}} > 10^{16} \text{ GeV}$  and  $v \sim 100 \text{ GeV}$ , we need the tree-level and the higher order corrections to match each other to one part in  $10^{28}$  (since it's  $v$ -squared).
- This really seems to be asking for a mechanism to make it work out.
  - The usual solutions work by adding something which cancels the loop corrections (or makes them small) so that the tree-level piece is dominant.
  - Whatever this new stuff is, it should have mass around  $v$ , or it will recreate the problem with the new stuff itself.

# Top and the Hierarchy Problem

- Its all well and good to believe in new physics, and worry about tuning away its contributions to  $M_W$ , but where does top fit into the picture?
- In fact, there is usually a clear link to top!
  - If the mechanism to solve the hierarchy problem involves new symmetries, top will generically be forced to participate as well.
  - In my GUT example, adding a symmetry to help with the hierarchy problem (SUSY?) adds partners for the GUT bosons, and also for the top quark. (In fact for the whole SM).
  - So a generic solution will usually extend the “top sector”.
- By looking at loops of SM particles, we can be more quantitative about where we expect new particles and what they should look like:



$$\sim \frac{g^2}{16\pi^2} M^2 \lesssim v^2$$

As the particle most strongly coupled to the Higgs, the top sector needs the **smallest** new particle mass  $M$  in order not to “over-correct”  $v^2$ !

# Summary: Top & Hierarchy

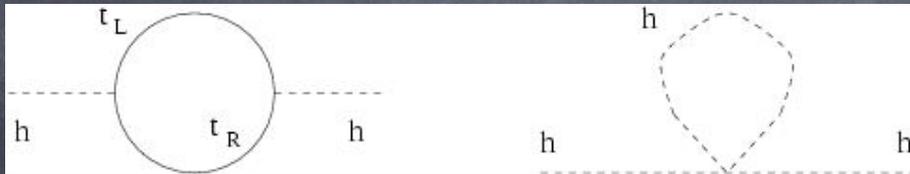
- Putting these arguments together, the hierarchy problem argues for new particles whose masses (to be natural) must be around the weak scale.
- Looking at the effect of the SM particles themselves on the Higgs mass, the more strongly coupled they are to the Higgs, the lighter their new partners must be in order not to destabilize the electroweak scale.
- Top is the most strongly coupled object to the Higgs we know about. Its partners must be light ( $<$  about 2 TeV).
- (Notice I didn't need to make any specific assumption about the actual solution of the hierarchy problem here – I just made some general assumptions about its nature!)

# A Role for Top in BSM

- Just as in the Standard Model, the large top mass (and thus strong coupling to the Higgs) is something a BSM model can take advantage of.
  - Supersymmetric (MSSM) Higgs Mass.
  - Radiative Electroweak Symmetry-breaking.
  - Balancing a Heavy Higgs.

# Supersymmetry

- Supersymmetry is the best-motivated and best-studied solution to the hierarchy problem.
- The super-partners cancel exactly (if SUSY was exact, anyway) the big contributions to  $v^2$ .



- As an added bonus, most SUSY theories contain a lightest super-partner which is neutral and stable – a dark matter candidate!
- New sources of  $CP$  violation and extra DOF can lead to EW baryogenesis!
- SUSY has a lot of model parameters (all related to how we break it and give masses to the super-partners).
  - We have some theoretical guidance as to the rough features, but even those arguments aren't infallible.

# Supersymmetry Breaking

- There are many, many ideas for SUSY-breaking on the market:
  - **SUGRA**: Gravity mediates SUSY breaking to the MSSM super-partners. Not clear why flavor would work out.
  - **GMSB**: Gauge interactions mediate SUSY breaking – a nice solution to the flavor problem. Less great for dark matter and EWSB.
  - **AMSB**: SUSY breaking is transmitted via the super-Weyl anomaly. Issues with negative slepton masses (squared).
  - $\tilde{g}$ **MSB**: Extra dimensional gauge interactions transmit SUSY breaking.
  - “**Orbifold SUSY breaking**”: SUSY breaking by boundary conditions in an extra dimension.
  - **????** : Theorists are constantly looking for new ways to mediate SUSY breaking! (Mirage mediation, direct gauge mediation,... )

# MSSM Higgs Mass

- The large top mass has turned out to be essential in the success of the Minimal Supersymmetric Standard Model (MSSM). Let's see how this works.
- We saw before that in the SM, the Higgs quartic  $\lambda$  is a free parameter. The physical Higgs mass is  $m_h^2 = \lambda v^2$ .
- We can adjust the Higgs mass to whatever we like by playing with the value of  $\lambda$  (up to about 1 TeV when the theory gets too strongly coupled and stops making sense).
- In the MSSM, the story is a little bit more complicated, because the theory has two Higgs doublets, so two CP even Higgs bosons which can share the VEV between them.

$$\tan \beta \equiv \frac{\langle H_1 \rangle}{\langle H_2 \rangle}$$

# SUSY Little Hierarchy Problem

- Remarkably, in the MSSM  $\lambda$  is not a free parameter. Its value is dictated by supersymmetry to be equal to a combination of the electroweak gauge couplings:

$$\lambda = \sqrt{g_W^2 + g_Y^2}$$

- We already know these couplings :  $\lambda = \frac{M_Z}{v}$
- This results in a tree-level prediction for the lighter Higgs:

$$m_h^2 = \lambda v^2 \cos^2 2\beta \leq M_Z^2$$

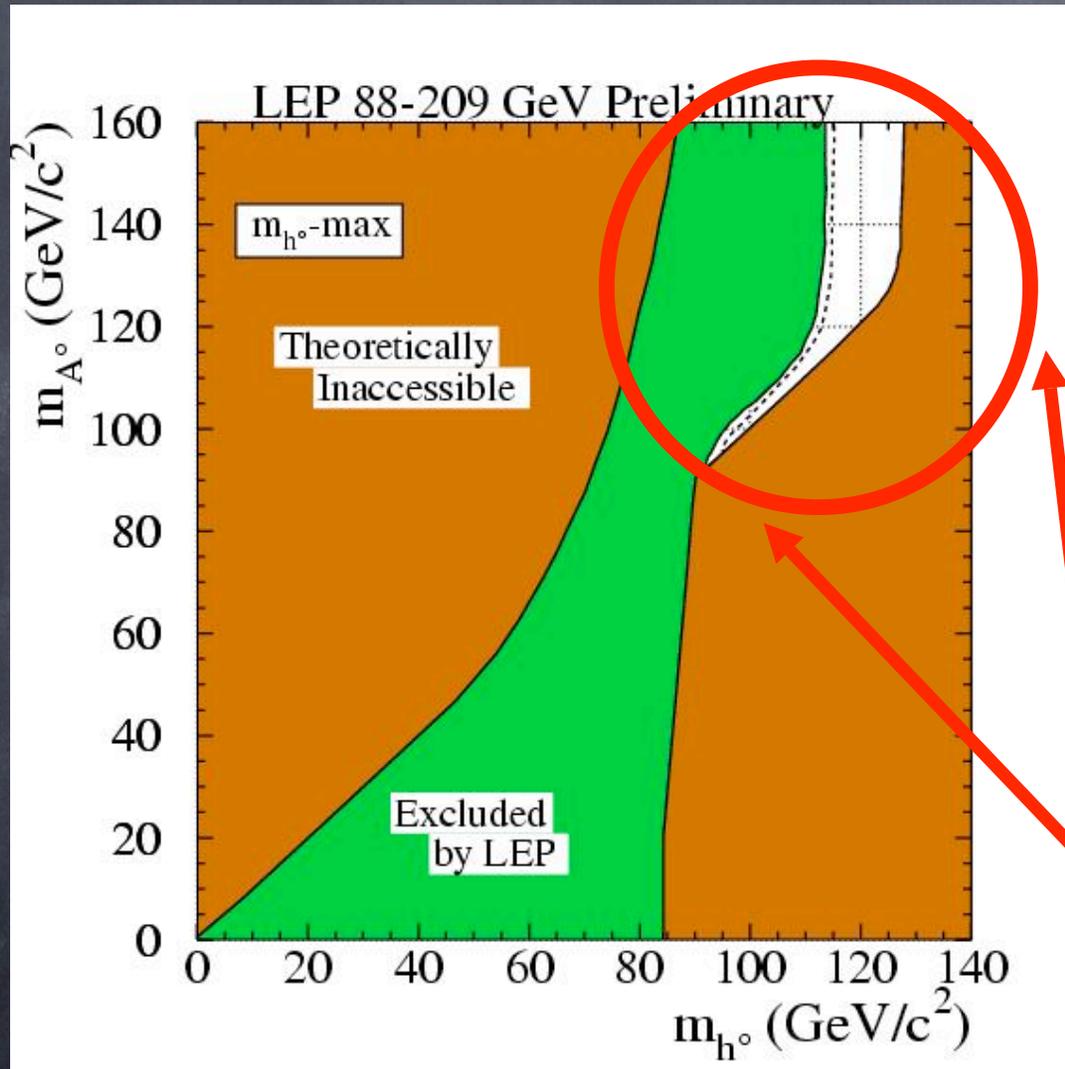
- Such a light Higgs is ruled out by LEP-II's Higgs searches.
- This is just the tree level result. Maybe loops can help? The best hope comes from top - the biggest coupling!

# Top to the Rescue!

- The corrections to the MSSM Higgs mass are largest for the top and its supersymmetric partners
  - two stops, one “right-handed” and one “left-handed”.
- The over-all size is set by the top Yukawa coupling, since SUSY requires stop and top have the same coupling to the Higgs bosons.
- The correction further depends on the stop masses and the amount of mixing between the left- and right-handed stops:

$$\delta m_h^2 = \frac{3m_t^4}{8\pi^2 v^2} \left[ \log \frac{m_{\tilde{t}}^2}{m_t^2} + \text{stop mixing} \right]$$

# LEP II Bounds



- LEP II rules out

$$m_h^{(SM)} \leq 115 \text{ GeV}$$

- The boundary on the right is the MSSM upper limit, assuming  $M \sim 1 \text{ TeV}$  and maximal stop mixing.
- The dashed curves are hypothetical exclusions assuming only SM backgrounds.

- The MSSM lives in the white sliver.

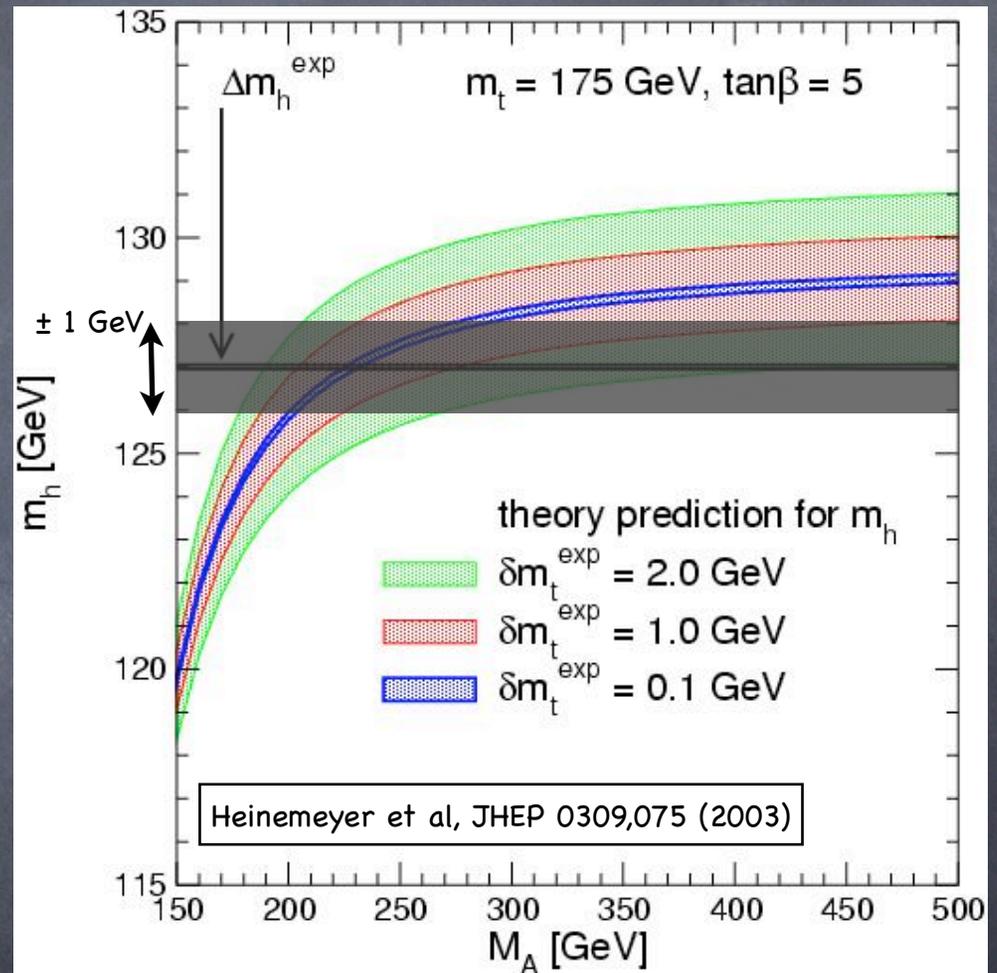
The fact that it is a "sliver" means top is barely able to do the job for reasonable values of the stop masses!

# MSSM and the Top Mass

- The MSSM Higgs has a love/hate relationship with the top mass:
  - The Higgs quartic corrections vary as  $m_t^4$ .
  - The mass parameter varies as  $m_t^2$ .
  - So in the MSSM, a larger top mass does result in a heavier Higgs, with less electroweak fine-tuning.
- The MSSM lives or dies by the top mass!
  - If the top mass were to increase by about  $2\sigma$ , the SUSY little hierarchy problem would completely cease to be an issue.
  - If the top mass were to decrease by about  $2\sigma$ , the MSSM would be fine-tuned much **below the % level**.
  - Below  $m_t \sim 160$  GeV, the stops would have to be higher than 1000 TeV to survive the LEP-II limit and I would safely say the MSSM was "**ruled out**".

# Top-Stop-Higgs

- There are a few ways to think about the importance of top in the MSSM:
- Precise measurements of  $m_t$  and the Higgs mass tell us about the stops (and  $M_A$ ).
- A mismatch between  $m_t$ ,  $m_h$ , and the stop parameters tells us we don't have the MSSM, and we need to think about extended SUSY models (like the NMSSM, for example).

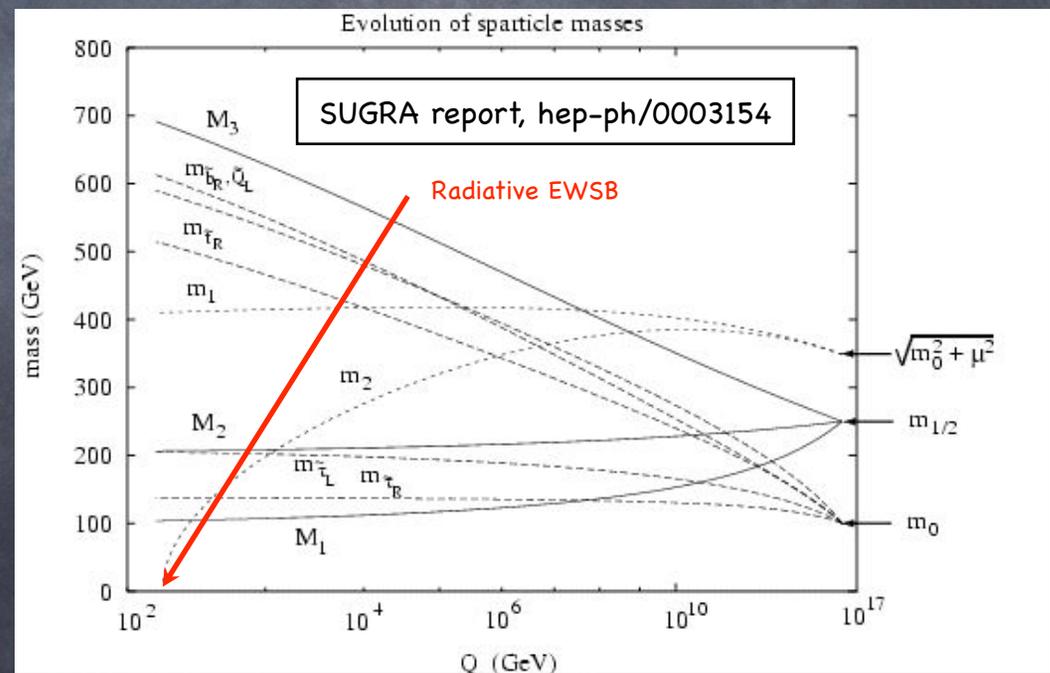


The Bottom Line:  $m_t$  is **IMPORTANT** for the MSSM!

# Radiative EWSB

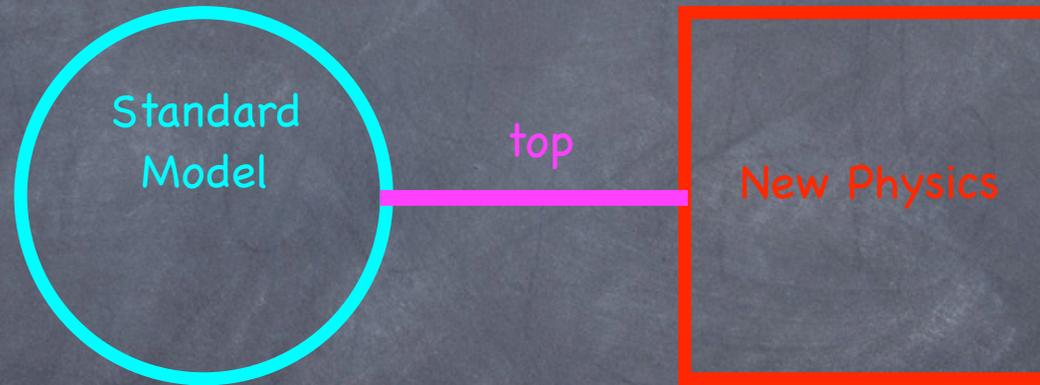
- The strong top coupling to the Higgs allows for radiative electroweak symmetry breaking.
- Even if the Higgs mass<sup>2</sup> is positive at some high scale, loops of the top quark can run it negative at low energies, triggering EWSB.
- This happens naturally in many SUSY theories.
  - In mSUGRA SUSY theories, one uses it to fix the  $\mu$  parameter.

It is also a phenomenon used by most little Higgs theories, and by theories in which the Higgs is a bound state of top quarks (like top color)



# Top as a Portal to BSM

- If the top mass is a clue that the top is special, it may be that top acts as a kind of “bridge” or portal between the Standard Model and the new physics.



- In that case, new physics may be produced in association with the top quark, or may manifest itself in top observables.

# Composite Higgs

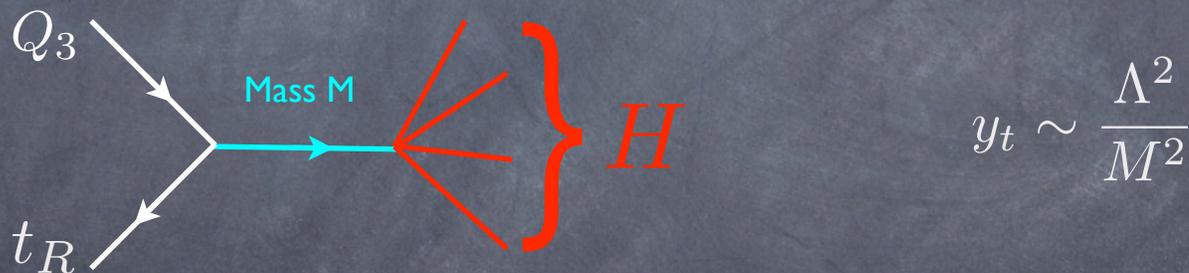
- One particular illustration of the top as a portal is furnished by theories where the Higgs is composite.
  - (This could itself be a connection to top, as in top-color where the Higgs is a bound state of tops themselves, or it could be a parallel construction, as in technicolor theories).
- But arguing very generically, imagine the Higgs is a composite, meaning it is made out of two or more fundamental particles (“preons”):

e.g. Higgs made of two fermions: 
$$H \leftrightarrow \bar{\psi}_1 \psi_2 / \Lambda^2$$

- The scale  $\Lambda$  is the “confinement scale” of the Higgs binding (something like the size of the composite Higgs).
- Such a composite Higgs presents no problem for generating the W and Z masses. (Because for them it is enough just to say how the symmetry is broken).
- The same hierarchy/naturalness arguments we had before argue that if the Higgs being composite is supposed to solve the hierarchy problem,  $\Lambda \sim \text{TeV!}$

# Fermion Masses from a Composite Higgs

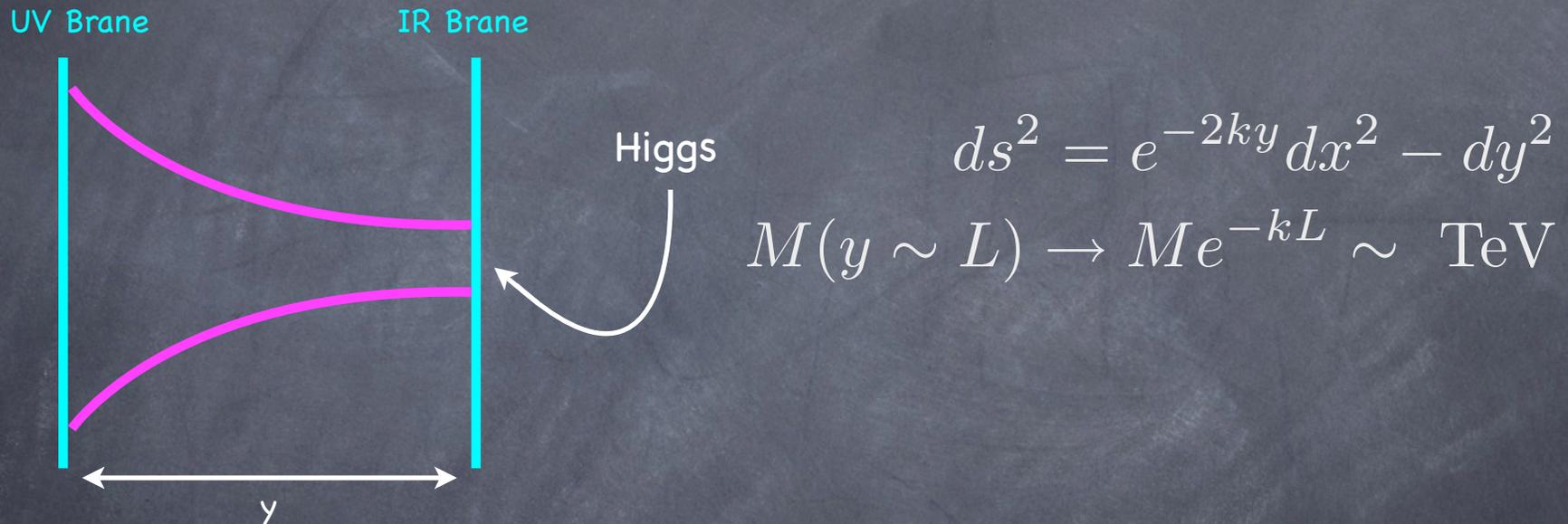
- Fermion masses are more tricky. Because the Higgs is fundamentally more than one particle, it can't have renormalizable interactions with  $t$   $\bar{t}$ .
- To generate the top mass, I need to introduce even more new physics, to communicate between the top and the preons that make up the Higgs.



- As the heavy quark in the SM, top requires that the extra physics scale  $M$  is basically the same scale as  $\Lambda \sim \text{TeV}$ . So there must be some kind of special dynamics for top if the Higgs is composite, in order to generate its mass.
- In Extended technicolor theories (ETC), this was a killer, because it led to too much flavor violation in Kaons - driven by the large top mass!

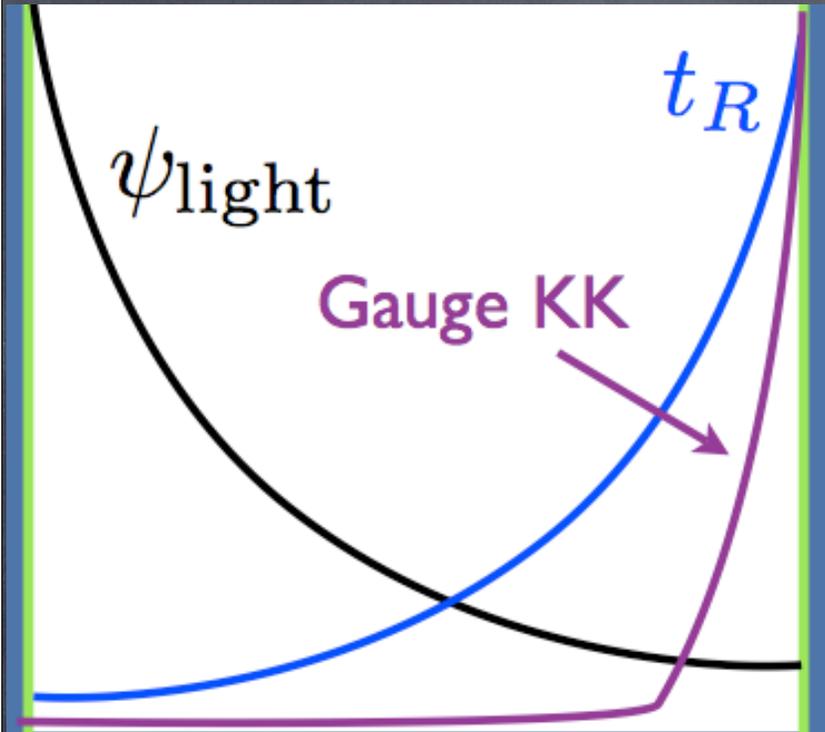
# Randall-Sundrum

- Another example of top as a portal comes up in RS models. The original RS model had gravity living on one end of an extra dimension, and the SM living on the other:



- This solved the hierarchy problem, because the space is warped enough that the fundamental scale of physics on the IR brane is TeV. The solution to the hierarchy problem doesn't really care where most of the SM is, just where the Higgs is.
- The versions theorists are excited about these days have the Higgs still near the IR brane, but the rest of the SM out in the bulk.

# Fermion Geography



Top couples most strongly to KK modes, and thus they dominantly decay into it!

- The way particles couple is given by the integral of their profiles in the extra dimensions:

$$g_{ijk} = \int_0^L dy f_i(y) f_j(y) f_k(y)$$

- So the way to understand couplings is to understand where different particles live in the extra dimension.
- The warped space implies that the KK modes always live near the IR brane. So they couple strongly to the Higgs.
- We can arrange the fermions as we like:
  - Light fermions do better close to the UV brane, so they have weaker couplings to KK modes and less constraints from EW.
  - The top MUST live close to the IR brane in order to have a strong enough coupling to the Higgs.

# Outlook

- Top is important in the Standard Model. But it is really exciting in relation to physics beyond the Standard Model.
  - It motivates the shape of new physics to address the hierarchy problem.
  - Its large mass gives it a unique role in BSM theories such as SUSY.
  - It may be our portal to access BSM physics.
- Tomorrow we'll look at top production and decay in the Standard Model and beyond - and see how all of the theories we discussed today manifest themselves in top observables.