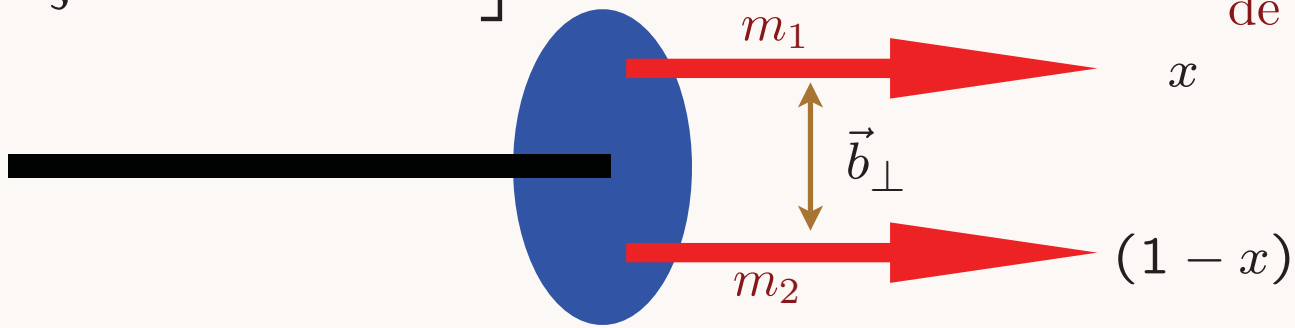
$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

de Teramond, sjb



$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

Holographic Variable

$$-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}$$

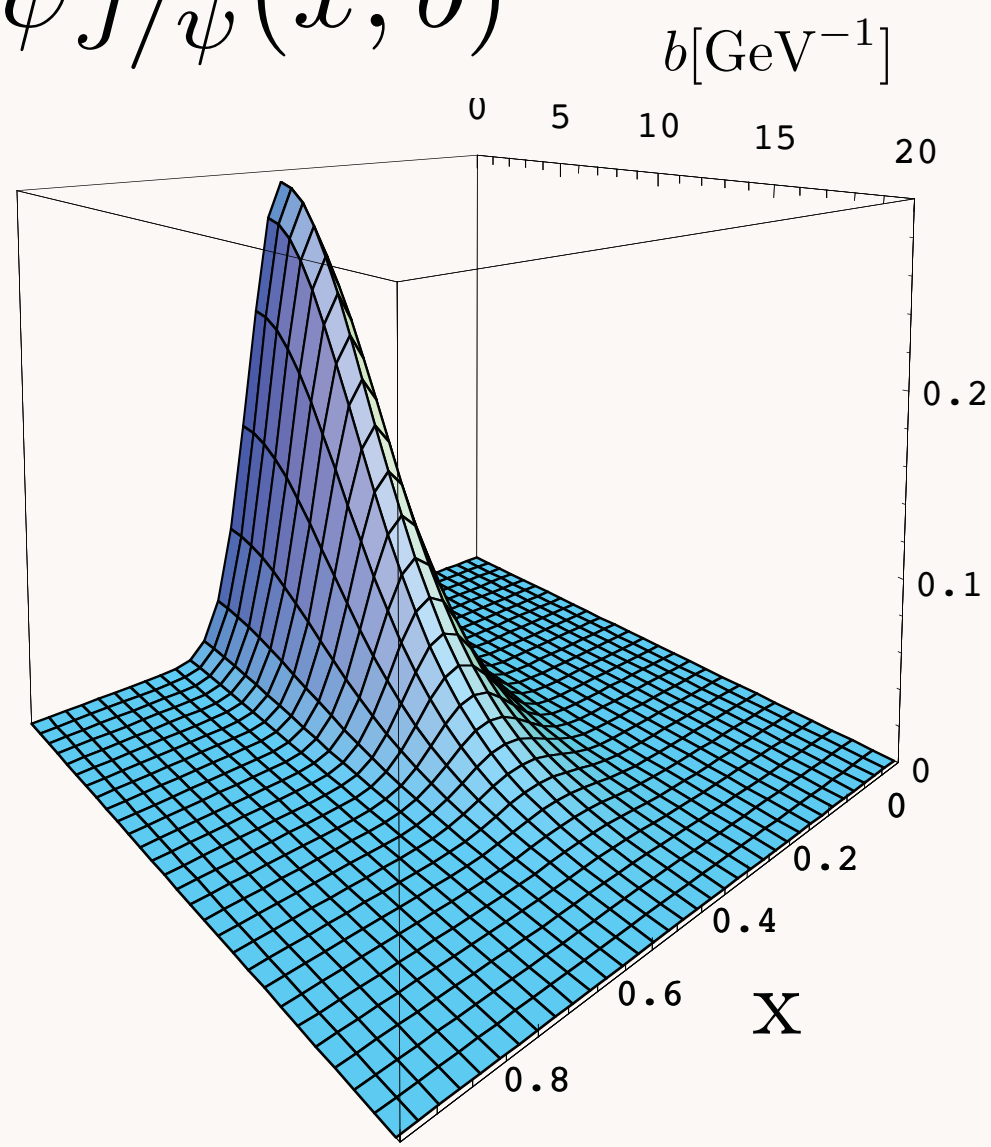
LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

$$-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}$$

J/ψ

$\psi_{J/\psi}(x, b)$



LFWF peaks at

$$x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

where

$$m_{\perp i} = \sqrt{m^2 + k_\perp^2}$$

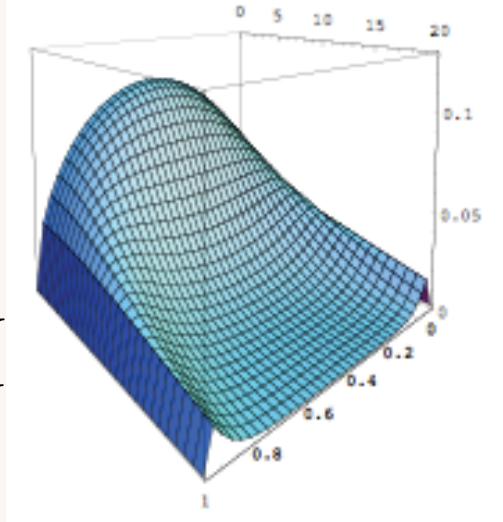
minimum of LF energy denominator

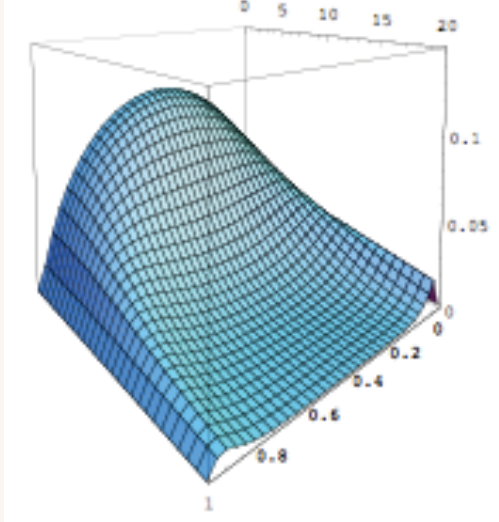
$$\kappa = 0.375 \text{ GeV}$$

$$m_a = m_b = 1.25 \text{ GeV}$$

$|\pi^+ >= |u\bar{d} >$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



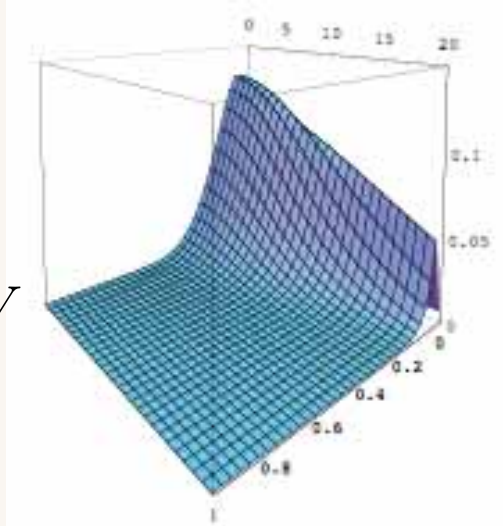


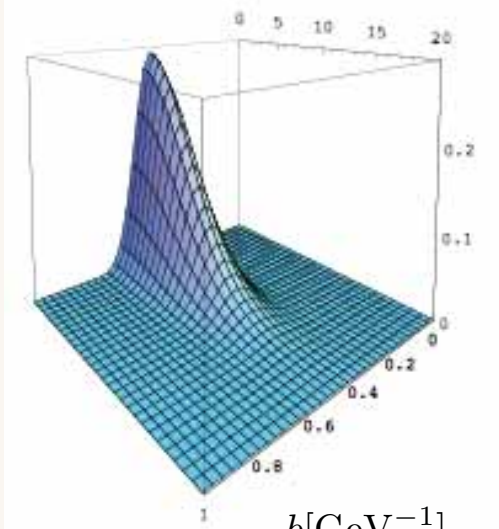
$|K^+ >= |u\bar{s} >$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d} >$

$m_c = 1.25\ GeV$

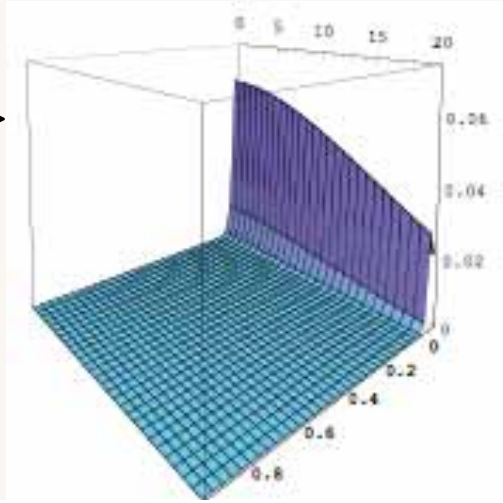


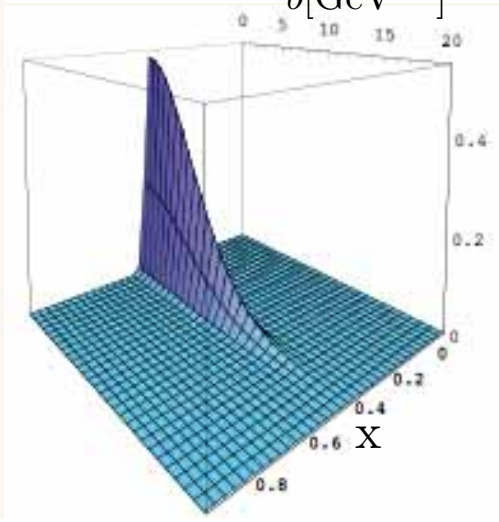


$|\eta_c >= |c\bar{c} >$

$|B^+ >= |u\bar{b} >$

$m_b = 4.2\ GeV$





$|\eta_b >= |b\bar{b} >$

$\kappa = 375\ MeV$

Meson LFWF (L=0) for massive quarks

$$\psi_{\bar{q}q/\pi}(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{\mathbf{k}_\perp^2 + m^2}{2\kappa^2 x(1-x)}},$$

$$\widetilde{\psi}_{q\bar{q}/\pi}(x, \mathbf{b}_\perp) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\, \exp\left(-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 - \frac{m^2}{2\kappa^2 x(1-x)}\right)$$

Key variable for n-parton LFWF with massive quarks:

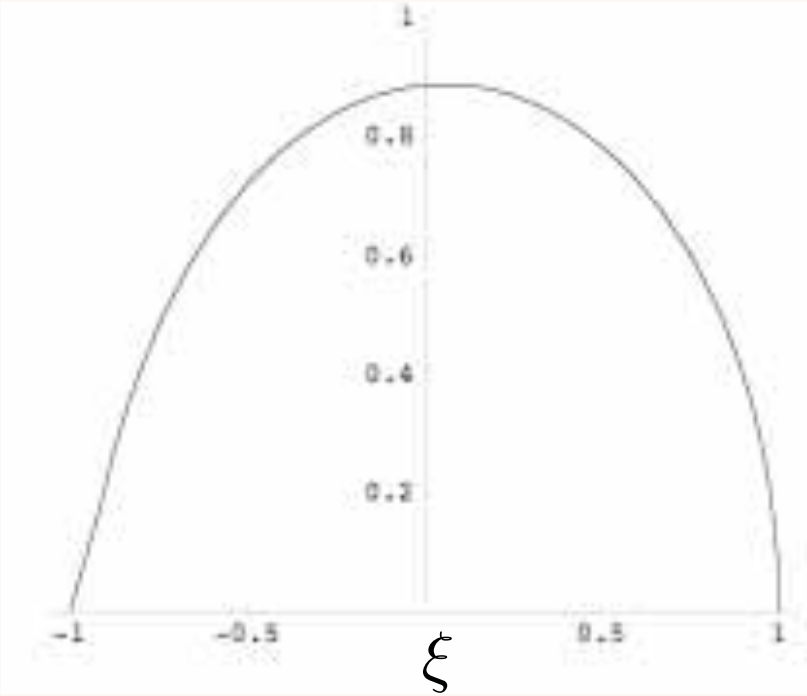
$$\boxed{\chi^2 = \zeta^2 + \frac{1}{\kappa^4}\sum_{i=1}^{n}\frac{m_i^2}{x_i},}$$

$$\zeta = \sqrt{\frac{x}{1-x}}\left|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right|$$

First Moment of Kaon Distribution Amplitude

$$< \xi >= \int_{-1}^{1} d\xi \; \xi \; \phi(\xi)$$

$$\xi = 1 - 2x$$



$$< \xi >_K = 0.04 \pm 0.02 \qquad \kappa = 375 \; MeV$$

Range from $m_s = 65 \pm 25$ MeV (PDG)

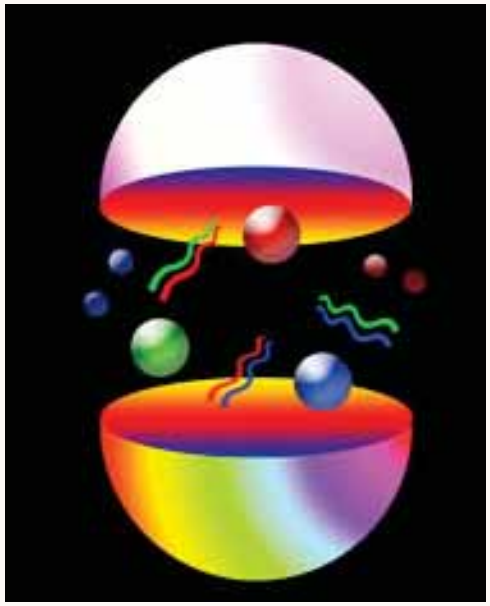
$$< \xi >_K = 0.029 \; \pm 0.002$$ Donnellan et al.

$$< \xi >_K = 0.0272 \pm 0.0005$$ Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD

  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

Baryons in AdS/CFT



- Action for massive fermionic modes on AdS$_{d+1}$:

$$S[\overline{\Psi},\Psi] = \int d^{d+1}x\,\sqrt{g}\,\overline{\Psi}(x,z)\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z).$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right)\Psi(x,z) = 0$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R\right]\Psi(x^\ell) = 0.$$

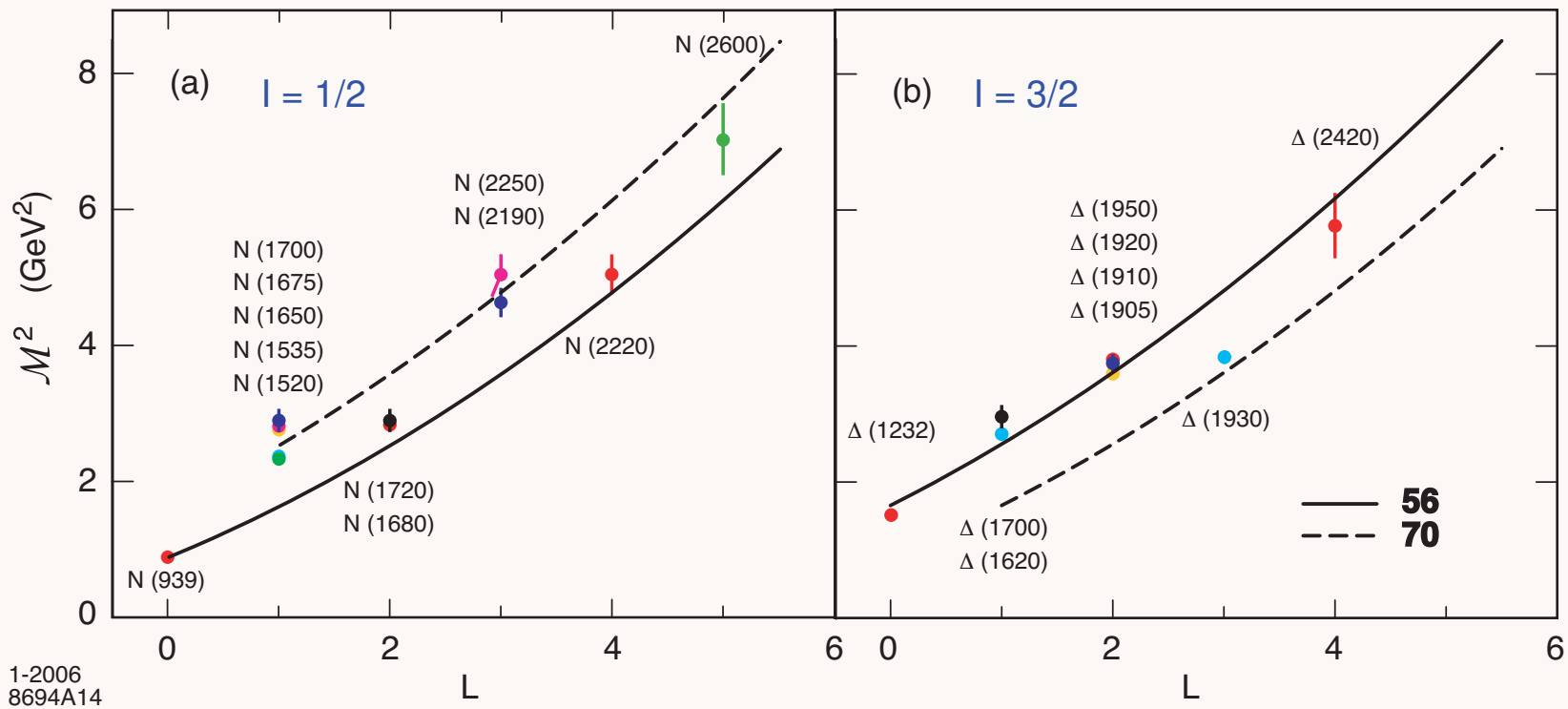


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu}e^{-\kappa^2\zeta^2/2}L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta)|\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

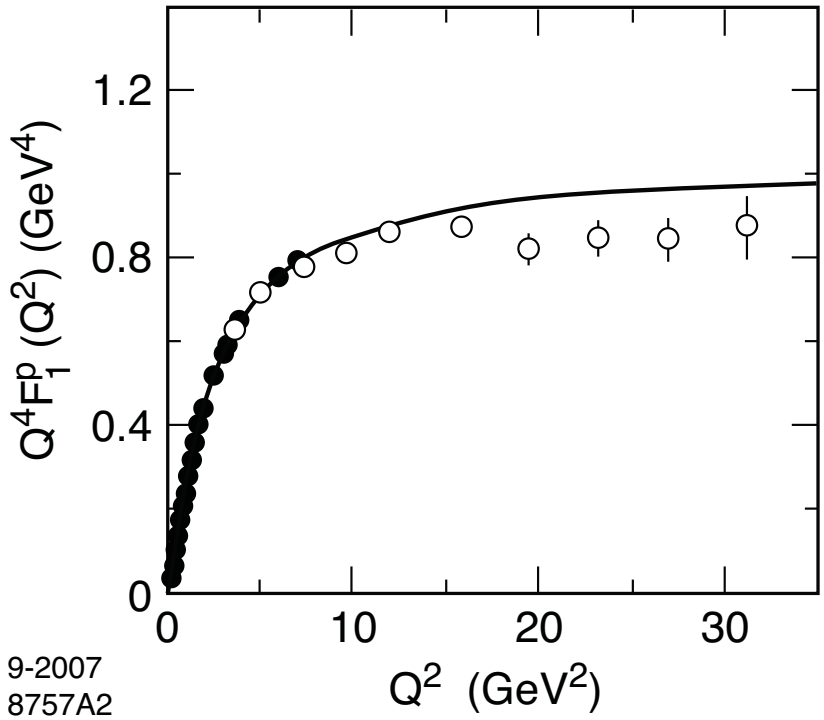
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3}\int d\zeta\, J(Q,\zeta)\left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

where $F_1^p(0) = 1,\ F_1^n(0) = 0.$

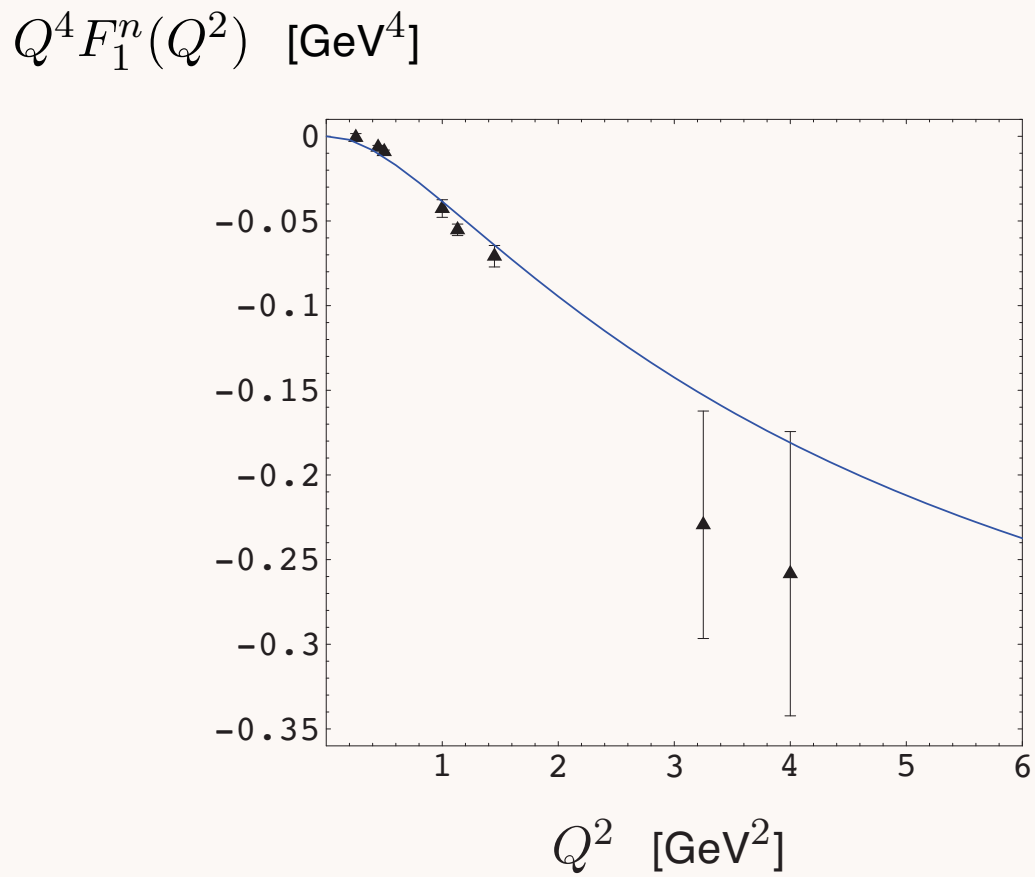
- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

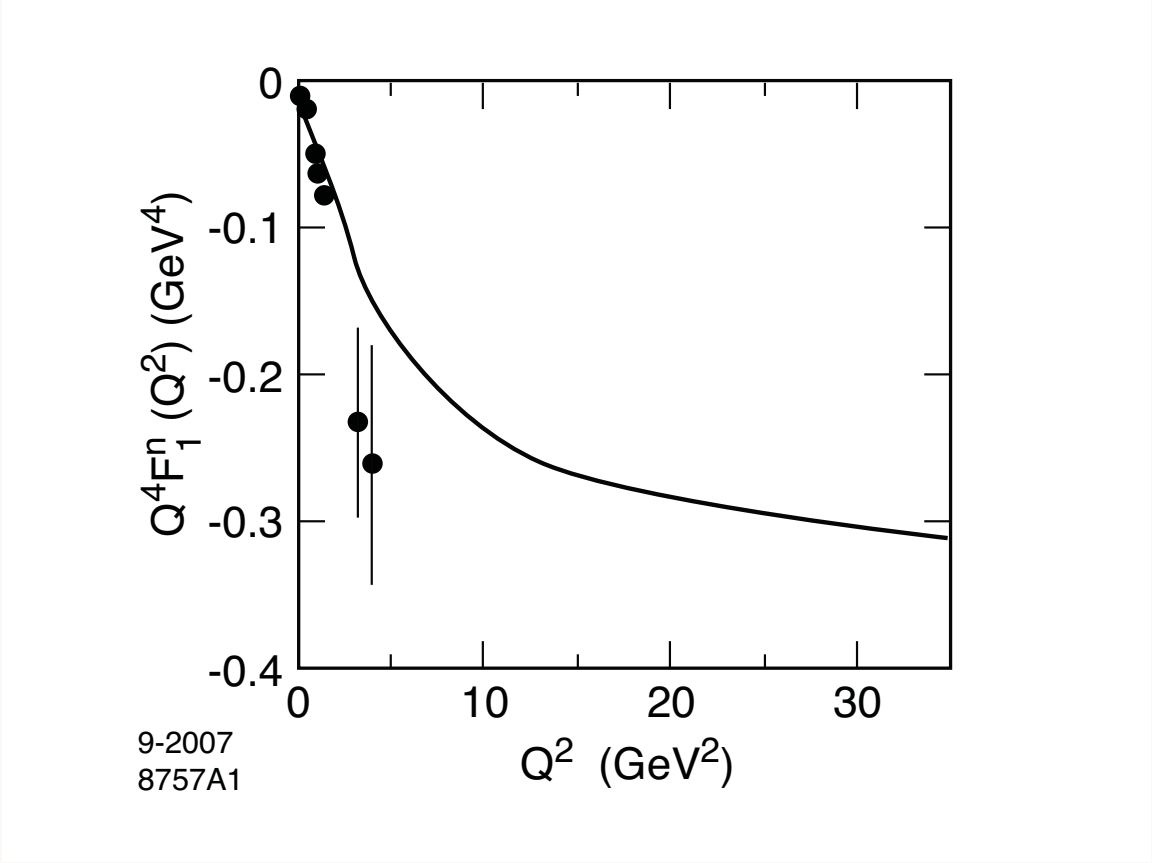
Dirac Neutron Form Factor
(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$

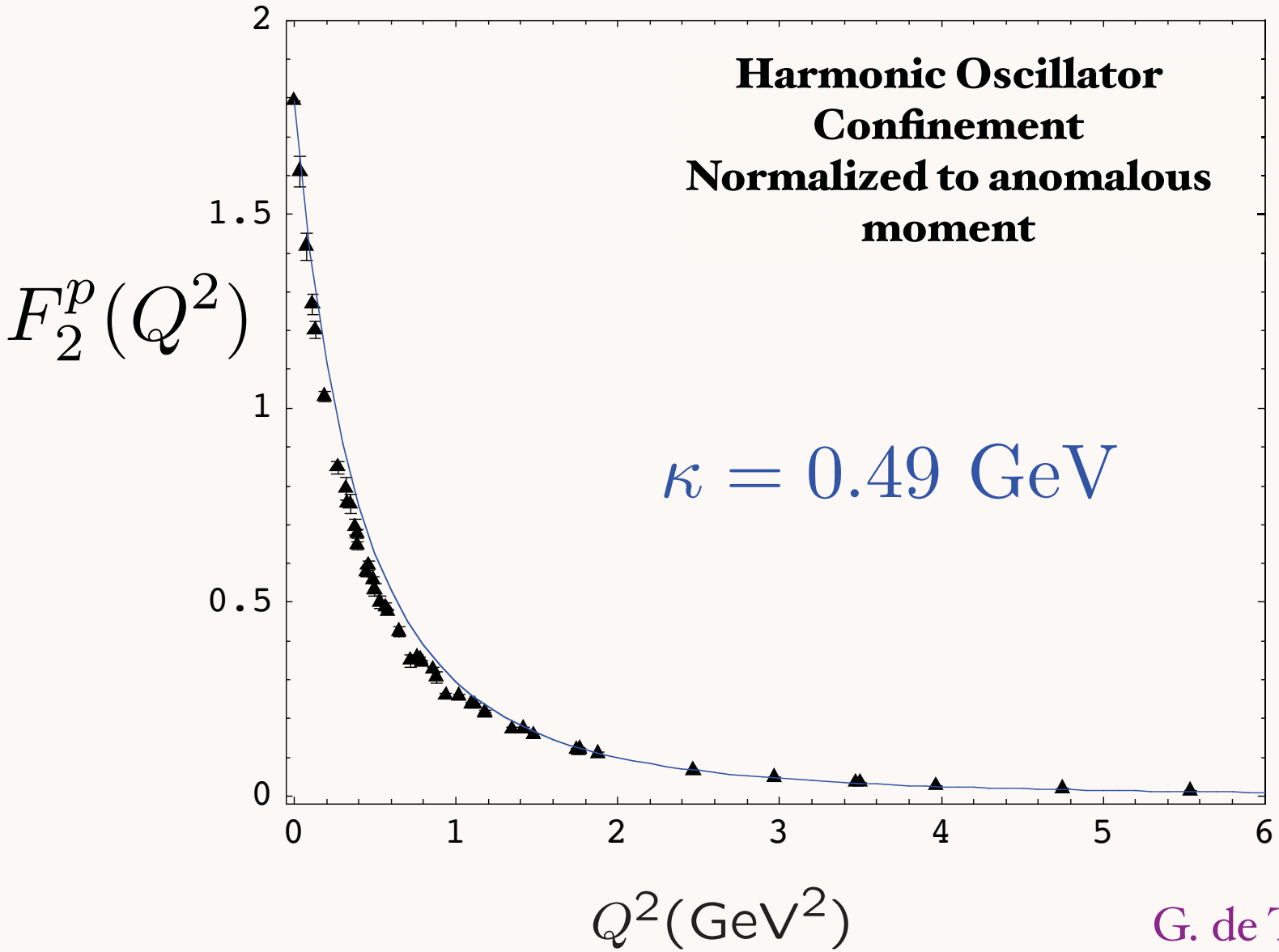


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Spacelike Pauli Form Factor

Preliminary

From overlap of $L = 1$ and $L = 0$ LFWFs



G. de Teramond, sjb

String Theory

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3 +1 space to AdS5 space

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

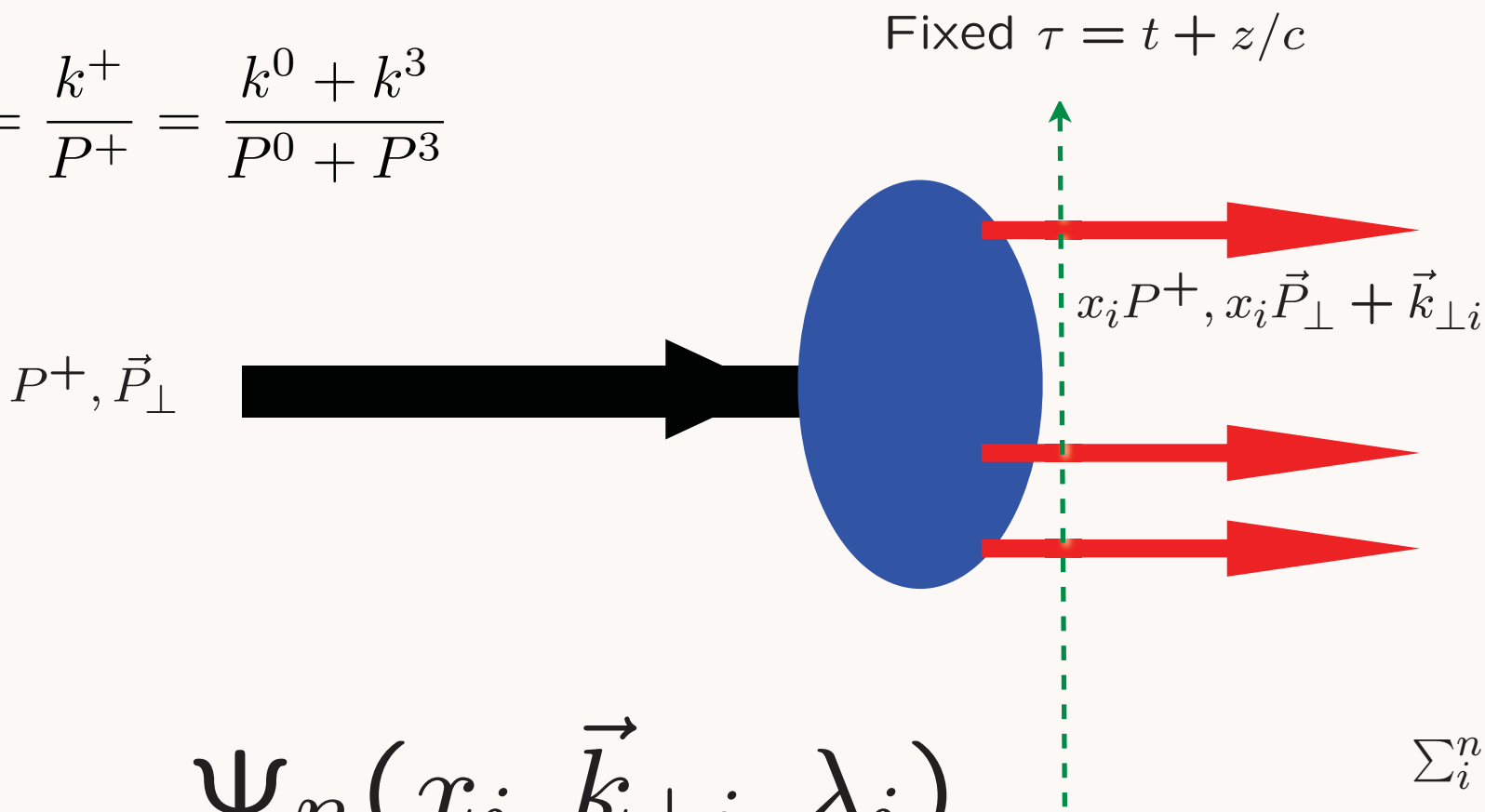
J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Invariant under boosts! Independent of P^μ

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

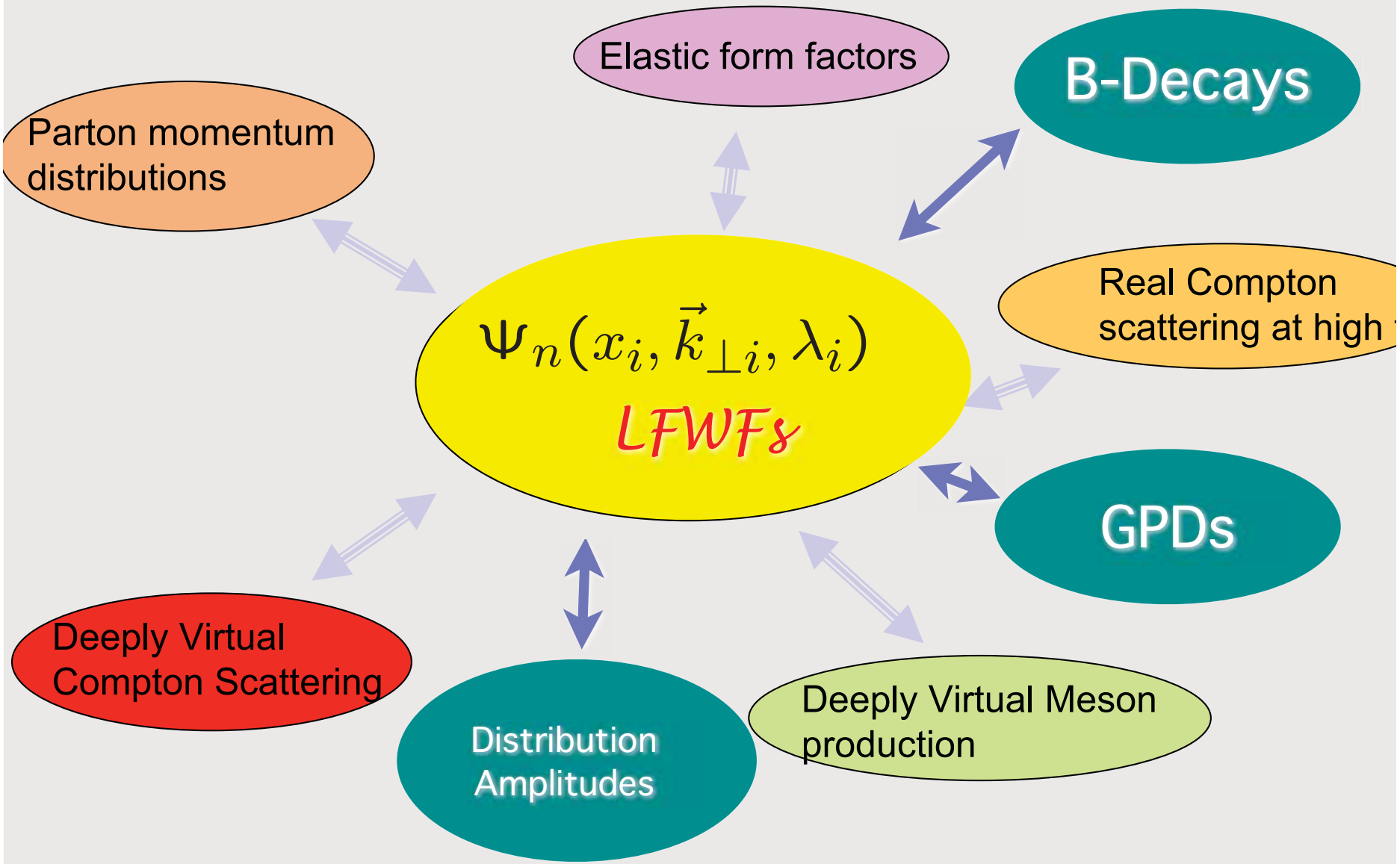
Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Nonzero Anomalous Moment -->Nonzero orbital angular momentum

A Unified Description of Hadron Structure



Light-Front QCD

Heisenberg Matrix Formulation

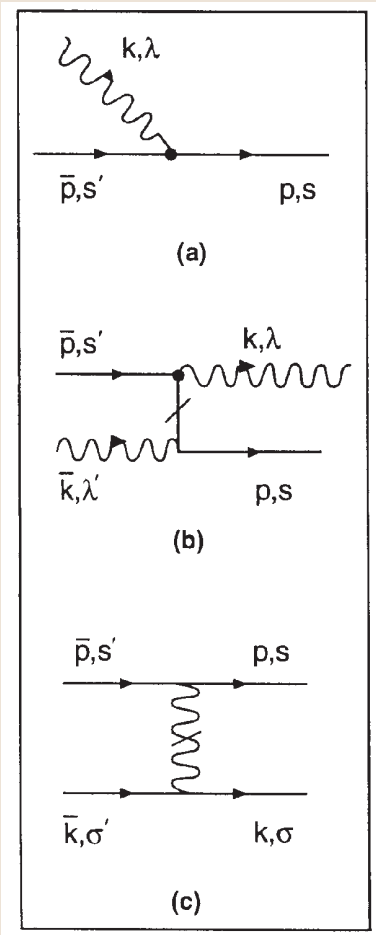
Physical gauge: $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H_{LF}^{int}$$

H_{LF}^{int}: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h >= \mathcal{M}_h^2 |\Psi_h >$$

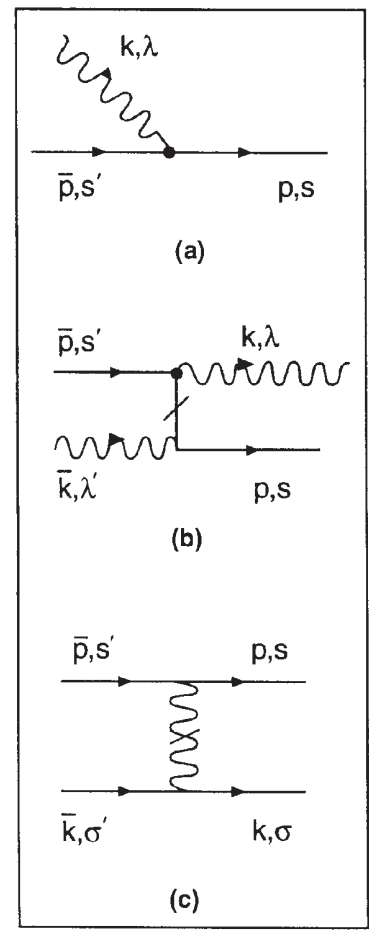


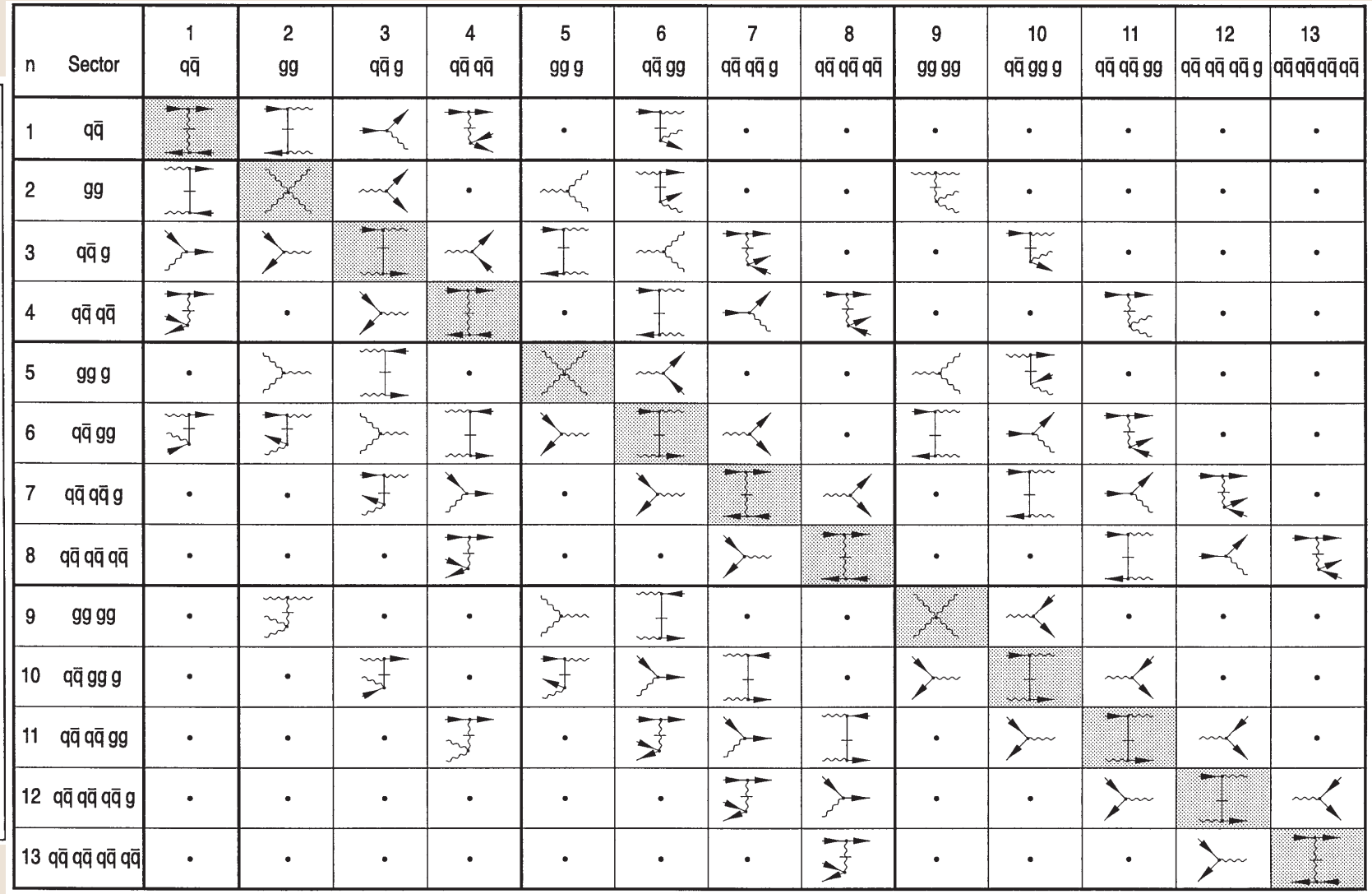
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in x^-. Discrete k^+; frame-independent truncation

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$





Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

Vary, Harinandrath, Maris, sjb

- Similar to Shell Model calculations

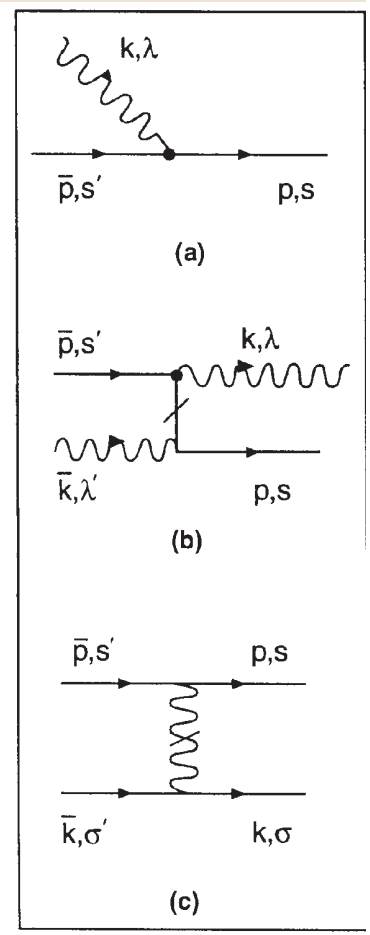
Light-Front QCD

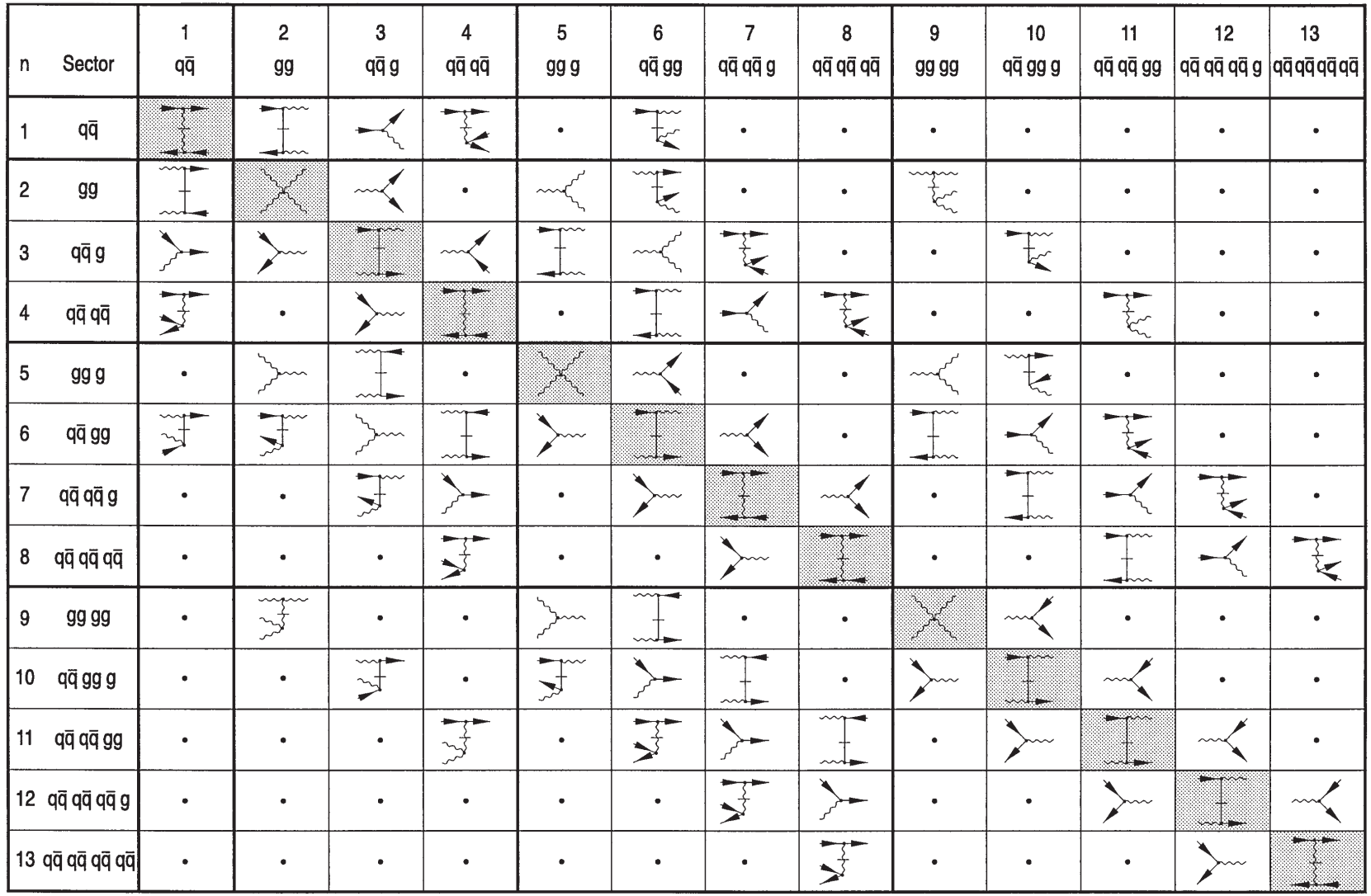
Heisenberg Matrix Formulation

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

DLCQ

Discretized Light-Cone Quantization





Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

H.C. Pauli & sjb

DLCQ: Frame-independent, No fermion doubling; Minkowski Space

Pauli, sjb

Discretized Light-Cone Quantization

- Periodic boundary condition in x-
- discrete positive plus momenta
- frame-independent formulation
- no fermion doubling
- covariant limit on Fock space

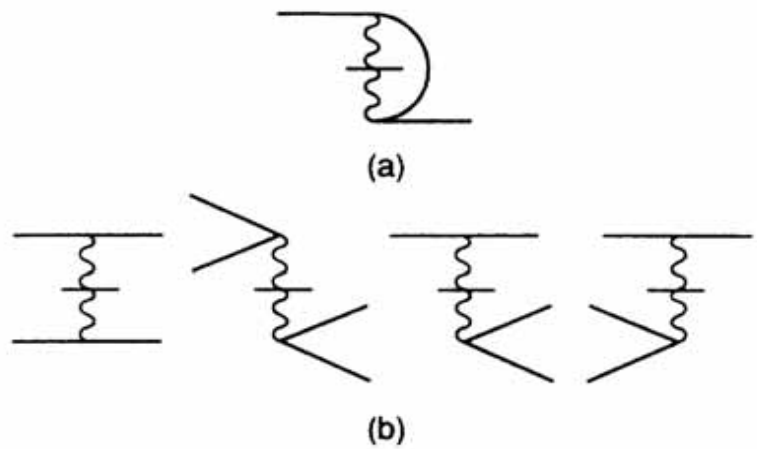
$$P^+ = \frac{2\pi}{L} K$$

$$k_i^+ = \frac{2\pi}{L} n_i$$

$$\sum_i k_i^+ = P^+$$

$$\sum_i n_i = K$$

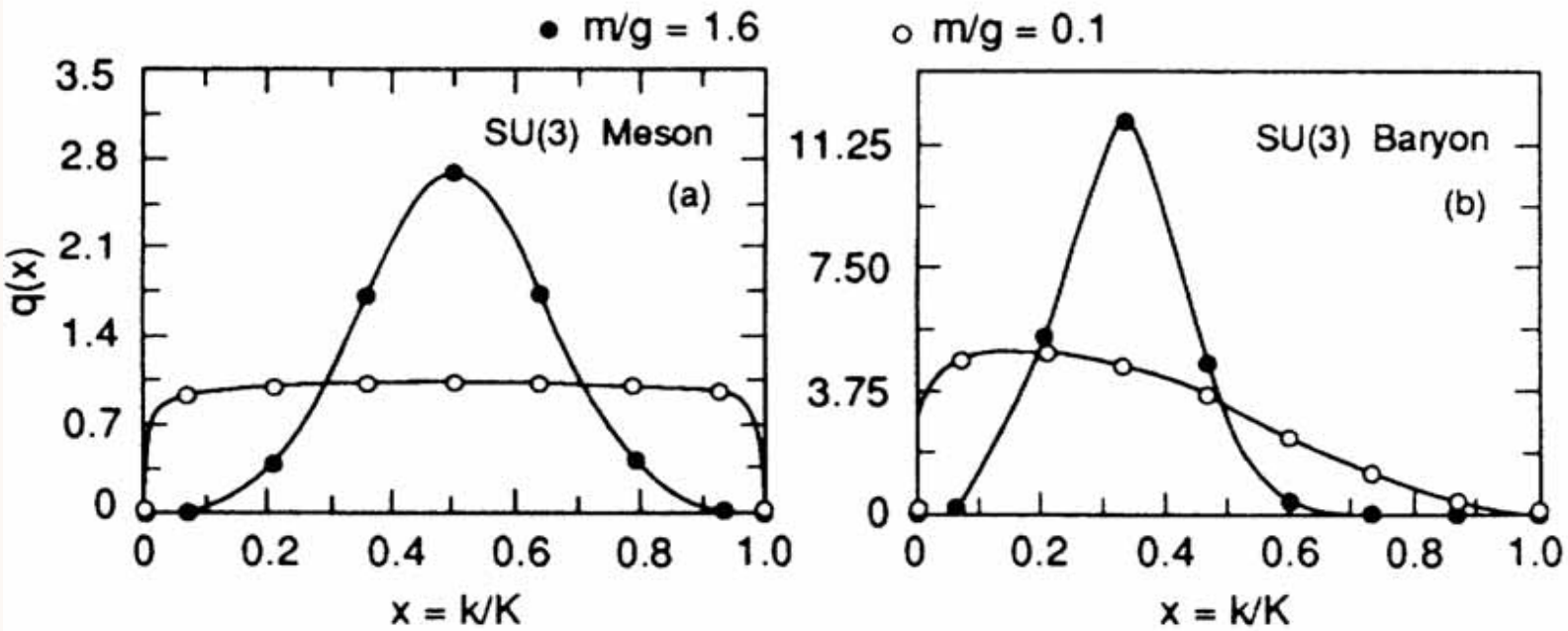
Hornbostel, Pauli, sjb



QCD(1+1) Interaction vertices.

$$\frac{L}{2\pi}\frac{g^2}{\pi}\frac{1}{2}\left[\delta^{c_2}_{c_4}\delta^{c_3}_{c_1}-\frac{1}{N}\delta^{c_3}_{c_4}\delta^{c_2}_{c_1}\right]$$

$$\times\sum_{n_i=1/2,3/2,\ldots}\frac{\delta_{n_1+n_2,n_3+n_4}}{(n_1+n_3)^2}b^{\dagger c_4}_{n_4}b_{n_3,c_3}d^{\dagger}_{n_2,c_2}d^{c_1}_{n_1}\ .$$


● m/g = 1.6
○ m/g = 0.1
SU(3) Meson
(a)
SU(3) Baryon
(b)
q(x)
3.5
2.8
2.1
1.4
0.7
0
11.25
7.50
3.75
0
0.2
0.4
0.6
0.8
1.0
x = k/K

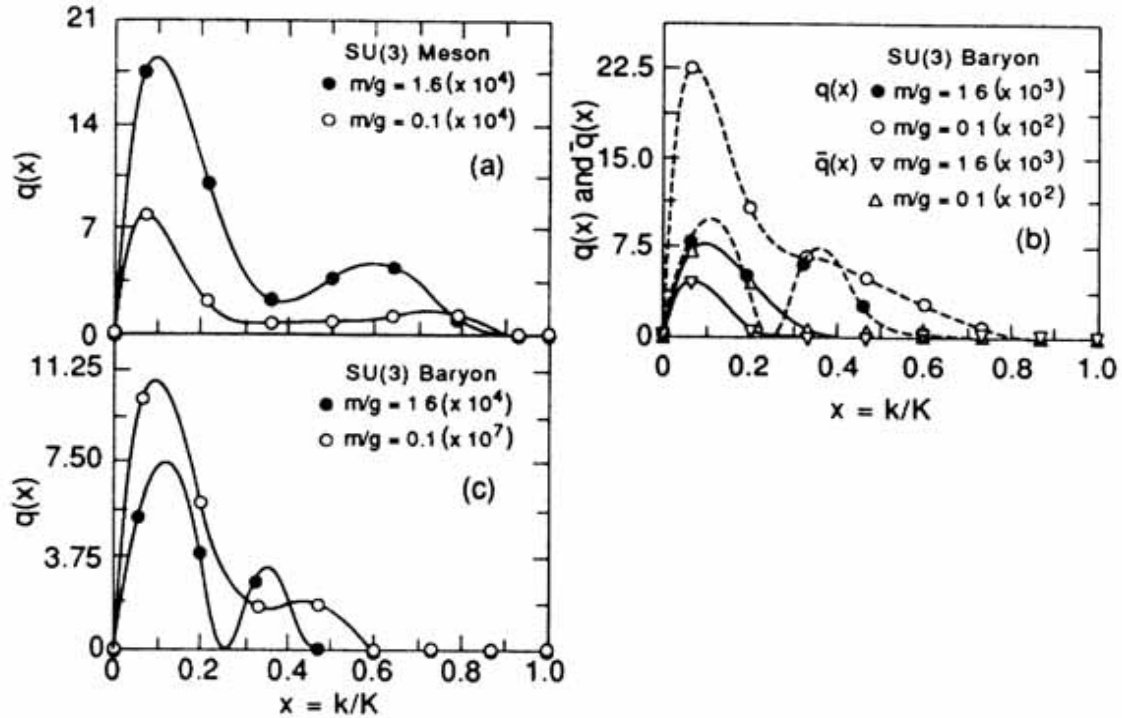
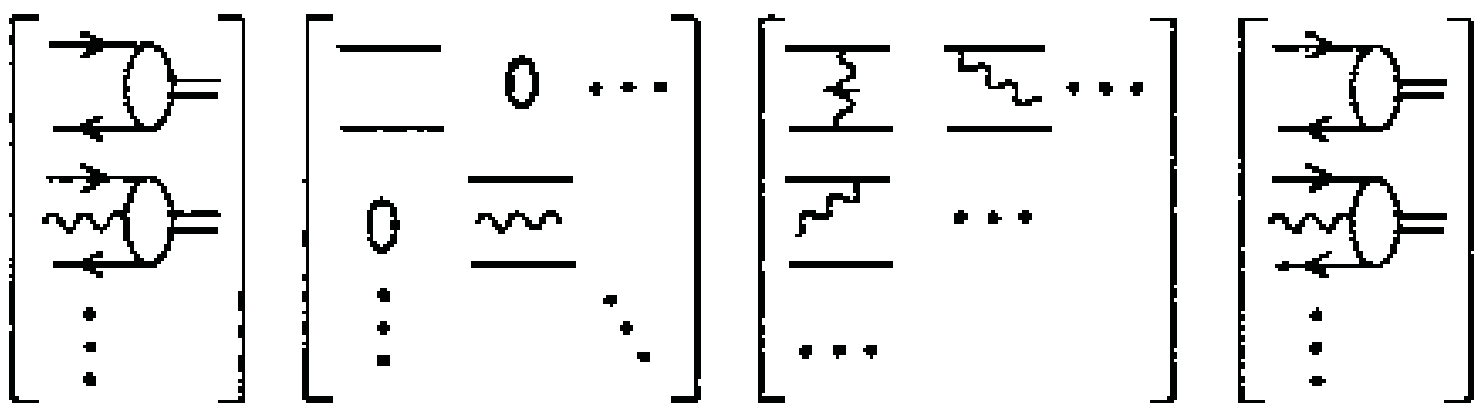


FIG. 4. Higher-Fock contributions to $N=3$ structure functions. (a) Lightest meson. (b) Lightest baryon, including antiquarks. (c) Baryon: contribution from two extra quark pairs. The curves are intended to guide the eye.

LIGHT-FRONT SCHRODINGER EQUATION

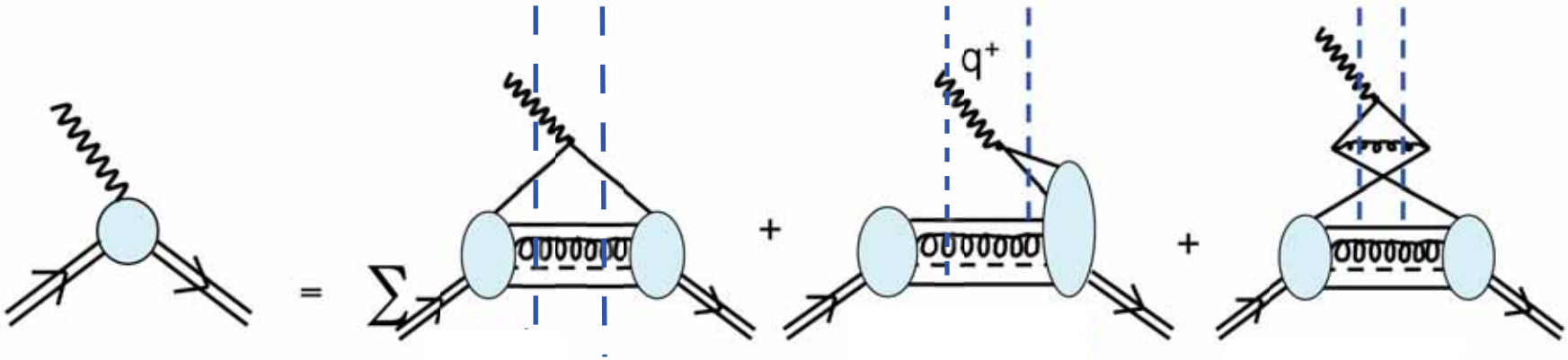
$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$A^+ = 0$

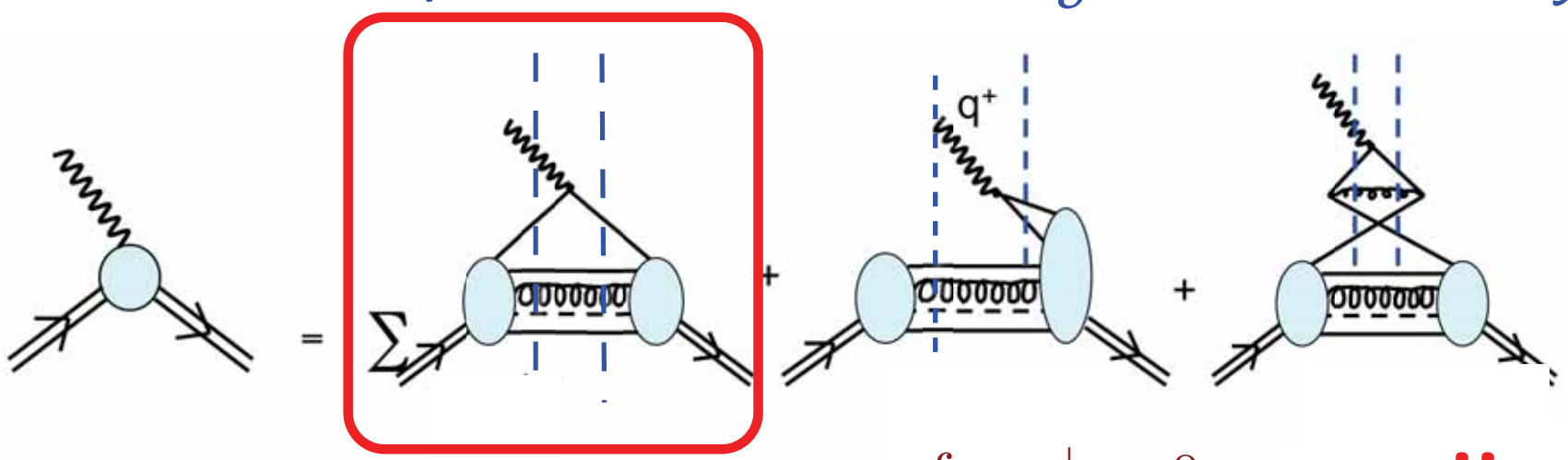
G.P. Lepage, sjb

Calculation of Form Factors in Equal-Time Theory



Need vacuum fluctuations

Calculation of Form Factors in Light-Front Theory



zero for $q^+ = 0$ ***zero !!***

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

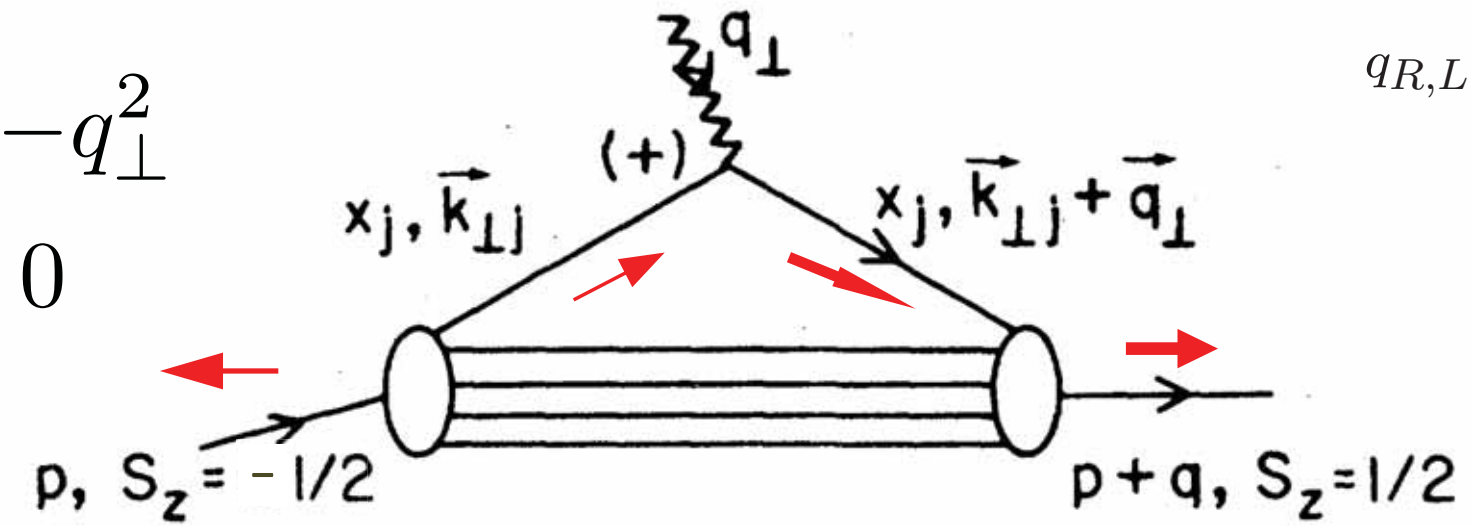
$$\Big[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

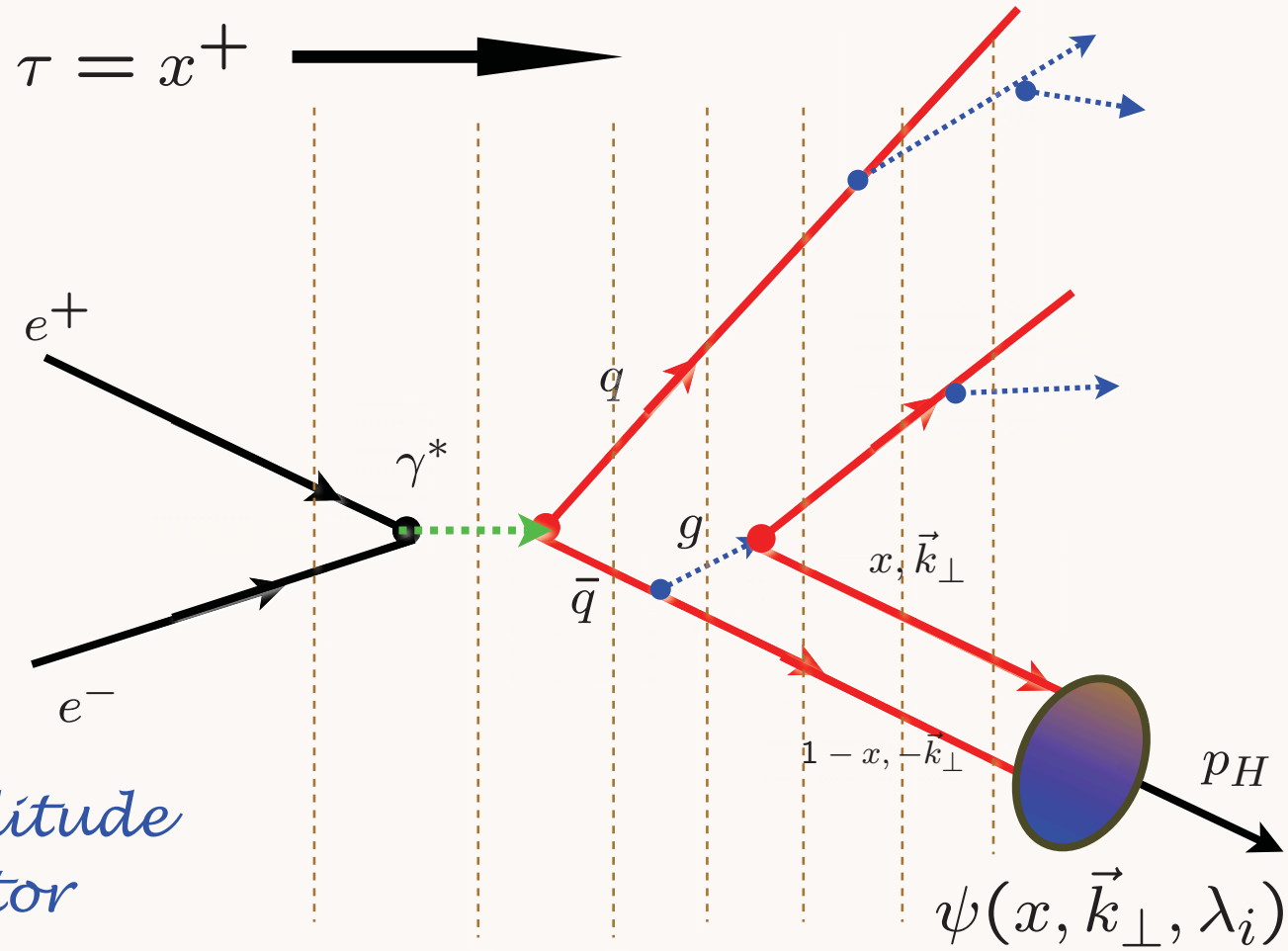
$$q^2 = -q_\perp^2$$

$$q^+ = 0$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

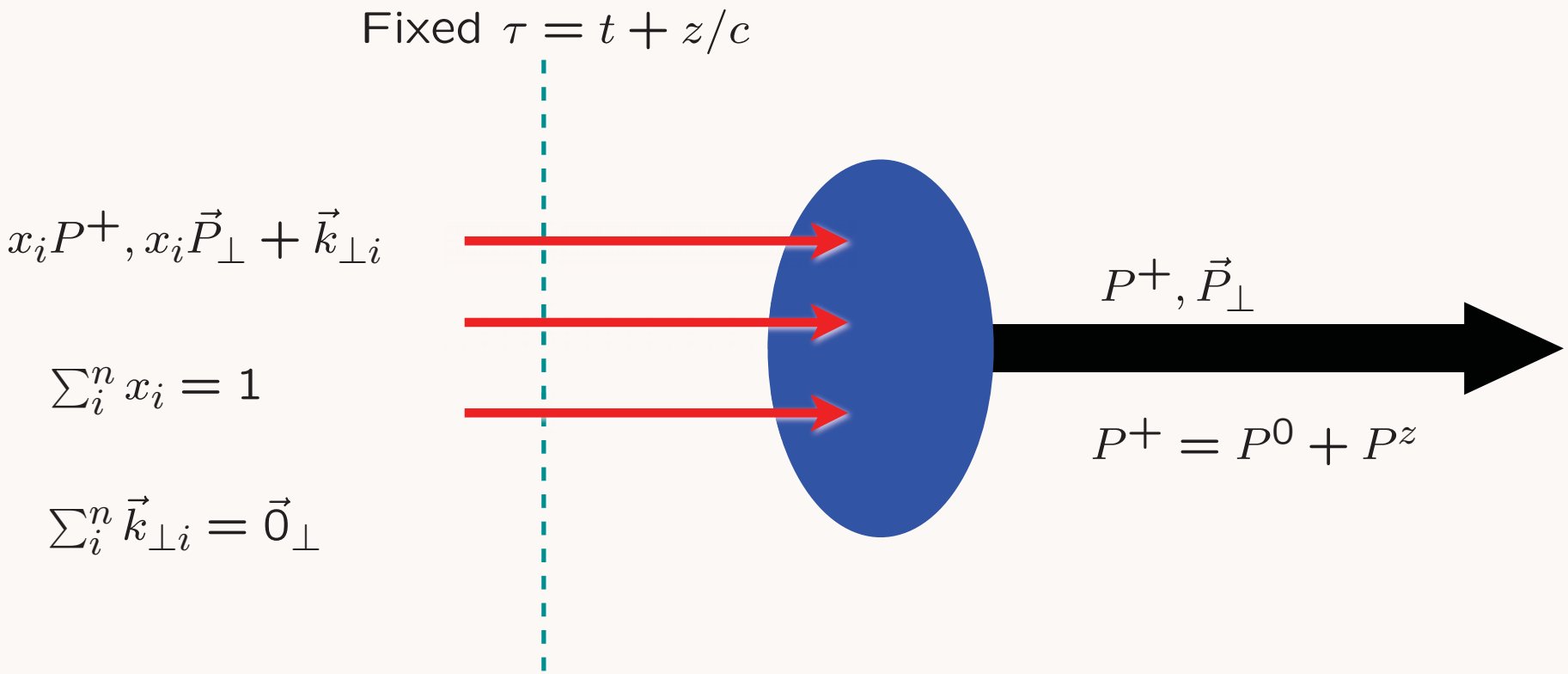
Hadronization at the Amplitude Level



Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Light-Front Wavefunctions



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Invariant under boosts! Independent of P^μ

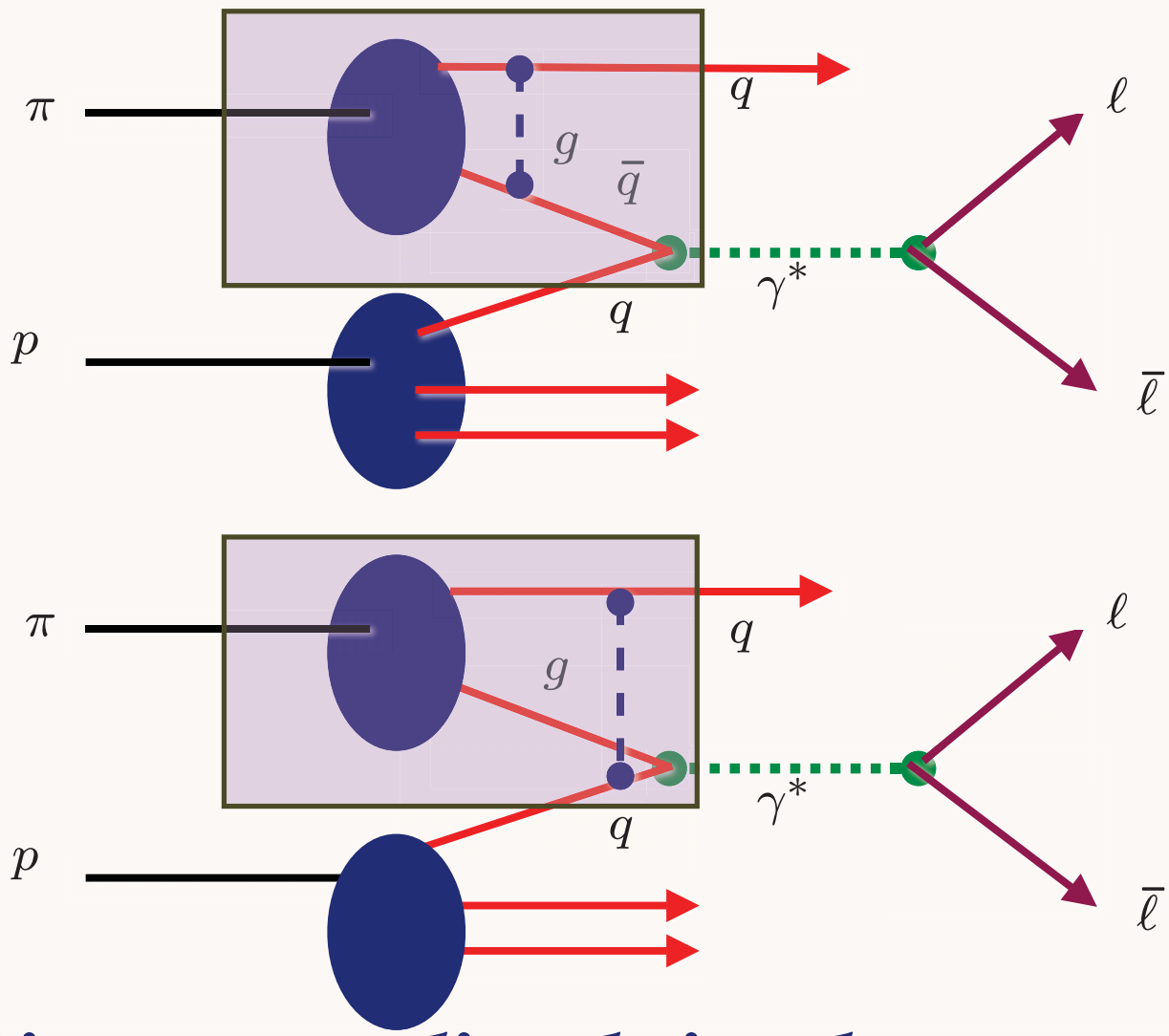
Direct Subprocesses

- Hadron produced directly in subprocess
- Higher twist in $1/Q^2$, but dominant at large x or z
- Observed in Drell-Yan reactions at large x_F: dramatic change in lepton angular distribution
- Merges with exclusive processes at exclusive boundary
- Strength amplified by trigger-bias effect in single-particle triggers
- Color transparent

Berger, Brodsky, Lepage

Brodsky, Hoyer, Mueller, Tang

Berger, Lepage, sjb



$$\pi q \to \gamma^* q$$

Pion appears directly in subprocess at large x_F

All of the pion momentum transferred to the lepton pair

Lepton Pair produced longitudinally polarized

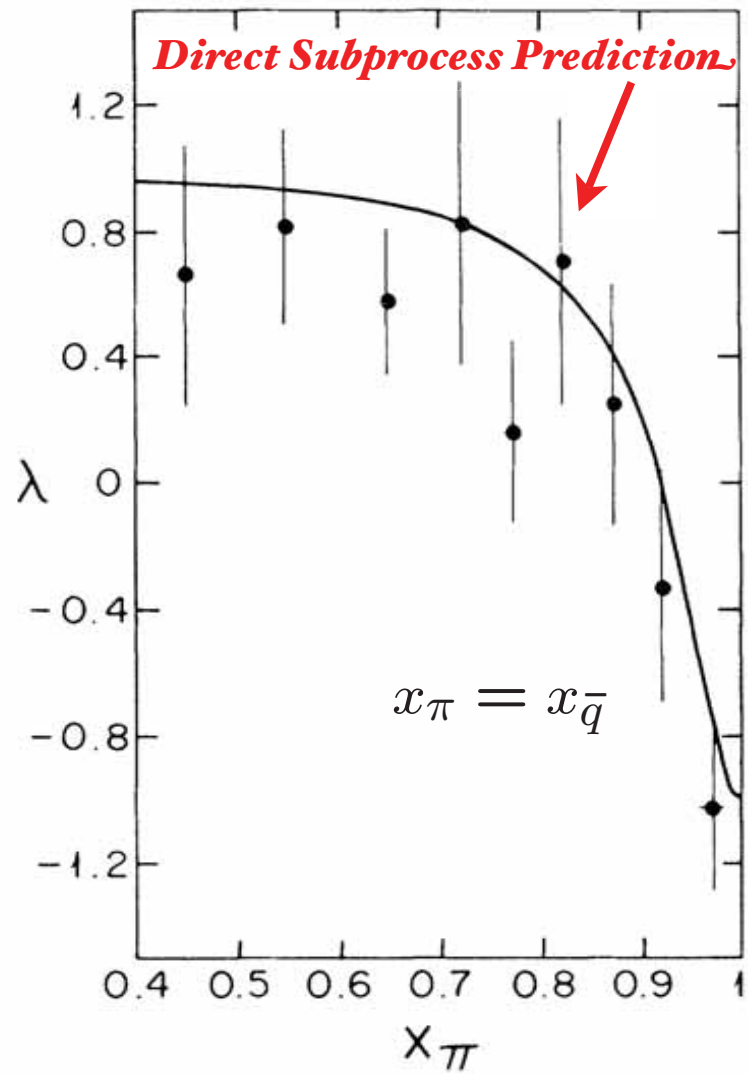
$$\pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2 (1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \ \text{GeV}^2/c^2$$

Dramatic change in angular distribution at large x_F



Chicago-Princeton Collaboration

Phys.Rev.Lett.55:2649,1985

Example of a higher-twist direct subprocess

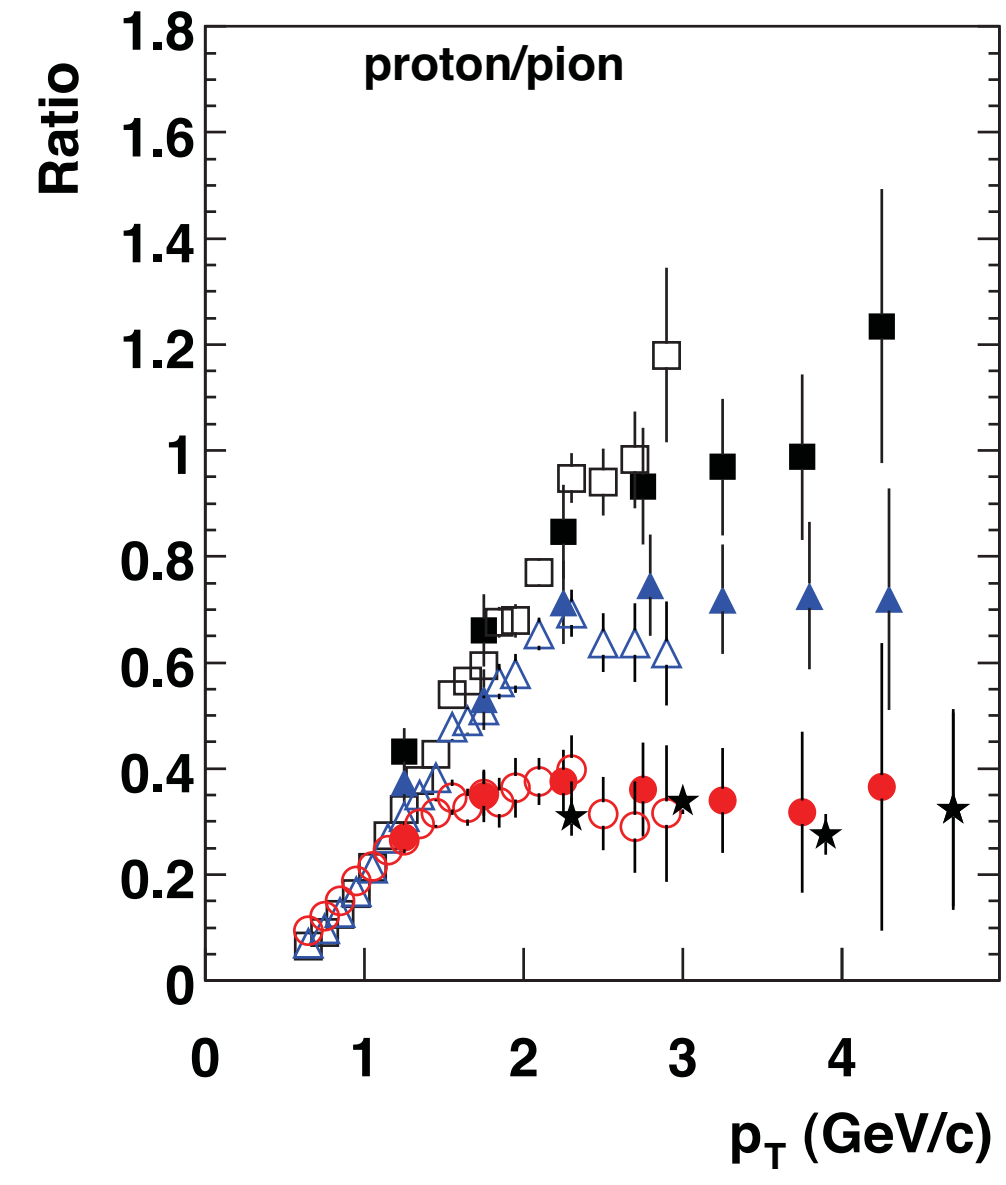
Parton Correlations contained in LFWFs

$$\frac{d\sigma}{dx_F dQ^2 d\cos\theta}(\pi N \to \ell^+\ell^- X) = A(1-x_F)^2(1+\cos^2\theta)+B(1-x_F)^0 \frac{\Lambda^2_{QCD}}{Q^2}\sin^2\theta$$

- Higher Twist contribution to DY, DIS
- Off-shell corrections to DGLAP at x ~ 1, z ~1
- Direct Production of Hadrons in Hard Subprocess
- Proton Decay

S. S. Adler *et al*. PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

Particle ratio changes with centrality!



Protons less absorbed in nuclear collisions than pions

← **Central**

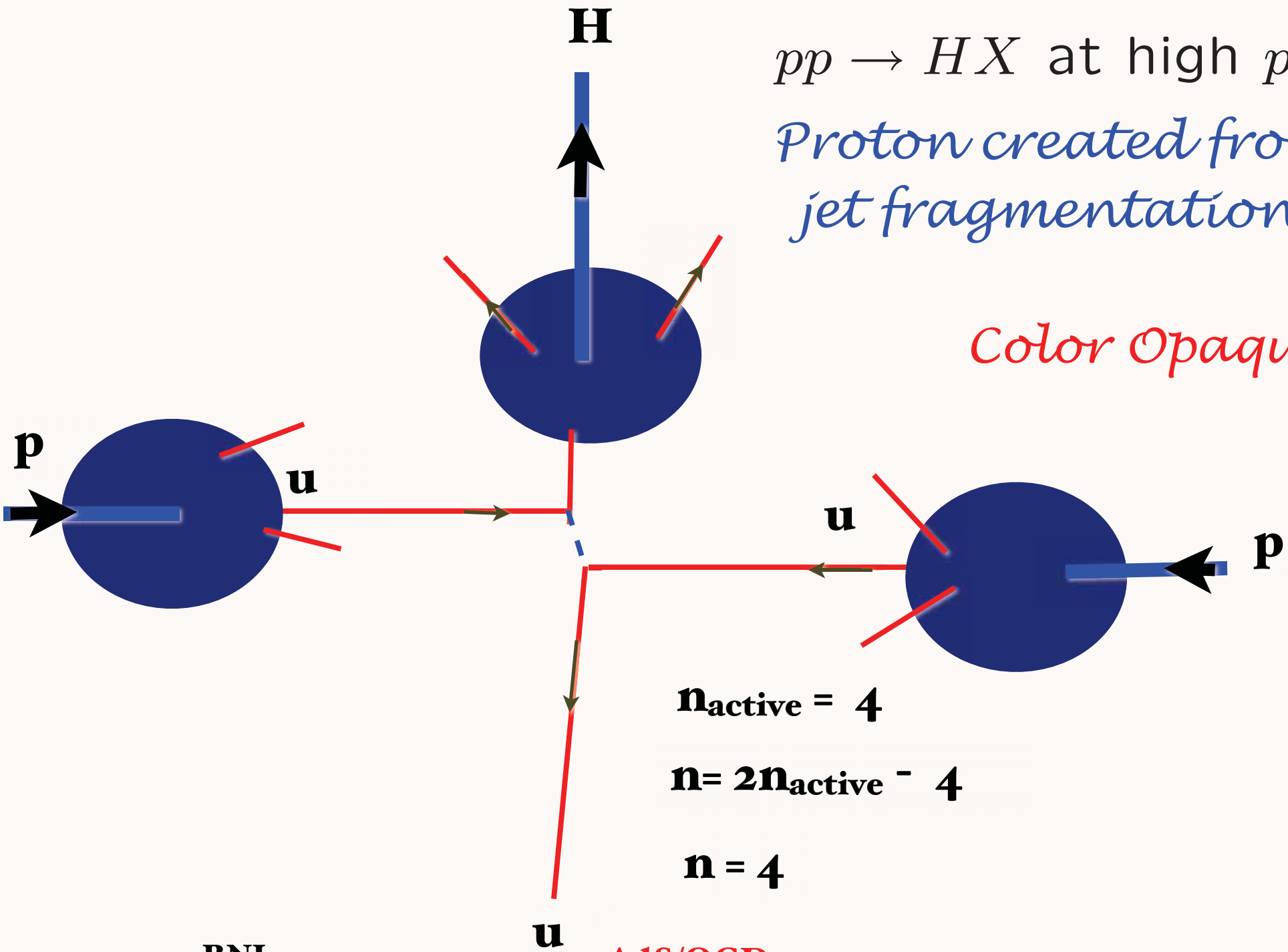
- □ ■ Au+Au 0-10%
- △ ▲ Au+Au 20-30%
- ○ ● Au+Au 60-92%
- ★ p+p, $\sqrt{s}$ = 53 GeV, ISR
- - - - e^+e^-, gluon jets, DELPHI
- ····· e^+e^-, quark jets, DELPHI

← **Peripheral**

$pp \to HX$ at high p_T

Proton created from jet fragmentation

Color Opaque



$n_{active} = 4$

$n = 2n_{active} - 4$

$n = 4$

Crucial Test of Leading -Twist QCD: Scaling at fixed x_T

$$x_T = \frac{2p_T}{\sqrt{s}}$$

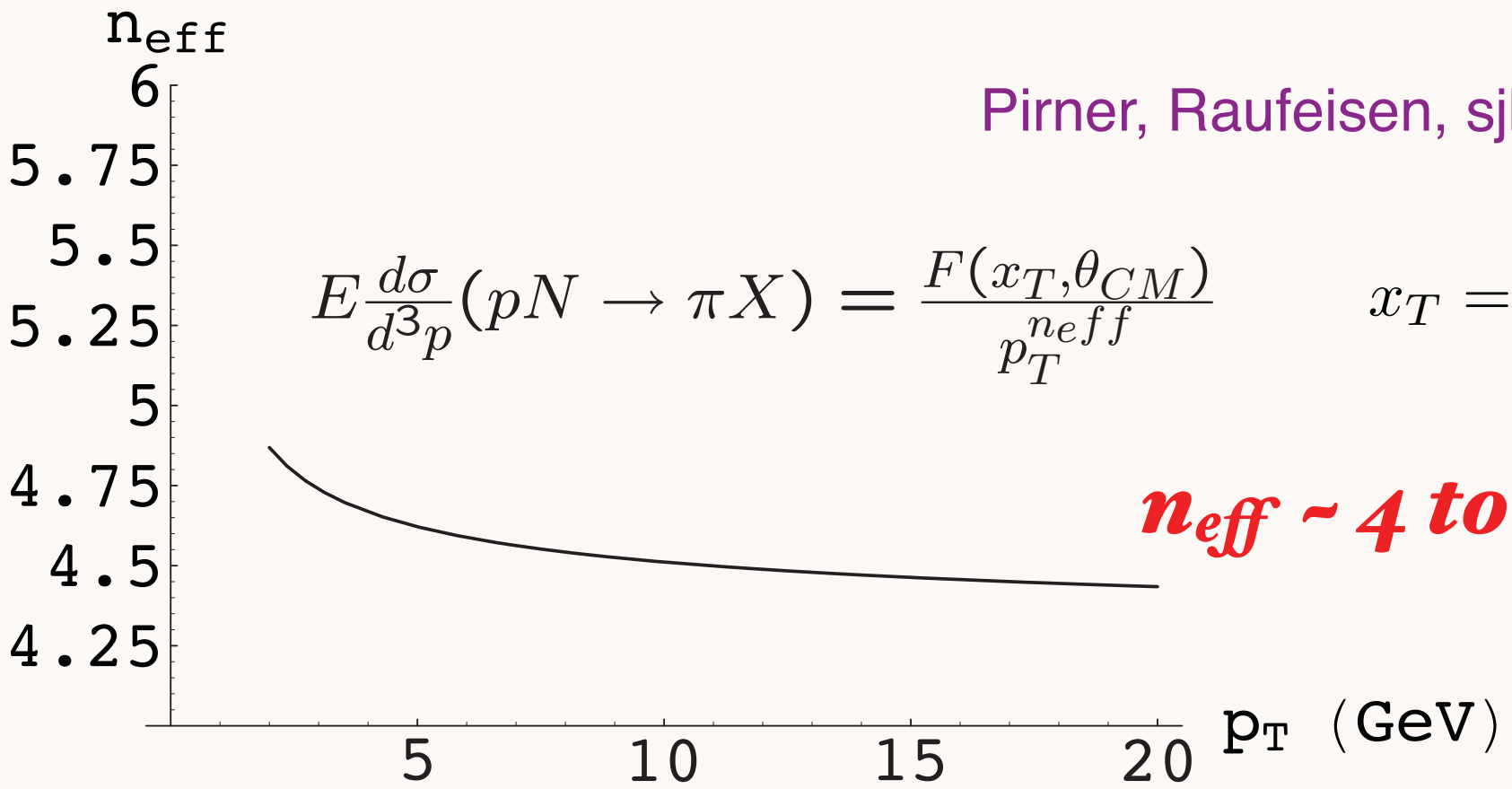
$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2\, n_{active} - 4$

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling



Pirner, Raufeisen, sjb

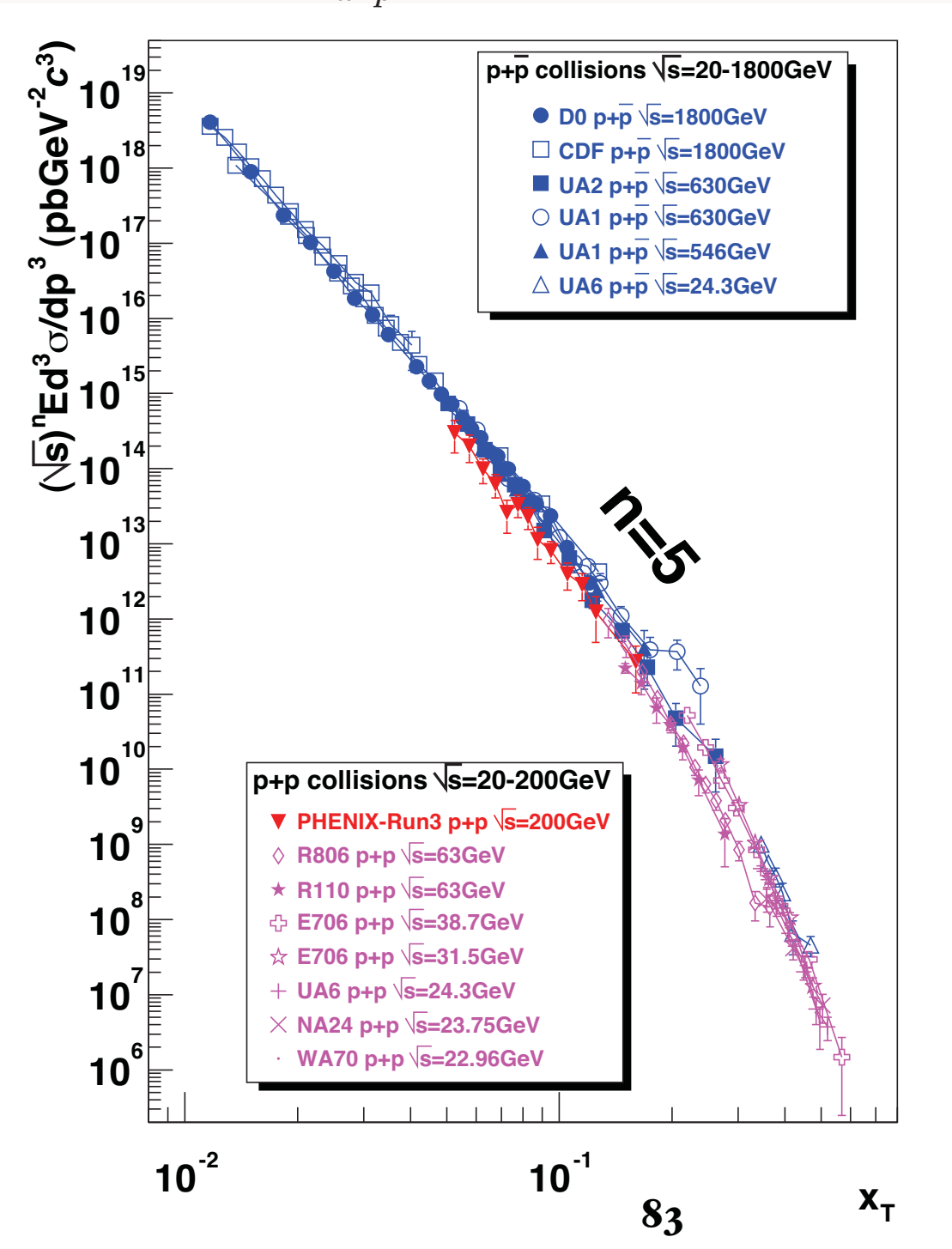
$$E\frac{d\sigma}{d^3p}(pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}} \qquad x_T = \frac{2p_T}{\sqrt{s}}$$

n_{eff} ~ 4 to 5

Key test of PQCD: power-law fall-off at fixed x_T

$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed x_T

Tannenbaum



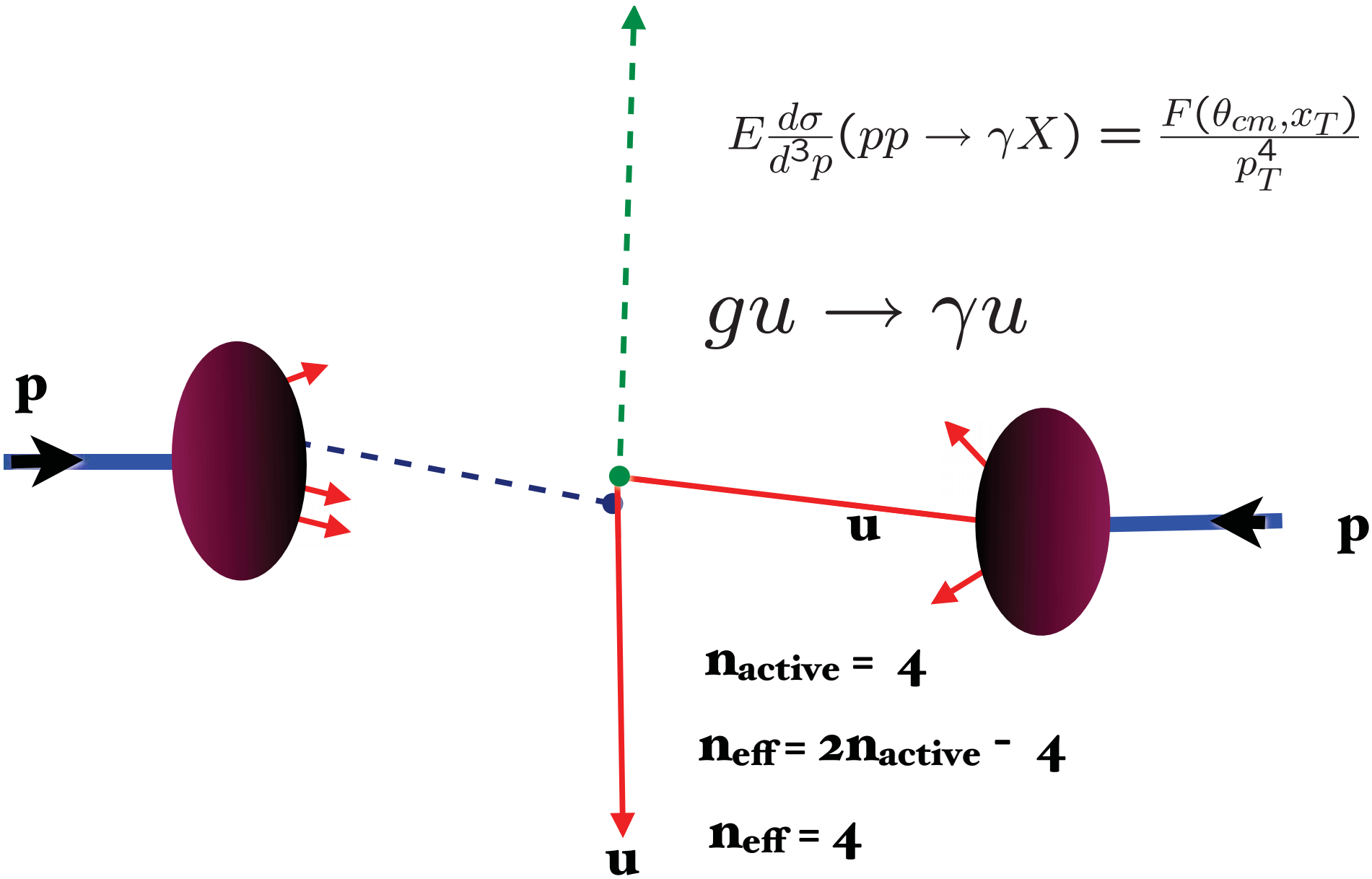
Scaling of direct photon production consistent with PQCD

Stan Brodsky, SLAC

$$pp \to \gamma X$$

$$E\frac{d\sigma}{d^3p}(pp \to \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \to \gamma u$$

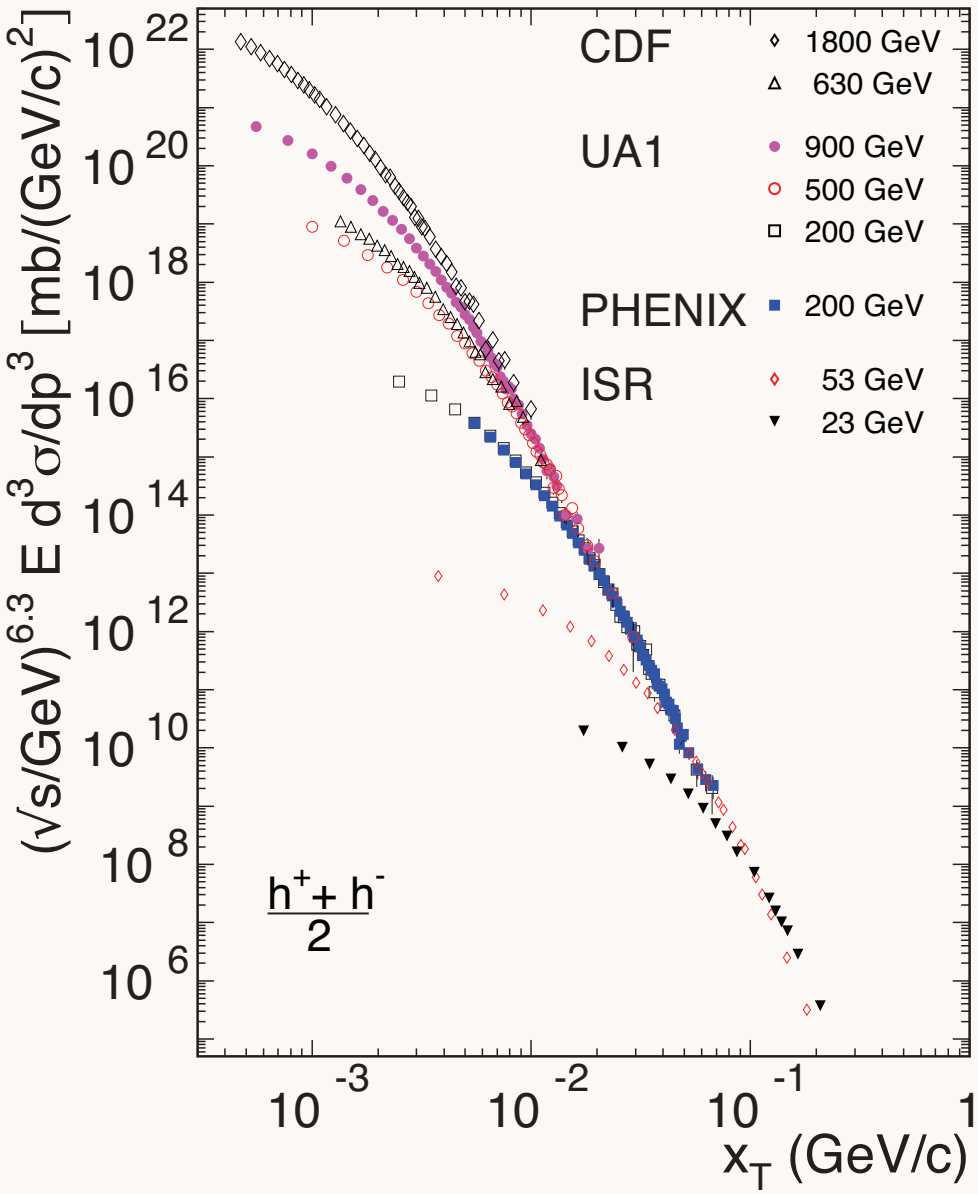


$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

$\sqrt{s}^{6.3} \times E\frac{d\sigma}{d^3p}(pp \to H^{\pm}X)$ at fixed x_T

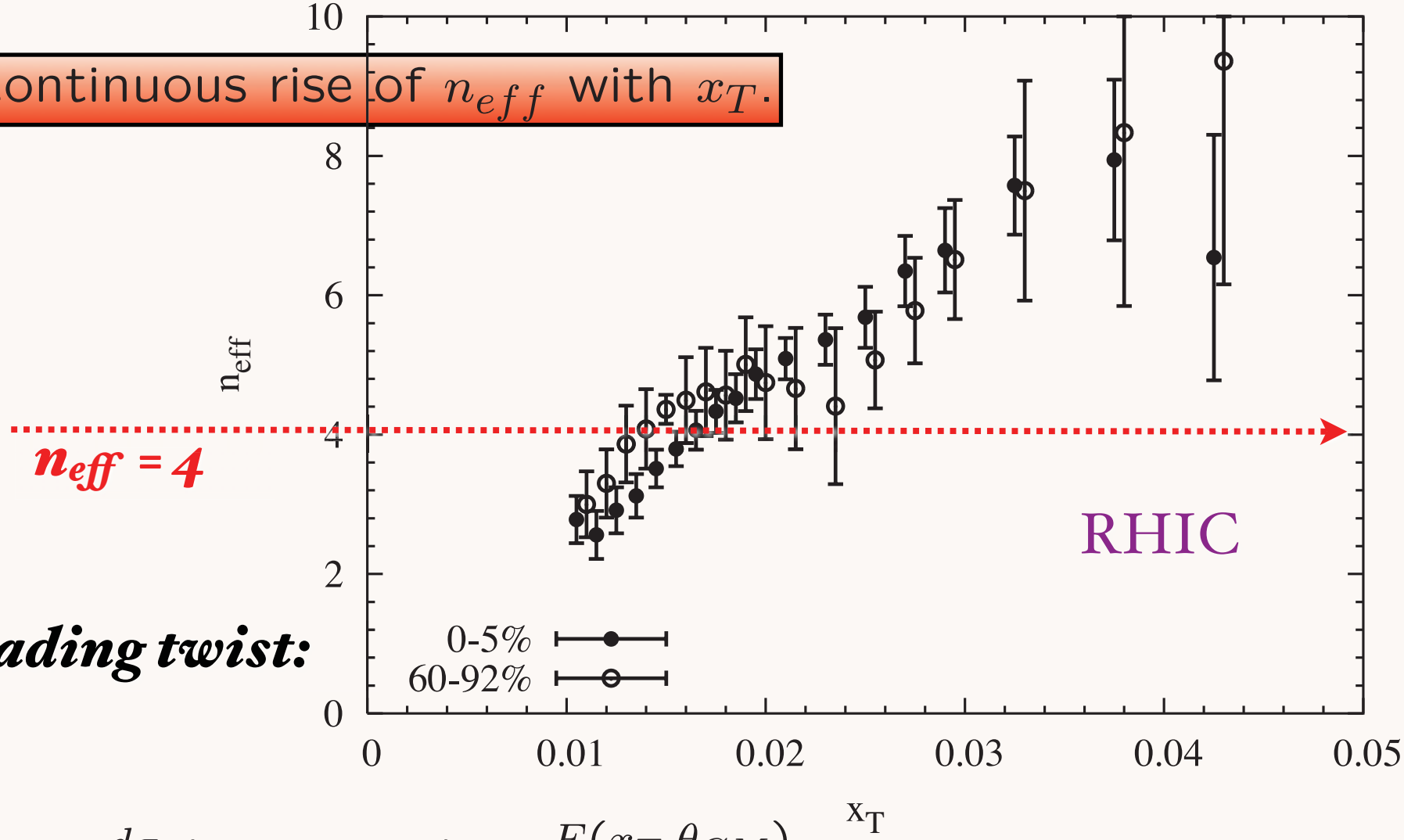


Tannenbaum

Scaling inconsistent with PQCD

Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available p_T range. Shown are data for central $(0-5\%)$ and for peripheral $(60-90\%)$ collisions.

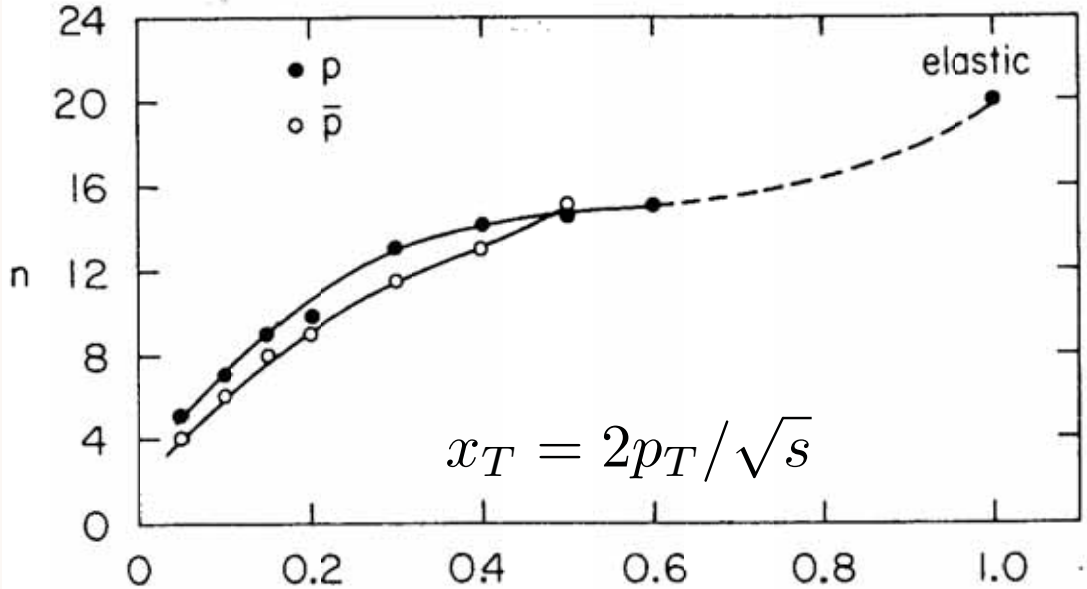
Continuous rise of n_{eff} with x_T.

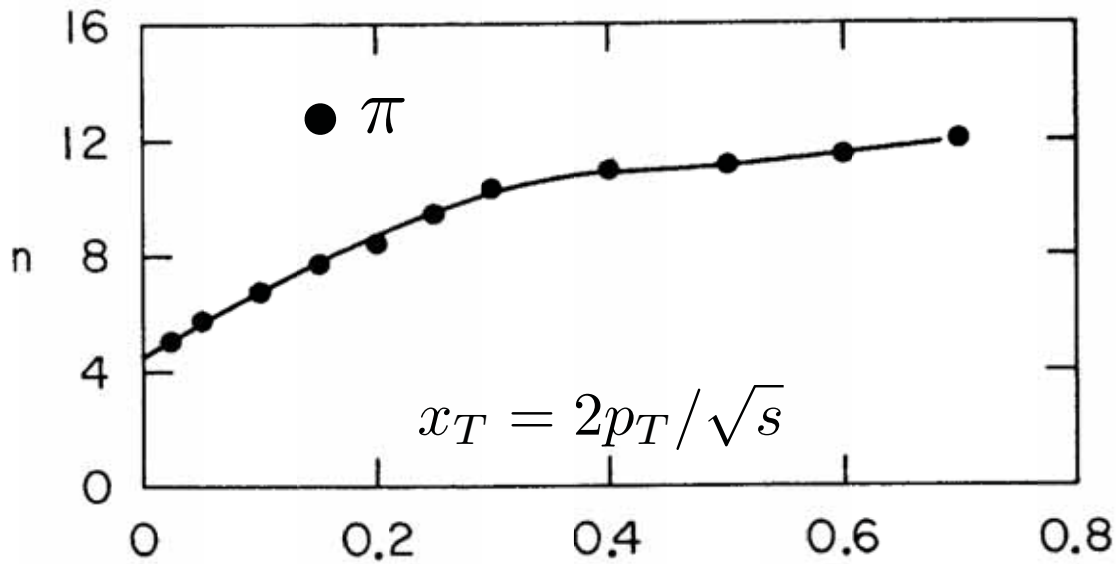


Leading twist:

$$E\frac{d\sigma}{d^3p}(pN \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{neff}}$$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n}$$



Clear evidence for higher-twist contributions

J. W. Cronin, SSI 1974