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Inclusive Higgs Production at Next-to-Next-to-Leading Order

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Phys. Lett. B 492, 74 (2000)

Motivation

The Standard Model is ~35 years old and its essential goal

To describe electroweak interactions with a spontaneously broken $SU(2) \otimes U(1)$ gauge symmetry

has been spectacularly confirmed

- Renormalizability
- Discovery of Neutral Currents
- Discovery of W and Z bosons
- Precision test of W/Z properties

But ...

The agent of electroweak symmetry breaking remains elusive

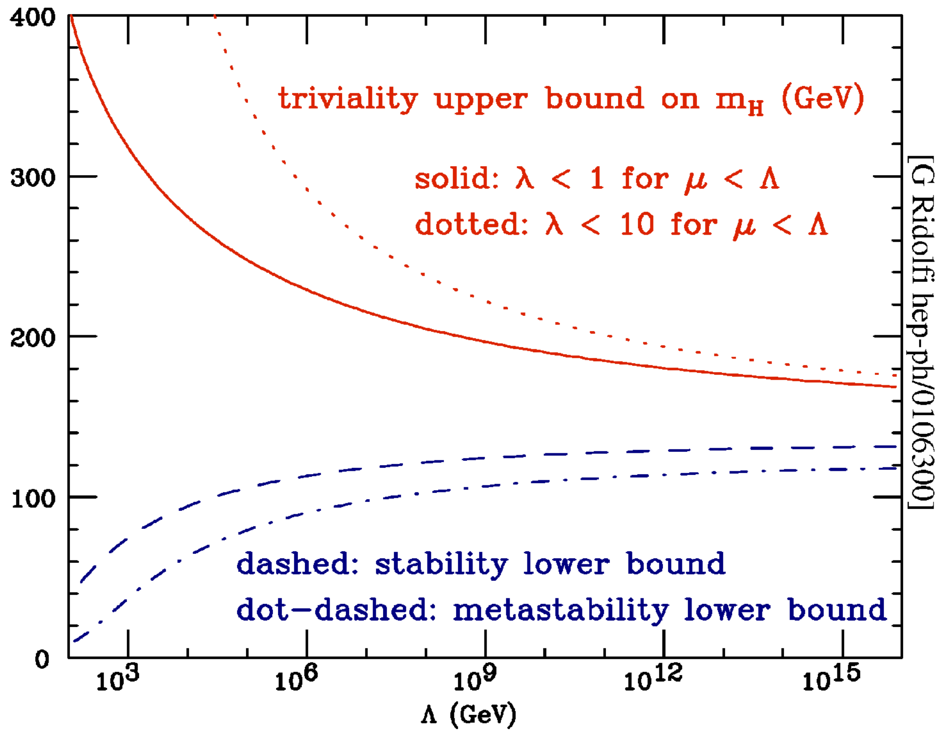
The Standard Model Higgs boson is the benchmark for studies of the symmetry breaking sector

Where should the Higgs be?

LEP Search:	$M_H \geq 114.1$ GeV	($M_H = 115$ GeV?)
Precision EW Fits:	$M_H = 85^{+54}_{-34}$ GeV	
95% CL upper limit:	$M_H < 196$ GeV	

Theoretical Bounds:

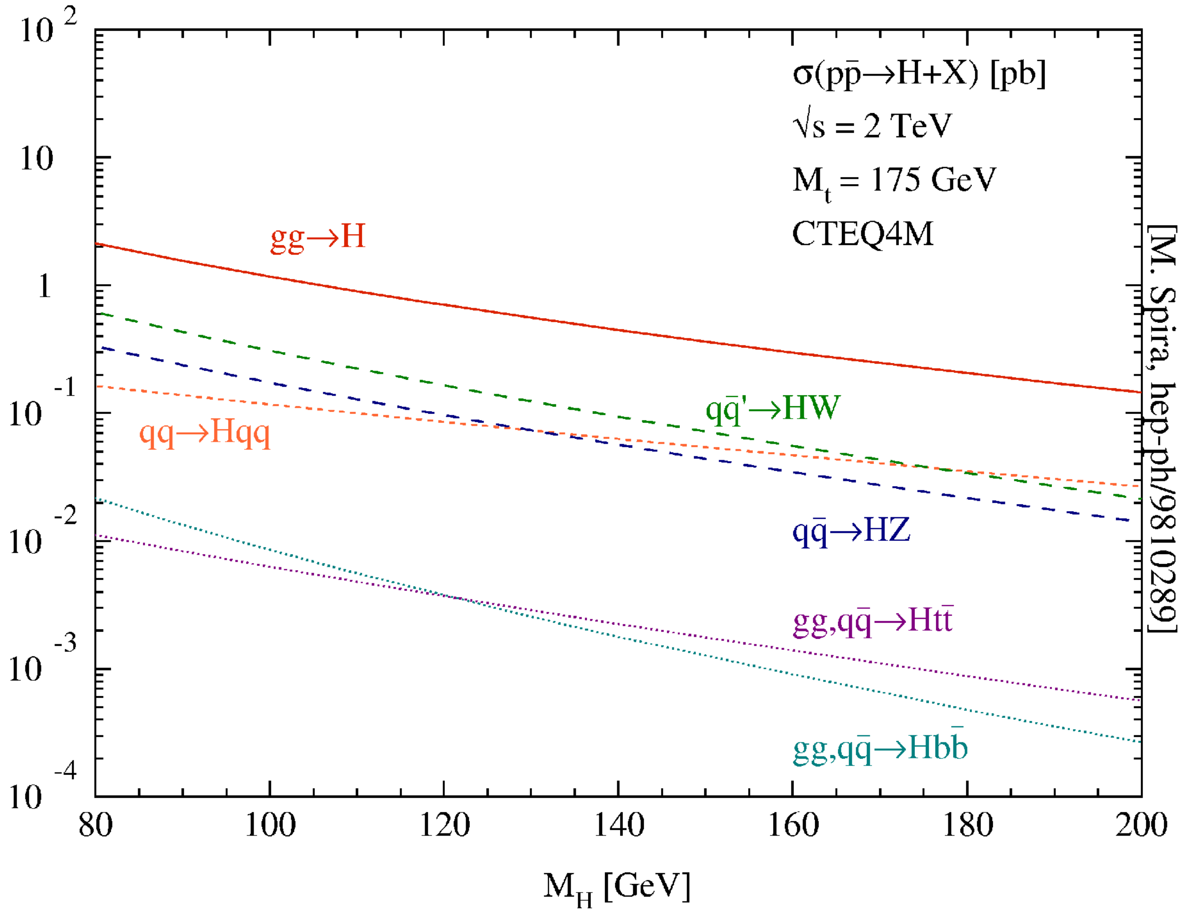
- Triviality
- Vacuum Stability



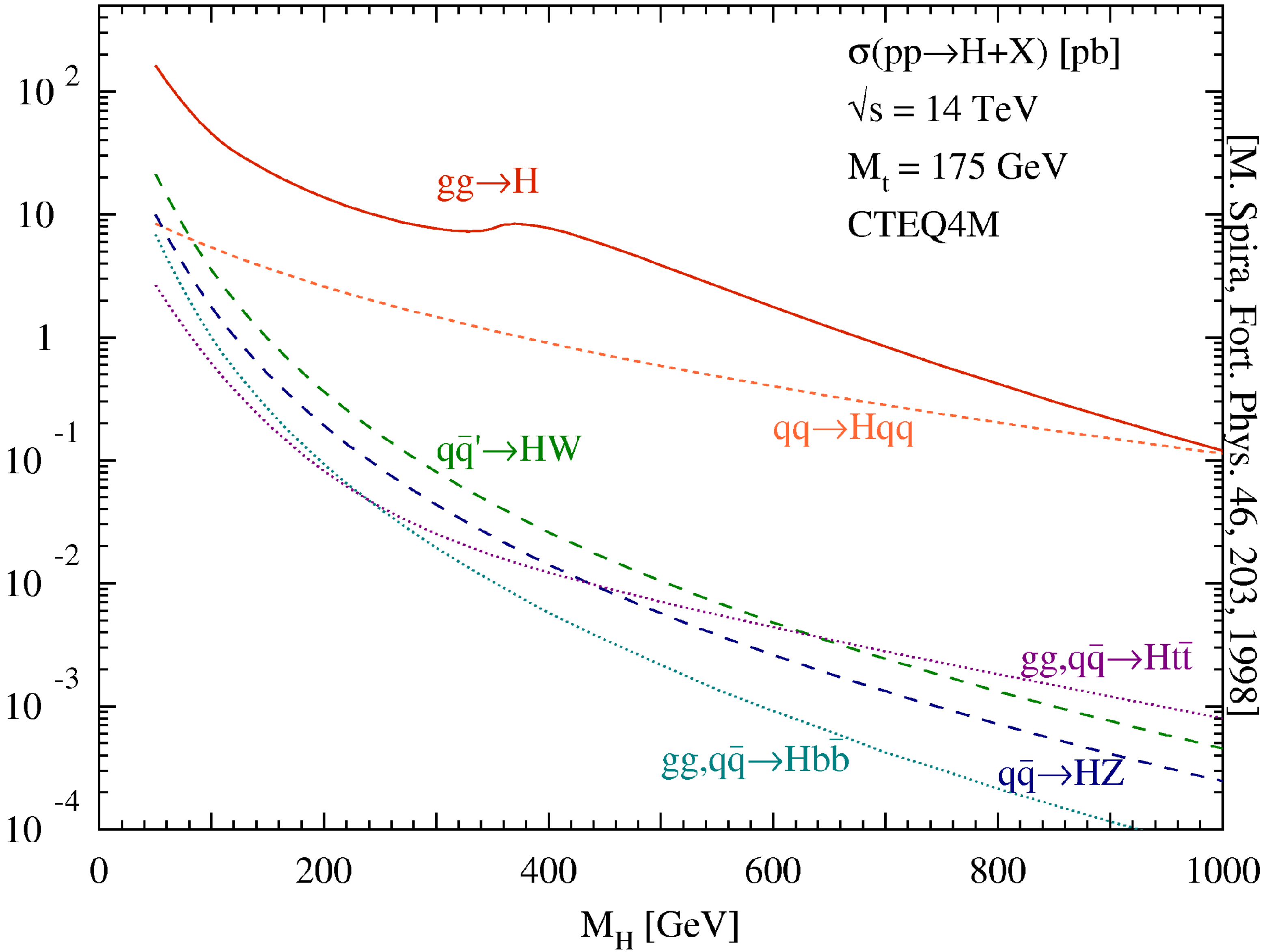
Higgs Production at Hadron Colliders

Gluon Fusion dominates Higgs production at Hadron Colliders

At the Tevatron:

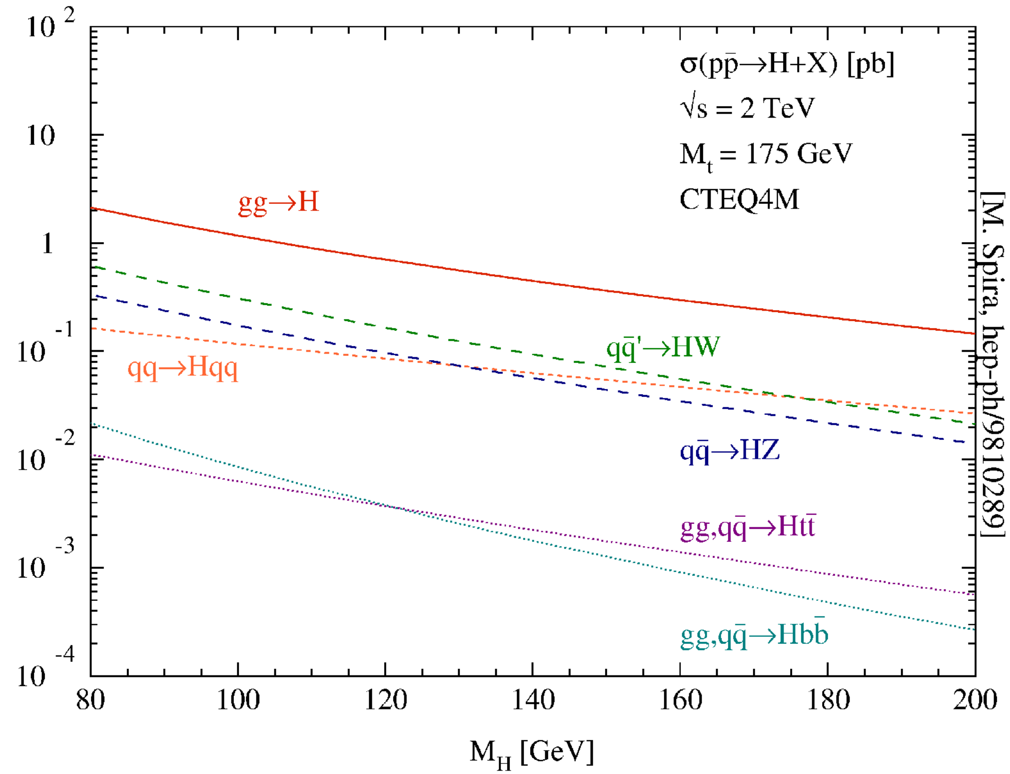


And at the LHC

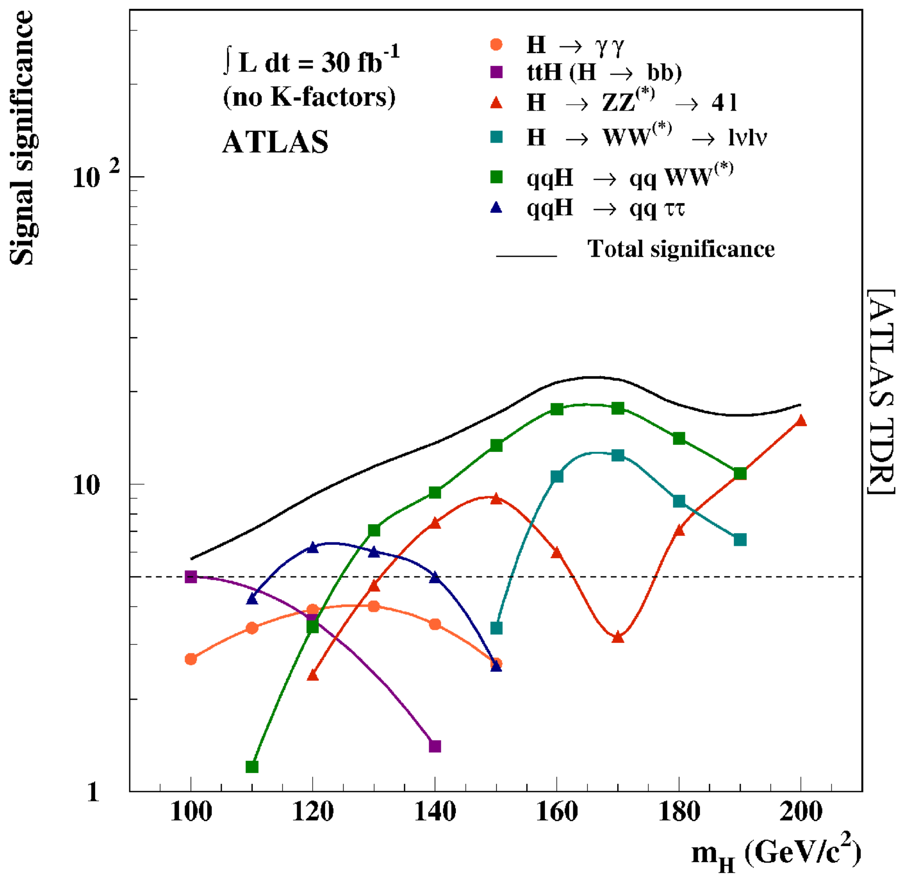


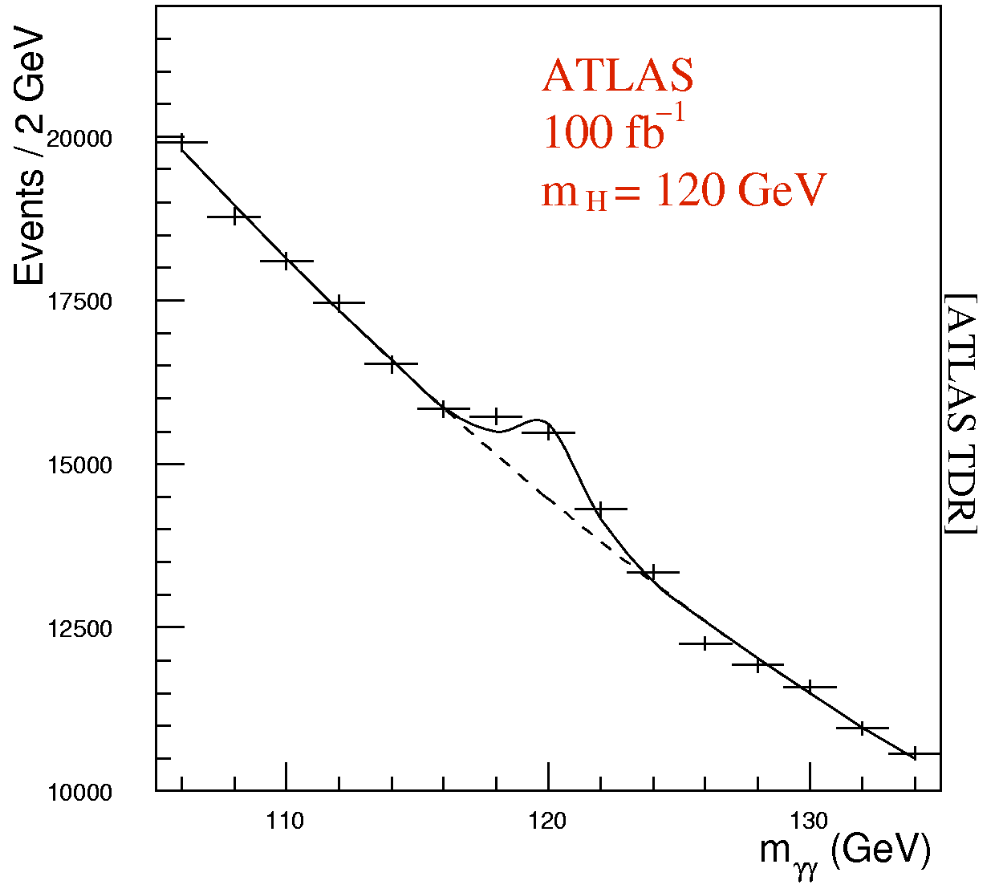
At the Tevatron, $H \rightarrow b\bar{b}$ decay is overwhelmed by QCD background and the rate is too low to observe rare decays like $H \rightarrow \gamma\gamma$.

Except near the $H \rightarrow WW$ threshold ($140 \text{ GeV} \leq M_H \leq 170 \text{ GeV}$) associated production ($q\bar{q} \rightarrow HW$) is better.



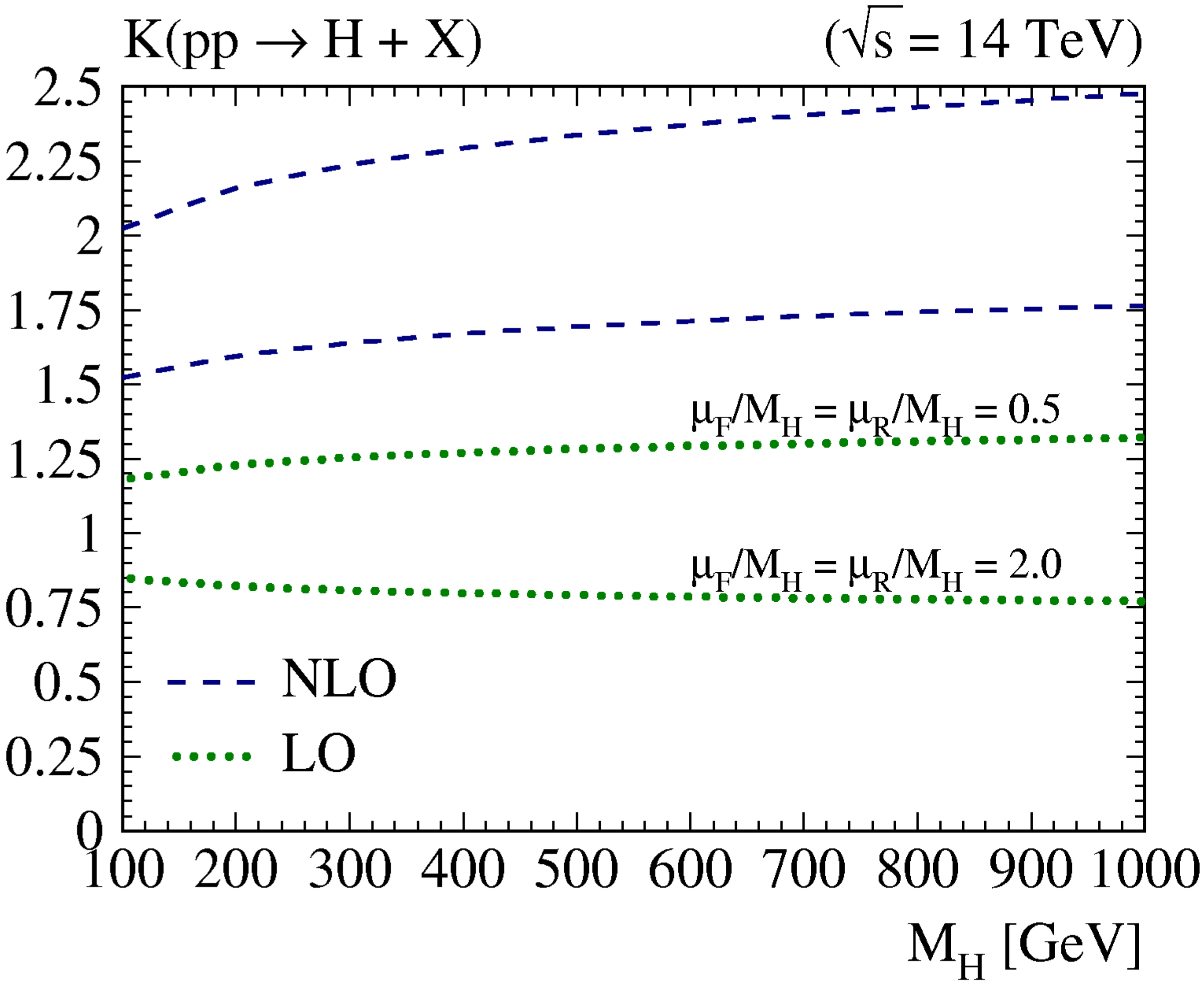
At LHC gg→H is the primary discovery mode



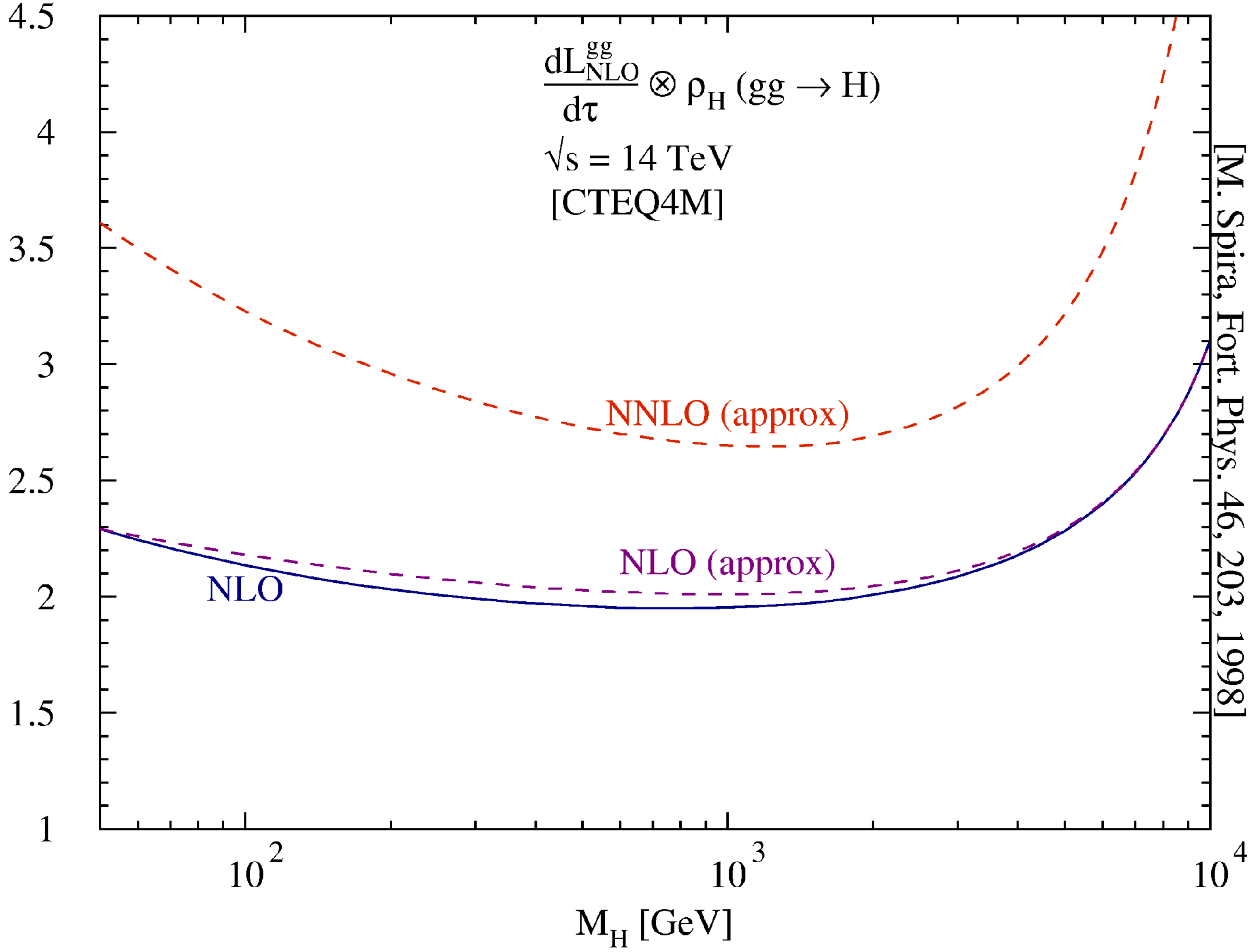


Why NNLO?

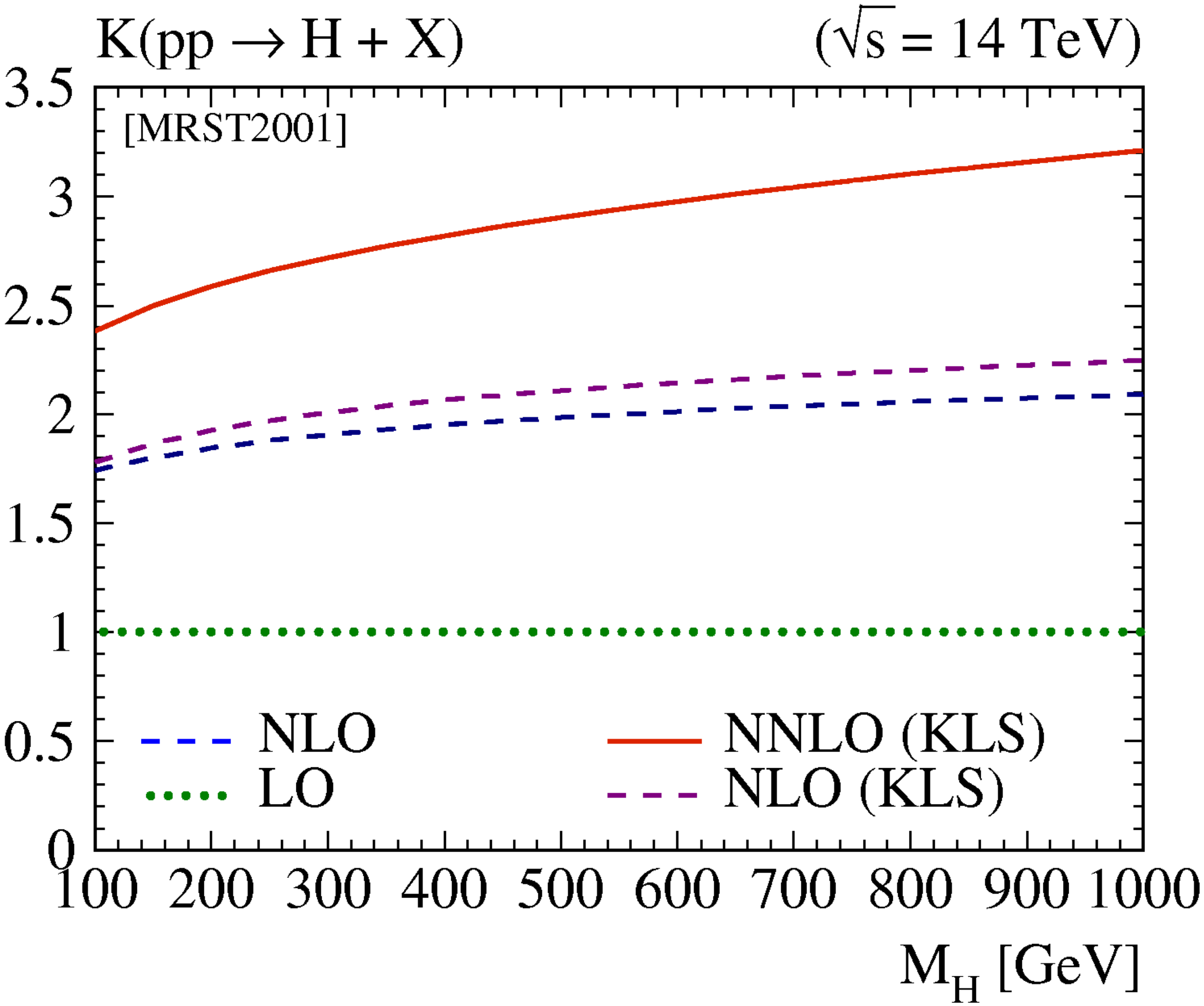
The NLO Corrections are very large



The NNLO corrections are also expected to be very large (KLS):



Using (consistent) MRST2001 PDFs:

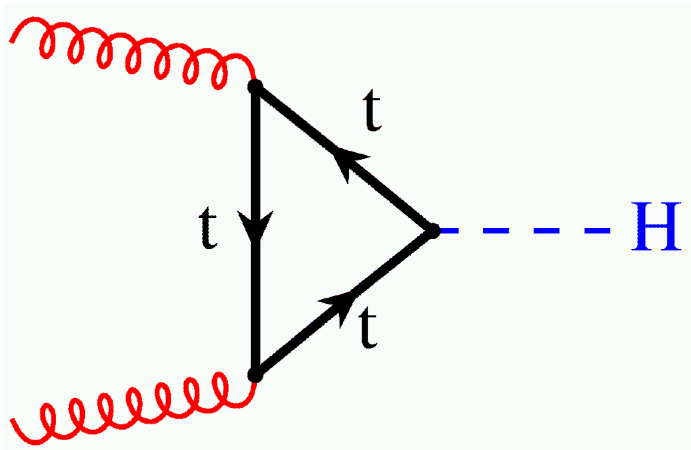


Methods

The Higgs boson couples to mass:

- The gluons have no direct coupling
- The quarks in the proton (u,d,s) have tiny couplings

Hadronic Higgs production is dominated by gluons interacting through virtual top quark loops.



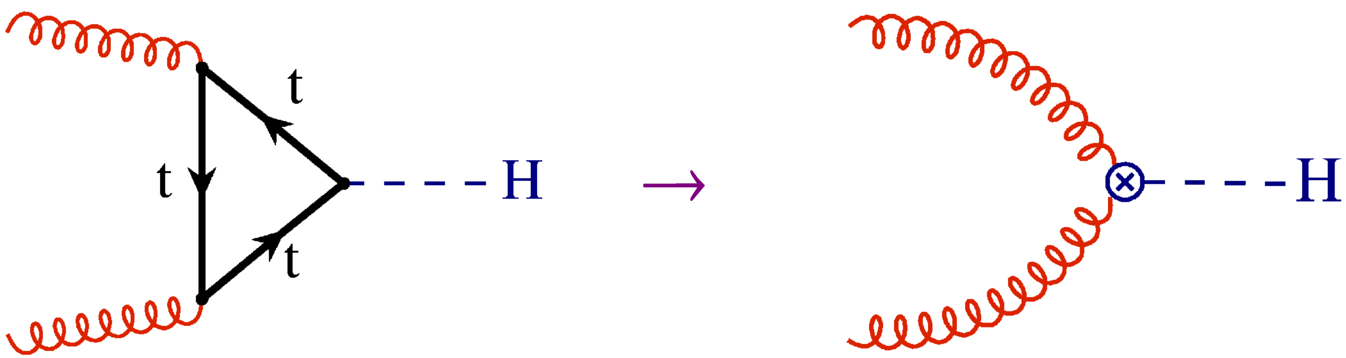
Effective Lagrangian

In the limit that the top quark is very heavy and all other quarks are massless, we can integrate out the top and formulate an effective Lagrangian coupling the Higgs to Gluons.

$$\mathcal{L} = C_1 \, H \, G^{\mu\nu} G_{\mu\nu}$$

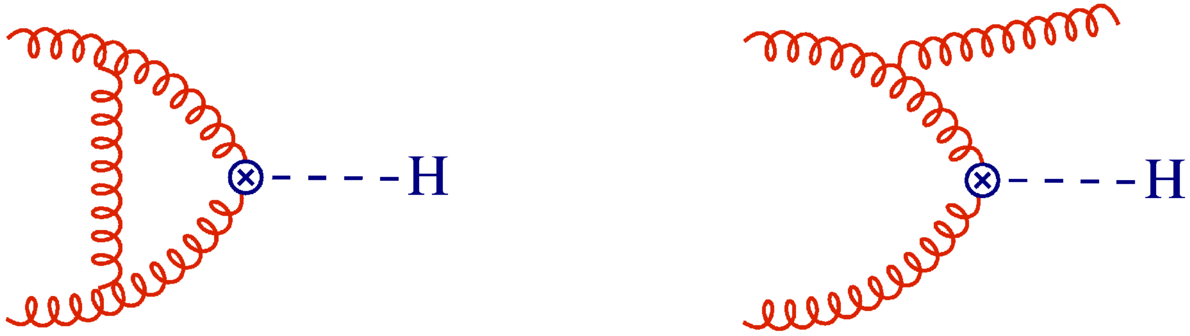
C_1 has been computed to order α_s^4 !

Using the effective Lagrangian greatly simplifies the calculation of radiative corrections.

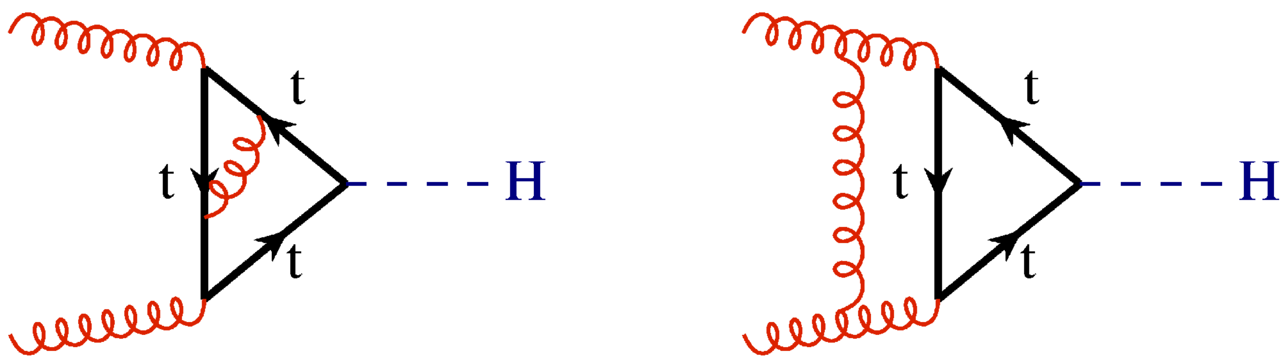


NLO Corrections

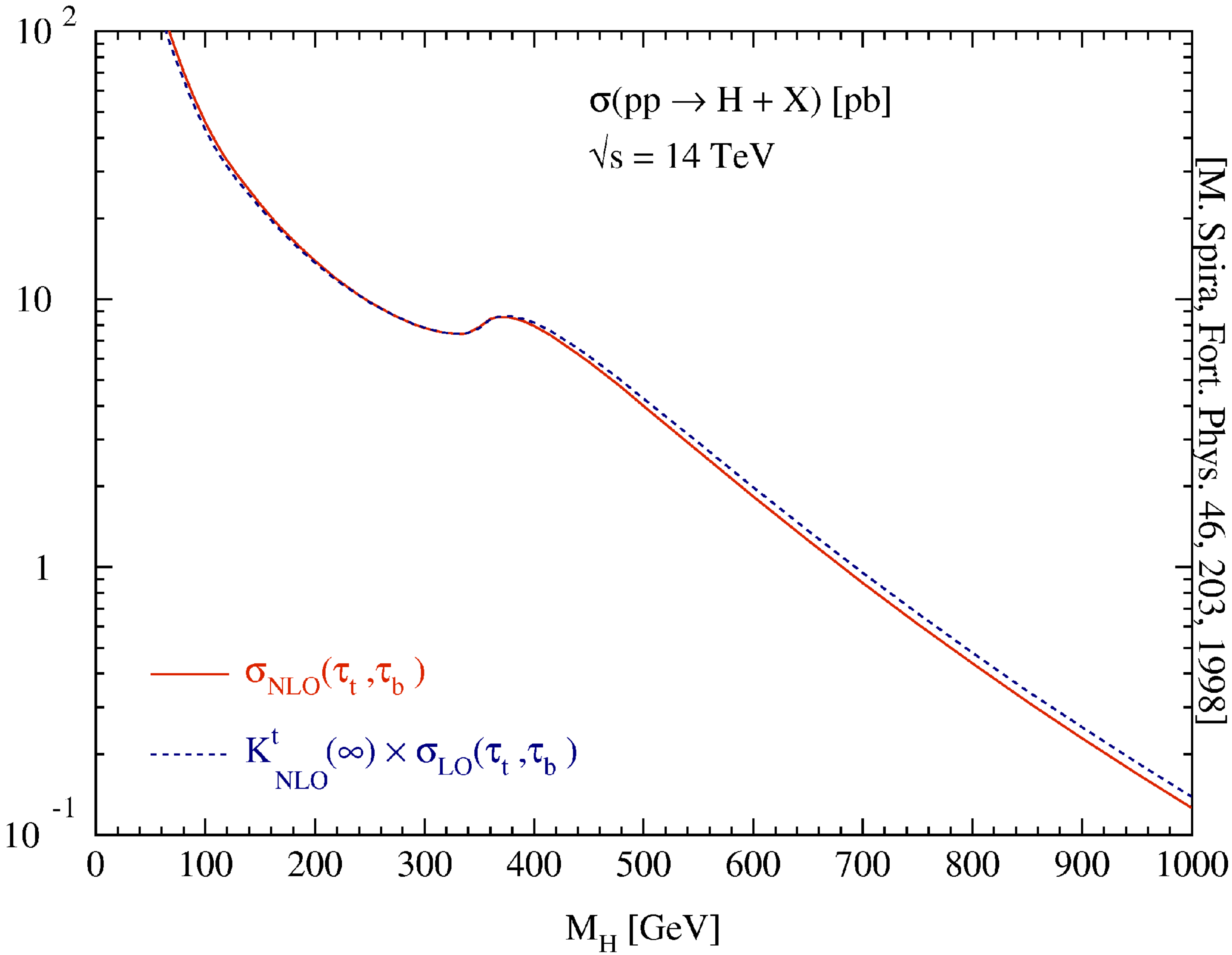
NLO Corrections have been computed in both the effective Lagrangian (Dawson; Djouadi et al.)



and in the full theory (Djouadi et al.)



They agree extremely well



[M. Spira, Fort. Phys. 46, 203, 1998]

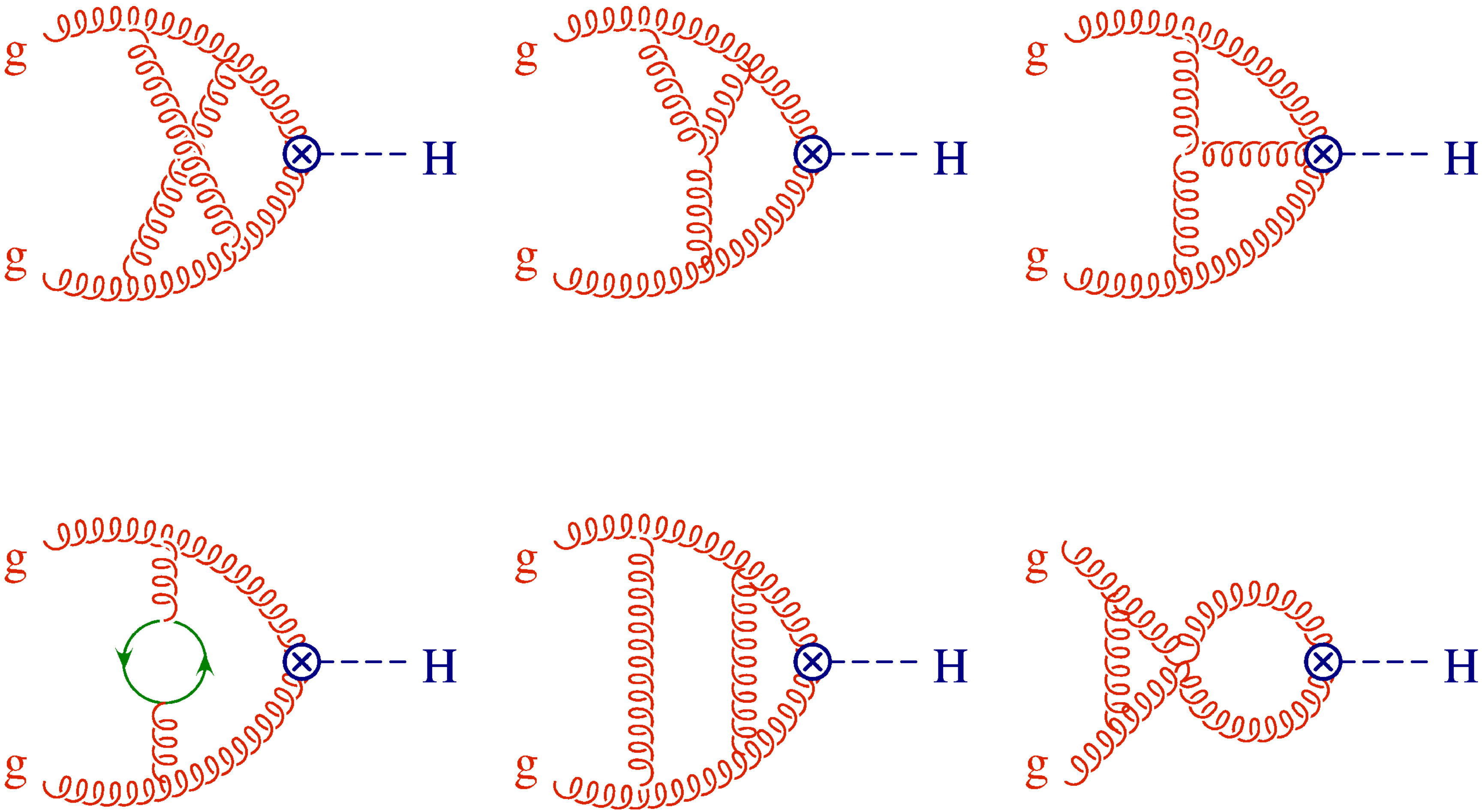
NNLO Corrections

For NNLO corrections, we assume that the Effective Lagrangian provides a good description of Higgs Production (especially in the most interesting mass range (< 200 GeV).

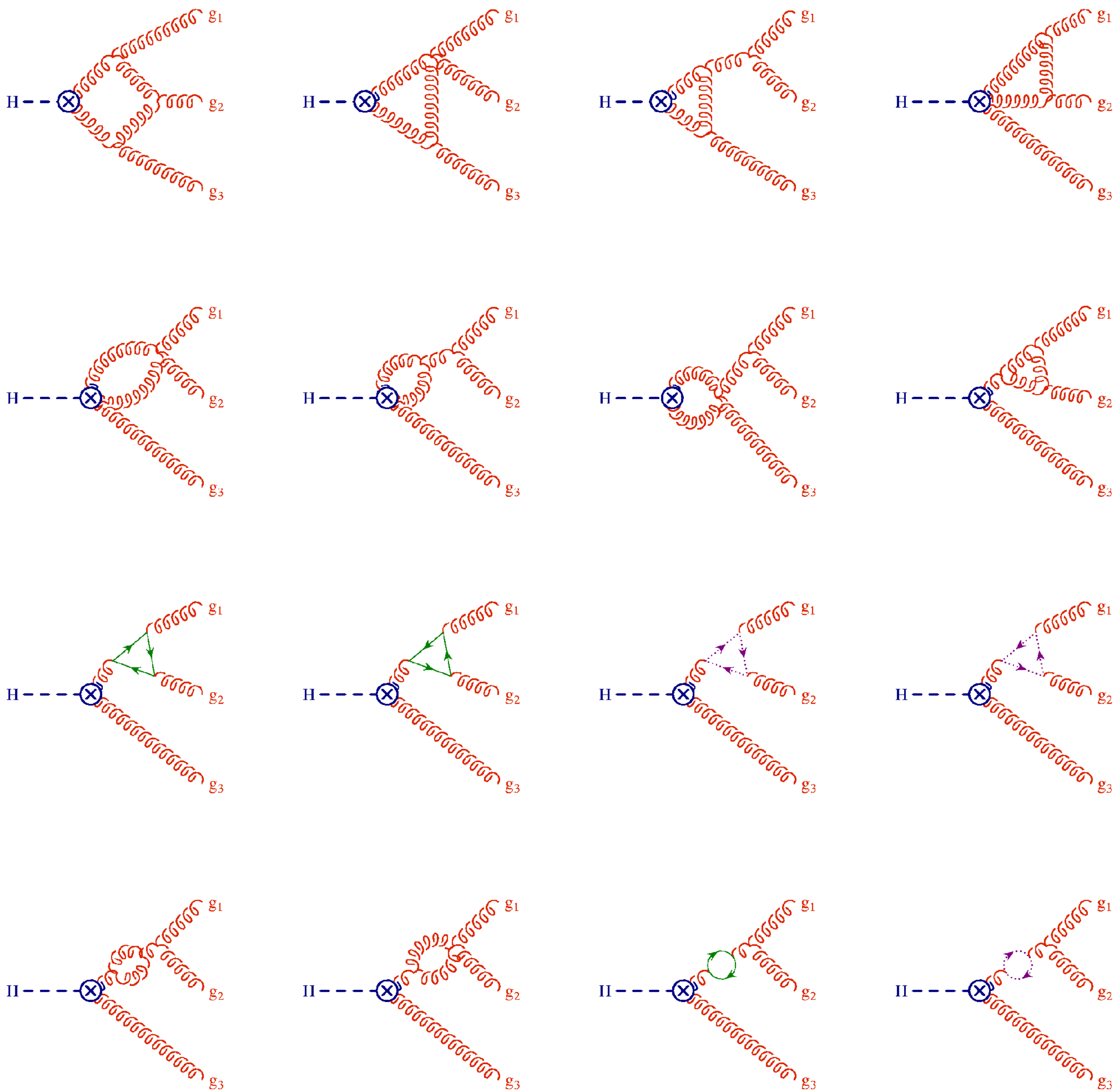
NNLO Corrections combine three components

- Virtual corrections to two loops
- Single Real Emission corrections to one loop
- Double Real Emission corrections

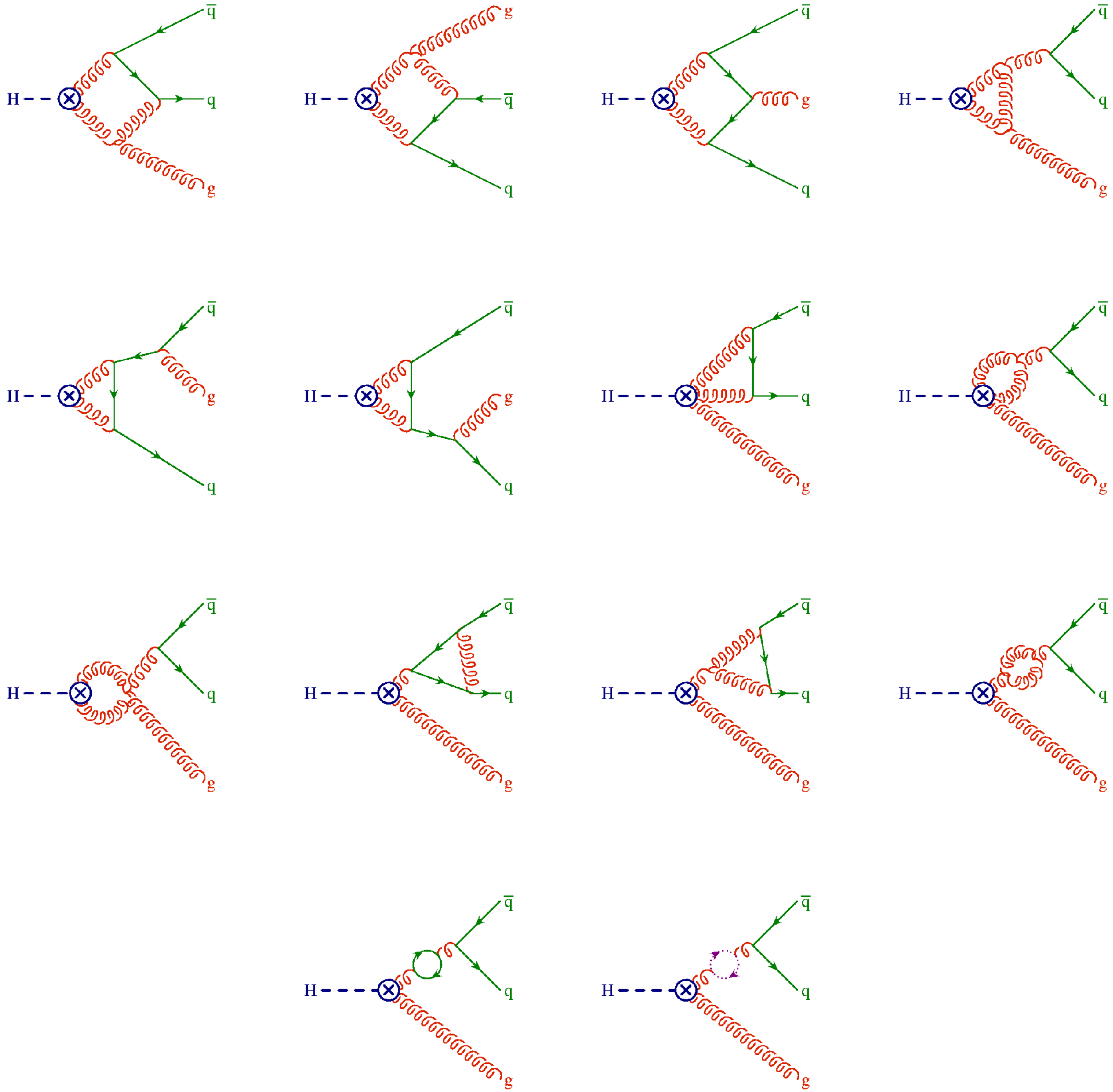
Virtual Corrections: Sample Diagrams



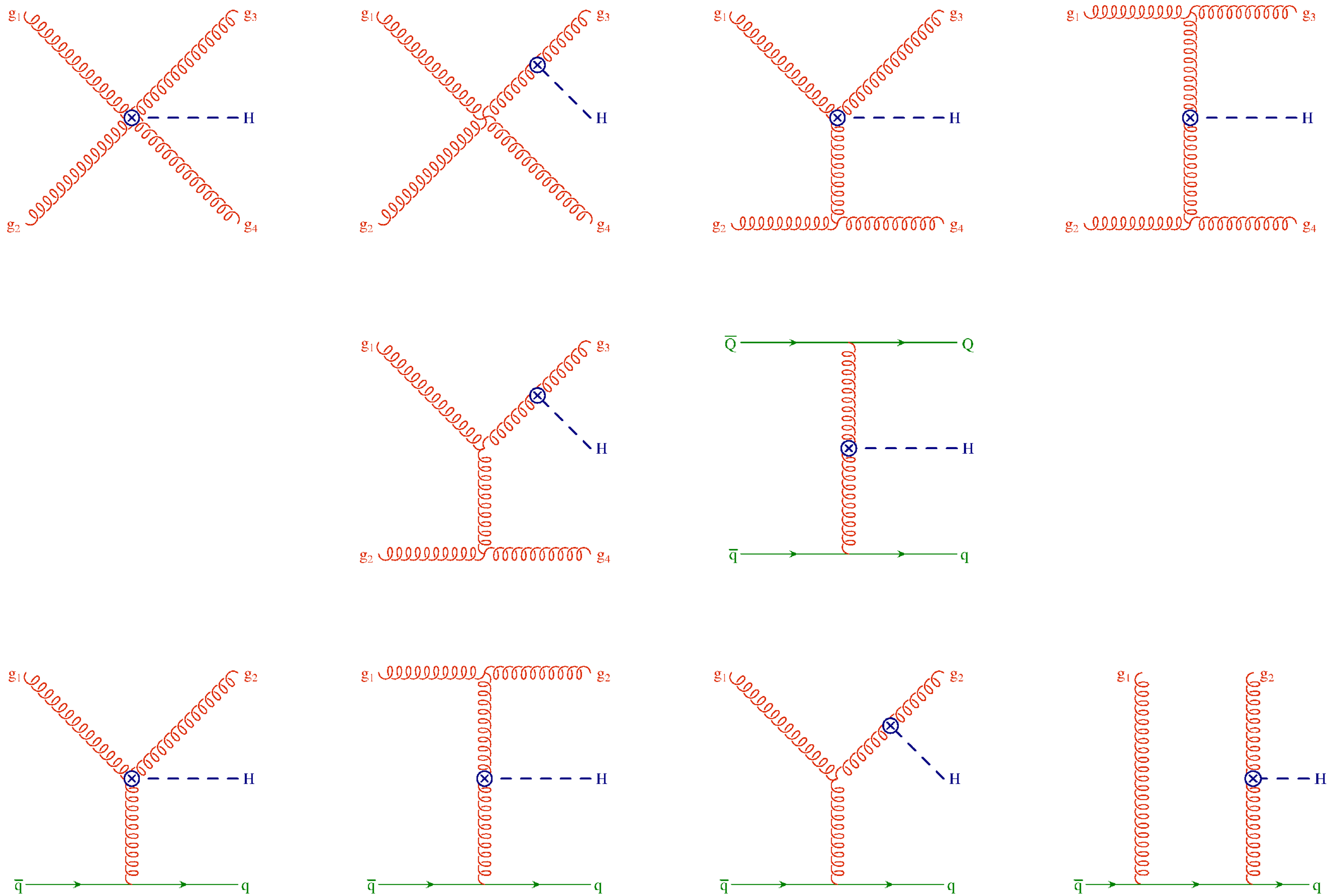
One-Loop Single Real Emission: $gg \to Hg$



One-Loop Single Real Emission: gq → Hq



Double Real Emission:



Mass Factorization

After combining all of the matrix element calculations and renormalizing, the sum still contains poles. These are removed by mass factorization.

$$\sigma_{ij} = \sum_{a,b} \hat{\sigma}_{ab} \otimes \left(x\Gamma_{ai}\right) \otimes \left(x\Gamma_{bj}\right)$$

$$\left[(f \otimes g)(x) = \int_0^1 dy \int_0^1 dz\, f(y)\, g(z)\, \delta(x - yz) \right]$$

$$\Gamma_{ij}(x) = \delta_{ij}\delta(1-x) - \frac{\alpha_s}{\pi}\frac{P_{ij}^{(0)}(x)}{\varepsilon} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{1}{2\varepsilon^2}\left(\left(P_{ik}^{(0)} \otimes P_{kj}^{(0)}\right)(x) + \beta_0 P_{ij}^{(0)}(x)\right) - \frac{1}{2\varepsilon}P_{ij}^{(1)}(x)\right] + \mathscr{O}(\alpha_s^3),$$

Phase Space Integration

Virtual Process: 2→1

$$\sigma|_{\mathrm{V}} = \frac{1}{2\hat{s}}\frac{1}{S}\frac{2\pi}{\hat{s}}\delta(1-x)\,|\mathcal{M}_{\mathrm{V}}|^2\ , \qquad x \equiv \frac{M_H^2}{\hat{s}}$$

Single Real Emission: 2→2

$$M_H^2 = x\,\hat{s}$$

$$s_{12} = \hat{s}$$

$$s_{13} = -\hat{s}(1-x)z$$

$$s_{23} = -\hat{s}(1-x)(1-z)$$

[redacted]

Double Real Emission: 2→3

$$k_1 = -\frac{\sqrt{s}}{2}(1, \quad \mathbf{0}, \quad 0, \quad 1)$$

$$k_2 = -\frac{\sqrt{s}}{2}(1, \quad \mathbf{0}, \quad 0, \quad -1)$$

$$k_3 = \frac{s - s_{4H}}{2\sqrt{s}}(1, \quad \mathbf{0}, \quad -\sin\theta, \quad \cos\theta)$$

$$k_4 = \frac{s - s_{3H}}{2\sqrt{s}}(1, \quad \mathbf{n}\sin\chi\sin\phi, \sin\chi\cos\theta\cos\phi - \cos\chi\sin\theta, \quad \sin\chi\sin\theta\cos\phi + \cos\chi\cos\theta)$$

$$k_H = -k_1 - k_2 - k_3 - k_4$$

Integrate over angles ϕ, θ and invariants s_{3H} and s_{4H}.

$$M_H^2 = x\hat{s}$$

$$s_{3H} = \hat{s}\,(1-(1-x)y)$$

$$s_{4H} = \hat{s}\left(\frac{1-(1-x)+(1-x)^2 y(1-y)(1-z)}{1-(1-x)y}\right)$$

The invariants are written as:

$$s_{12} = \hat{s} \qquad s_{34} = \hat{s}\frac{(1-x)^2 y(1-y)z}{1-(1-x)y}$$

$$s_{13} = -\frac{\hat{s}(1-x)(1-y)}{2}\frac{1-(1-x)y(1-z)}{1-(1-x)y}\Phi_1 \qquad s_{14} = -\frac{\hat{s}(1-x)(1-y)}{2}\frac{1-(1-x)y(1-z)}{1-(1-x)y}\Phi_2$$

$$s_{23} = -\frac{\hat{s}(1-x)y}{2}\Psi_1 \qquad s_{24} = -\frac{\hat{s}(1-x)y}{2}\Psi_2$$

$$s_{1H} = -\hat{s}(1-x)y\frac{x+(1-x)(1-y)(1-z)}{1-(1-x)y} \qquad s_{2H} = -\hat{s}(1-x)(1-y),$$

where

$$\Phi_1 = 1-\cos\theta, \qquad \Phi_2 = 1+\cos\theta,$$

$$\Psi_1 = 1-\sin\chi\sin\theta\cos\phi-\cos\chi\cos\theta, \qquad \Psi_2 = 1+\sin\chi\sin\theta\cos\phi+\cos\chi\cos\theta$$

$$\cos^2\frac{\chi}{2} = (1-z)\frac{1-(1-x)y}{1-(1-x)y(1-z)} \qquad \sin^2\frac{\chi}{2} = \frac{z}{1-(1-x)y(1-z)}$$

$$\sigma|_{\mathrm{DR}} = \frac{1}{2\hat{s}}\frac{1}{S}\frac{\hat{s}}{256\pi^4\Gamma(1-2\epsilon)}\left(\frac{4\pi x\mu^2}{M_H^2}\right)^{2\epsilon}(1-x)^{3-4\epsilon}\int_0^\pi d\theta\int_0^\pi d\phi\,(\sin\theta)^{d-3}(\sin\phi)^{d-4}$$
$$\times\int_0^1 dy\int_0^1 dz\,[1-(1-x)y]^{-1+\epsilon}y^{1-2\epsilon}(1-y)^{1-2\epsilon}z^{-\epsilon}(1-z)^{-\epsilon}|\mathcal{M}_{\mathrm{DR}}|^2$$

Power Series Expansion in (1-x)

The cross section can be written as a power series in (1-x), ln(1-x):

$$x \equiv \frac{M_H^2}{\hat{s}}$$

$$\hat{\sigma}_{ij} = \sum_{n \geq 0} \left(\frac{\alpha_s}{\pi} \right)^n \hat{\sigma}_{ij}^{(n)} ,$$

$$\hat{\sigma}_{ij}^{(n)} = x \left[a^{(n)} \delta(1-x) + \sum_{k=0}^{2n-1} b_k^{(n)} \left[\frac{\ln^k(1-x)}{1-x} \right]_+ + \sum_{l=0}^{\infty} \sum_{k=0}^{2n-1} c_{lk}^{(n)} (1-x)^l \ln^k(1-x) \right]$$

In the Soft limit, one keeps only the $a^{(n)}$ and $b_k^{(n)}$ terms.

Real emission generates terms like:

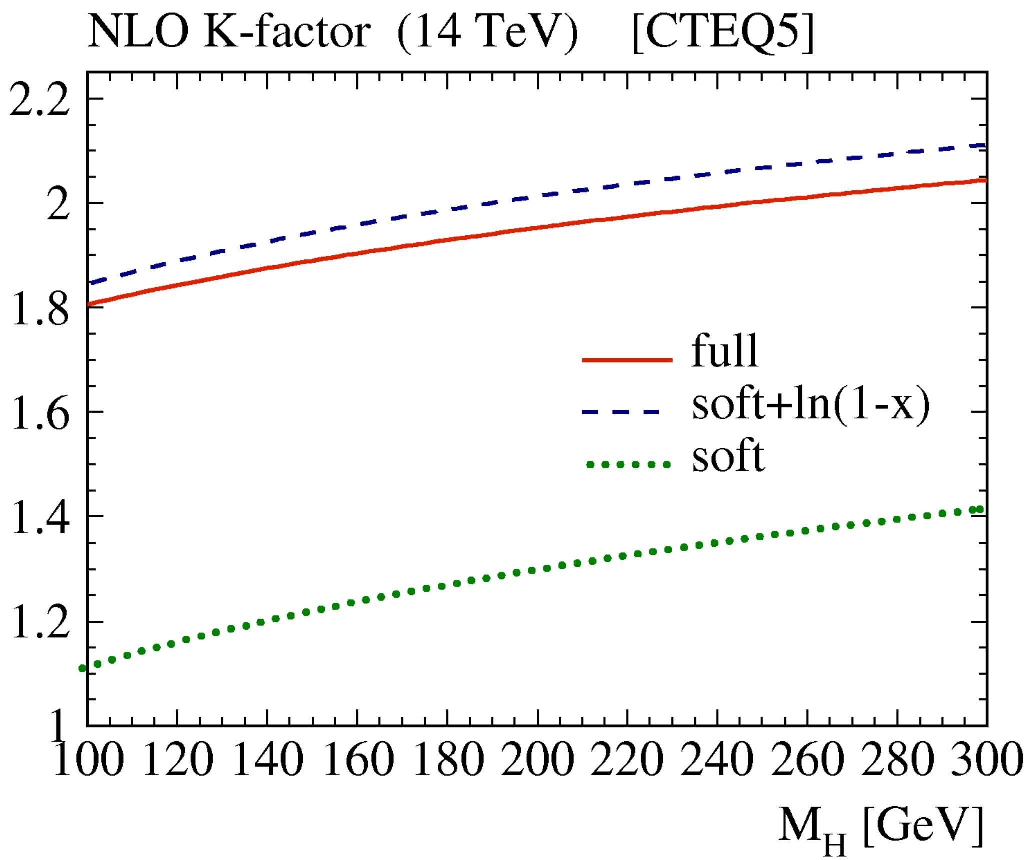
$$(1-x)^{-1-m\varepsilon} = -\frac{\delta(1-x)}{m\varepsilon} + \sum_{n=0}^{\infty} \frac{(-m\varepsilon)^n}{n!} \left[\frac{\ln^n(1-x)}{1-x}\right]_+$$

so the $b_k^{(n)}$ terms come for free.

In the Soft + Collinear limit (SVC), one also keeps $c_{03}^{(2)}$

The Soft Limit is not enough!

At NLO, it was found that the soft approximation is inadequate. The leading $c_{lk}^{(n)}$ term dominates!



Recall:

$$s_{12} = \hat{s} \qquad s_{34} = \hat{s}\frac{(1-x)^2 y(1-y)z}{1-(1-x)y}$$

$$s_{13} = -\frac{\hat{s}(1-x)(1-y)}{2}\frac{1-(1-x)y(1-z)}{1-(1-x)y}\Phi_1 \qquad s_{14} = -\frac{\hat{s}(1-x)(1-y)}{2}\frac{1-(1-x)y(1-z)}{1-(1-x)y}\Phi_2$$

$$s_{23} = -\frac{\hat{s}(1-x)y}{2}\Psi_1 \qquad s_{24} = -\frac{\hat{s}(1-x)y}{2}\Psi_2$$

$$s_{1H} = -\hat{s}(1-x)y\frac{x+(1-x)(1-y)(1-z)}{1-(1-x)y} \qquad s_{2H} = -\hat{s}(1-x)(1-y),$$

Where

$$\Phi_1 = 1-\cos\theta, \qquad \Phi_2 = 1+\cos\theta,$$

$$\Psi_1 = 1-\sin\chi\sin\theta\cos\phi-\cos\chi\cos\theta, \qquad \Psi_2 = 1+\sin\chi\sin\theta\cos\phi+\cos\chi\cos\theta$$

$$\cos^2\frac{\chi}{2} = (1-z)\frac{1-(1-x)y}{1-(1-x)y(1-z)} \qquad \sin^2\frac{\chi}{2} = \frac{z}{1-(1-x)y(1-z)}$$

$$\sigma|_{\mathrm{DR}} = \frac{1}{2\hat{s}}\frac{1}{S}\frac{\hat{s}}{256\pi^4\Gamma(1-2\epsilon)}\left(\frac{4\pi x\mu^2}{M_H^2}\right)^{2\epsilon}(1-x)^{3-4\epsilon}\int_0^\pi d\theta\int_0^\pi d\phi\ (\sin\theta)^{d-3}\ (\sin\phi)^{d-4}$$
$$\times\int_0^1 dy\int_0^1 dz\ [1-(1-x)y]^{-1+\epsilon}\,y^{1-2\epsilon}(1-y)^{1-2\epsilon}z^{-\epsilon}(1-z)^{-\epsilon}\,|\mathcal{M}_{\mathrm{DR}}|^2$$

and

$$\mathrm{Li}_n(1-x) = \sum_{m=1}^{\infty} \frac{(1-x)^m}{m^n}$$

$$\frac{1}{x} = \frac{1}{1-(1-x)} = \sum_{m=0}^{\infty} (1-x)^m$$

$$\ln(x) = -\mathrm{Li}_1(1-x)$$

$$\mathrm{Li}_2(x) = -\mathrm{Li}_2(1-x) - \ln(x)\ln(1-x) + \zeta_2$$

$$\mathrm{Li}_3(x) = -\mathrm{Li}_3(1-x) - \mathrm{Li}_3\left(-\frac{1-x}{x}\right) + \zeta_3 + \zeta_2 \ln(x) - \frac{1}{2}\ln^2(x)\ln(1-x) + \frac{1}{6}\ln^3(x)$$

$$S_{1,2}(1-x) = \mathrm{Li}_3(1-x) + \mathrm{Li}_3\left(-\frac{1-x}{x}\right) - \frac{1}{6}\ln^3(x) - \mathrm{Li}_2(1-x)\ln(x)$$

Summing the Series

With enough terms in the expansion, one can invert the series and obtain the result in closed form if one knows the basis functions:

Prefactors	Functions	
$\frac{1}{1-x}$	1	$\ln(x)$
$\frac{1}{1+x}$	$\ln^2(x)$	$\ln^3(x)$
1	$\mathrm{Li}_2(1-x)$	$\mathrm{Li}_2(1-x)\ln(x)$
	$\mathrm{Li}_2(1-x^2)$	$\mathrm{Li}_2(1-x^2)\ln(x)$
$1-x$	$\mathrm{Li}_3(1-x)$	$\mathrm{Li}_3\left(-\frac{1-x}{x}\right)$
$(1-x)^2$	$\mathrm{Li}_3(1-x^2)$	$\mathrm{Li}_3\left(-\frac{1-x^2}{x^2}\right)$
$(1-x)^3$	$\mathrm{Li}_3\left(\frac{1-x}{1+x}\right)-\mathrm{Li}_3\left(-\frac{1-x}{1+x}\right)$	

So, with $6\times 13 = 78$ terms, the series can be inverted.

Even at low orders in the expansion, we were able to find an error in the NNLO Drell-Yan correction

$$\Delta^{(2),C_A}_{q\bar{q}} = \Delta^{(2),C_A}_{q\bar{q}}\Big|_{\mathrm{HMN}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A C_F \Big\{ - 8x\left(2\mathrm{Li}_2(1-x) + 2\ln(x)\ln(1-x) - \ln^2(x)\right)\Big\} ,$$

$$\Delta^{(2),C_F}_{q\bar{q}} = \Delta^{(2),C_F}_{q\bar{q}}\Big|_{\mathrm{HMN}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F^2 \Big\{ - 16\ln(x) - 8(3+x)\left(2\mathrm{Li}_2(1-x) + 2\ln(x)\ln(1-x) - \ln^2(x)\right)\Big\} ,$$

$$\Delta^{(2),C_A}_{qg} = \Delta^{(2),C_A}_{qg}\Big|_{\mathrm{HMN}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A T_f \Big\{ - 8x\ln(x) + 4x\left(2\mathrm{Li}_2(1-x) + 2\ln(x)\ln(1-x) - \ln^2(x)\right)\Big\} ,$$

$$\Delta^{(2),C_F}_{qg} = \Delta^{(2),C_F}_{qg}\Big|_{\mathrm{HMN}} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_f \Big\{ - 4(3-x)\left(2\mathrm{Li}_2(1-x) + 2\ln(x)\ln(1-x) - \ln^2(x)\right) + 12(1-x)(1-2\ln(1-x)) + (28-44x)\ln(x)\Big\} .$$

NNLO Correction terms: gg initiated processes

$$
\begin{aligned}
\Delta_{ggA}(x) = {} & \left[\frac{741}{8} + \frac{139}{2}\zeta_2 - \frac{165}{4}\zeta_3 - \frac{9}{20}\zeta_2^2\right]\delta(1-x) - \left[\frac{101}{3} - 33\,\zeta_2 - \frac{351}{2}\,\zeta_3\right]\mathcal{D}_0(x) + \left[139 - 90\,\zeta_2\right]\mathcal{D}_1(x) - 33\,\mathcal{D}_2(x) + 72\,\mathcal{D}_3(x) \\
& - (144x - 72x^2 + 72x^3)\ln^3(1-x) - (297 - 381x + 348x^2 - 330x^3)\ln^2(1-x) - \frac{9}{2}\,\frac{(31 - 30x + 93x^2 - 94x^3 + 31x^4)}{1-x}\ln^2(1-x)\ln(x) \\
& + \left[\frac{(2027 - 2735x + 2182x^2 - 2583x^3)}{4} + (180x - 90x^2 + 90x^3)\zeta_2\right]\ln(1-x) + 3\,\frac{(88 - 211x + 312x^2 - 365x^3 + 187x^4)}{1-x}\ln(1-x)\ln(x) \\
& + 9\,\frac{(7 + 3x + 19x^2 - 3x^3 - 19x^4 + 9x^5)}{1-x^2}\ln(1-x)\ln^2(x) + 36\,\frac{(1 - 6x - 13x^2 - 6x^3 + x^4)}{1+x}\ln(1-x)\mathrm{Li}_2(1-x) \\
& - 18\,\frac{(1 + 2x + 3x^2 + 2x^3 + x^4)}{1+x}\ln(1-x)\mathrm{Li}_2(1-x^2) - \frac{9}{2}\,\frac{(24 - 38x + 8x^2 + 54x^3 - 19x^4 + 9x^5)}{1-x^2}\mathrm{Li}_3(1-x) \\
& - \frac{9}{2}\,\frac{(27 + 35x + 75x^2 - 29x^3 - 78x^4 + 6x^5)}{1-x^2}\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right) + \frac{9}{8}\,\frac{(1 + 2x + 3x^2 + 24x^3 + 16x^4)}{1+x}\mathrm{Li}_3(1-x^2) \\
& - \frac{9}{8}\,\frac{(1 + 2x + 3x^2 - 8x^3 - 8x^4)}{1+x}\mathrm{Li}_3\left(-\frac{(1-x^2)}{x^2}\right) - \frac{9}{2}\,\frac{(7 + 14x + 21x^2 + 8x^3 + 4x^4)}{1+x}\left[\mathrm{Li}_3\left(\frac{1-x}{1+x}\right) - \mathrm{Li}_3\left(-\frac{1-x}{1+x}\right)\right] \\
& - \frac{3}{4}\,\frac{(317 - 398x - 87x^2 + 300x^3 - 121x^4)}{1-x}\mathrm{Li}_2(1-x) + \frac{9}{2}\,\frac{(11 + 31x + 59x^2 - 25x^3 - 65x^4 + 11x^5)}{1-x^2}\mathrm{Li}_2(1-x)\ln(x) \\
& + \frac{(42 + 36x - 63x^2 - 33x^3)}{4}\mathrm{Li}_2(1-x^2) + \frac{9}{4}\,\frac{(5 + 10x + 15x^2 - 2x^4)}{1+x}\mathrm{Li}_2(1-x^2)\ln(x) + \frac{3}{4}\,\frac{(21 + 23x + 41x^2 - 37x^3 - 44x^4 + 4x^5)}{1-x^2}\ln^3(x) \\
& - \frac{3}{8}\,\frac{(154 - 365x + 675x^2 - 827x^3 + 374x^4)}{1-x}\ln^2(x) - \frac{1}{8}\,\frac{(2213 - 5599x + 6603x^2 - 7003x^3 + 4342x^4)}{1-x}\ln(x) \\
& + 9\,\frac{(9 - 2x + 27x^2 - 34x^3 + 9x^4)}{1-x}\zeta_2\ln(x) - \frac{(16309 - 20611x + 23819x^2 - 22749x^3)}{48} + \frac{3}{4}(319 - 277x + 233x^2 - 363x^3)\zeta_2 - \frac{351}{2}(2x - x^2 + x^3)\zeta_3 \\
& + n_f\Bigg\{\left[-\frac{689}{72} + l_{Ht} - \frac{5}{3}\zeta_2 + \frac{5}{6}\zeta_3\right]\delta(1-x) + \left[\frac{14}{9} - 2\,\zeta_2\right]\mathcal{D}_0(x) - \frac{10}{3}\mathcal{D}_1(x) + 2\,\mathcal{D}_2(x) - \frac{2}{9}(8 - 12x + 3x^2 - 17x^3)\,\zeta_2 \\
& + \frac{2}{9}(8 - 12x + 3x^2 - 17x^3)\ln^2(1-x) + \frac{8}{3}(x + x^2)\ln^2(1-x)\ln(x) - \frac{(922 - 294x + 249x^2 - 1570x^3)}{108}\ln(1-x) \\
& - \frac{2}{9}\,\frac{(17 + 7x + 21x^2 - 61x^3 + 25x^4)}{1-x}\ln(1-x)\ln(x) - \frac{8}{3}(x + x^2)\ln^2(x)\ln(1-x) + \frac{16}{3}(x + x^2)\ln(1-x)\mathrm{Li}_2(1-x) \\
& - \frac{(2 + 14x + 17x^2)}{6}\mathrm{Li}_3(1-x) - \frac{(2 - 34x - 31x^2)}{12}\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right). + \frac{1}{36}\,\frac{(68 - 302x + 21x^2 + 227x^3 + 4x^4)}{1-x}\mathrm{Li}_2(1-x) \\
& + \frac{(2 - 50x - 47x^2)}{12}\mathrm{Li}_2(1-x)\ln(x) + \frac{(2 + 6x + 9x^2)}{72}\ln^3(x) + \frac{1}{72}\,\frac{(68 + 100x + 69x^2 - 351x^3 + 132x^4)}{1-x}\ln^2(x) \\
& + \frac{1}{216}\,\frac{(1282 - 382x + 117x^2 - 3041x^3 + 2384x^4)}{1-x}\ln(x) - \frac{8}{3}(x + x^2)\zeta_2\ln(x) + \frac{(12707 - 606x + 1641x^2 - 17774x^3)}{1296}\Bigg\}
\end{aligned}
$$

NNLO Correction terms: gq initiated processes

$$
\begin{aligned}
\Delta_{gqA}(x) =\; & \frac{367}{54}(2-2x+x^2)\ln^3(1-x) - \frac{(2592-2278x-111x^2-288x^3)}{36}\ln^2(1-x) - \frac{(642+190x+553x^2)}{18}\ln^2(1-x)\ln(x) \\
& + \left[\frac{(23887-17388x-2538x^2-784x^3)}{162} - \frac{50}{9}(2-2x+x^2)\zeta_2\right]\ln(1-x) + \frac{(1665-2040x+174x^2-400x^3)}{27}\ln(1-x)\ln(x) \\
& + \frac{4(38+21x+39x^2)}{9}\ln(1-x)\ln^2(x) - \frac{2}{9}(46+298x+139x^2)\ln(1-x)\,\mathrm{Li}_2(1-x) - 2(2+2x+x^2)\ln(1-x)\,\mathrm{Li}_2(1-x^2) \\
& - \frac{2}{9}(42-142x-x^2)\mathrm{Li}_3(1-x) - \frac{(302+474x+339x^2)}{9}\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right) - \frac{(2+2x+x^2)}{2}\mathrm{Li}_3(1-x^2) - \frac{(2+2x+x^2)}{2}\mathrm{Li}_3\left(-\frac{(1-x^2)}{x^2}\right) \\
& - 4(2+2x+x^2)\left[\mathrm{Li}_3\left(\frac{1-x}{1+x}\right) - \mathrm{Li}_3\left(-\frac{1-x}{1+x}\right)\right] - \frac{(979-144x-215x^2+52x^3)}{18}\mathrm{Li}_2(1-x) + \frac{(142+374x+245x^2)}{9}\mathrm{Li}_2(1-x)\ln(x) \\
& + \frac{(166+222x+33x^2+4x^3)}{54}\mathrm{Li}_2(1-x^2) + 2(2+2x+x^2)\ln(x)\,\mathrm{Li}_2(1-x^2) + \frac{(133+202x+115x^2)}{27}\ln^3(x) \\
& - \frac{(837-1296x+234x^2-226x^3)}{54}\ln^2(x) - \left[\frac{(22042-32040x-5847x^2-2464x^3)}{324} - \frac{(194+222x+213x^2)}{9}\zeta_2\right]\ln(x) \\
& - \frac{(173719-156324x-12687x^2-6148x^3)}{1944} + \frac{(1071-710x-130x^2-144x^3)}{18}\zeta_2 + \frac{311}{18}(2-2x+x^2)\zeta_3 \\
& + n_F\left\{\frac{(2-2x+x^2)}{18}\ln^2(1-x) - \frac{(13-16x+9x^2)}{9}\ln(1-x) - \frac{(4-4x+2x^2)}{9}\ln(1-x)\ln(x)\right. \\
& \left. + \frac{(2-2x+x^2)}{9}\ln^2(x) + \frac{(29-38x+19x^2)}{27}\ln(x) + \frac{(265-418x+179x^2)}{162}\right\}
\end{aligned}
$$

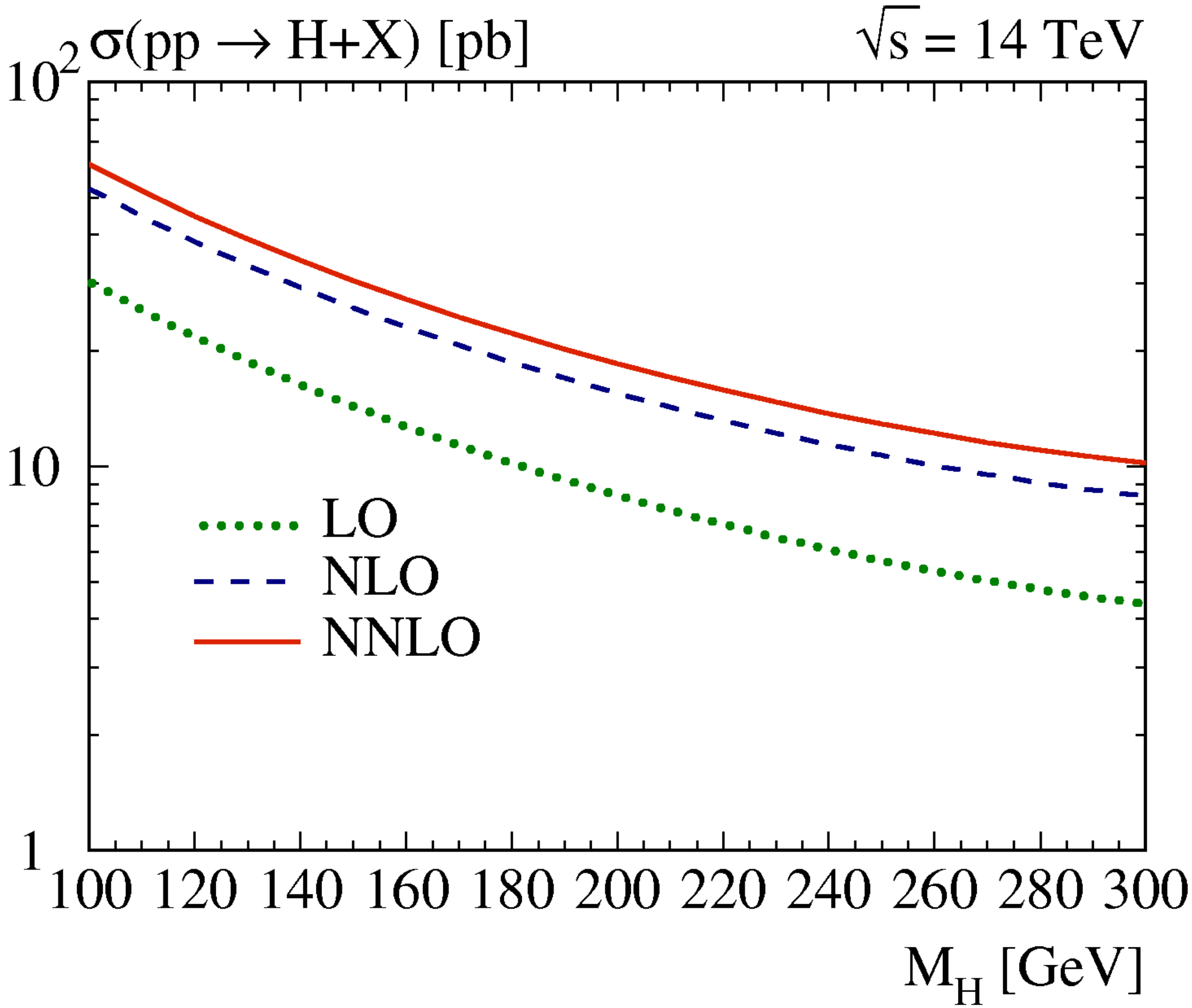
NNLO Correction Terms: qq and q$\bar{\text{q}}$ initiated processes

$$\Delta_{qqA}(x) = -\frac{32}{9}(3-2x-x^2)\ln^2(1-x) - \frac{16}{9}(4+4x+x^2)\ln^2(1-x)\ln(x) + \frac{4}{3}(17-12x-5x^2)\ln(1-x) + \frac{8}{9}(12-8x-5x^2)\ln(1-x)\ln(x)$$
$$+\frac{8}{9}(4+4x+x^2)\ln(1-x)\ln^2(x) - \frac{32}{9}(4+4x+x^2)\ln(1-x)\mathrm{Li}_2(1-x) - \frac{8}{27}(2-2x+x^2)\mathrm{Li}_3(1-x) - \frac{8}{27}(50+46x+13x^2)\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right)$$
$$-\frac{8}{9}(6-4x-x^2)\mathrm{Li}_2(1-x) + \frac{16}{27}(19+17x+5x^2)\mathrm{Li}_2(1-x)\ln(x) + \frac{8}{81}(19+17x+5x^2)\ln^3(x) - \frac{4}{27}(18-10x-9x^2)\ln^2(x)$$
$$-\frac{2}{27}(129-212x-69x^2)\ln(x) + \frac{16}{9}(4+4x+x^2)\zeta_2\ln(x) + \frac{4}{27}(-86+53x+33x^2) + \frac{32}{9}(3-2x-x^2)\zeta_2$$

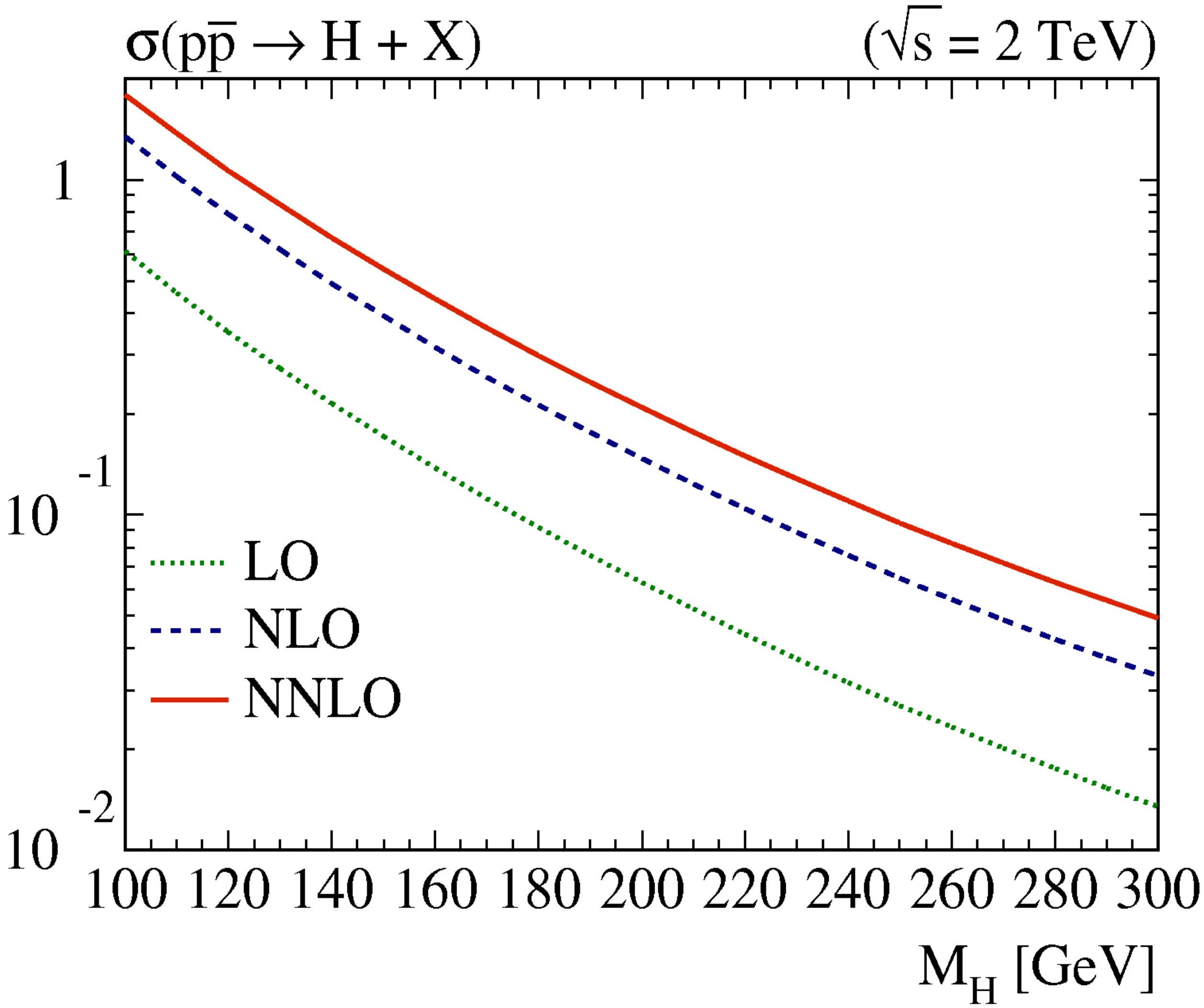
$$\Delta_{qq'A}(x) = -\frac{32}{9}(3-2x-x^2)\ln^2(1-x) - \frac{16}{9}(4+4x+x^2)\ln^2(1-x)\ln(x) + \frac{4}{3}(17-12x-5x^2)\ln(1-x) + \frac{8}{9}(12-8x-5x^2)\ln(1-x)\ln(x)$$
$$+\frac{8}{9}(4+4x+x^2)\ln(1-x)\ln^2(x) - \frac{32}{9}(4+4x+x^2)\ln(1-x)\mathrm{Li}_2(1-x) - \frac{32}{9}(4+4x+x^2)\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right)$$
$$-\frac{8}{9}(6-4x-x^2)\mathrm{Li}_2(1-x) + \frac{8}{3}(4+4x+x^2)\mathrm{Li}_2(1-x)\ln(x) + \frac{4}{9}(4+4x+x^2)\ln^3(x) - \frac{8}{9}(3-2x-x^2)\ln^2(x)$$
$$-\frac{2}{9}(43-68x-29x^2)\ln(x) + \frac{16}{9}(4+4x+x^2)\zeta_2\ln(x) - \frac{2}{9}(61-46x-15x^2) + \frac{32}{9}(3-2x-x^2)\zeta_2$$

$$\Delta_{q\bar{q}A}(x) = -\frac{32}{81}(14+21x-48x^2+13x^3)\ln^2(1-x) - \frac{16}{9}(4+4x+x^2)\ln^2(1-x)\ln(x) + \frac{4}{81}(75+892x-1351x^2+384x^3)\ln(1-x)$$
$$+\frac{8}{81}(76+72x-189x^2+64x^3)\ln(1-x)\ln(x) + \frac{8}{9}(4+4x+x^2)\ln(1-x)\ln^2(x) - \frac{32}{9}(4+4x+x^2)\ln(1-x)\mathrm{Li}_2(1-x)$$
$$-\frac{40}{27}(2+2x+x^2)\mathrm{Li}_3(1-x) - \frac{8}{9}(18+18x+5x^2)\mathrm{Li}_3\left(-\frac{(1-x)}{x}\right) + \frac{10}{27}(2+2x+x^2)\mathrm{Li}_3(1-x^2) + \frac{2}{9}(2+2x+x^2)\mathrm{Li}_3\left(-\frac{(1-x^2)}{x^2}\right)$$
$$+\frac{8}{27}(2+2x+x^2)\left[\mathrm{Li}_3\left(\frac{1-x}{1+x}\right) - \mathrm{Li}_3\left(-\frac{1-x}{1+x}\right)\right] - \frac{8}{81}(12+30x-93x^2-26x^3)\mathrm{Li}_2(1-x) + \frac{16}{9}(7+7x+2x^2)\mathrm{Li}_2(1-x)\ln(x)$$
$$-\frac{8}{27}(2x+x^2+6x^3)\mathrm{Li}_2(1-x^2) - \frac{4}{9}(2+2x+x^2)\mathrm{Li}_2(1-x^2)\ln(x) + \frac{8}{27}(5+5x+x^2)\ln^3(x) - \frac{8}{81}(27+27x-81x^2+44x^3)\ln^2(x)$$
$$+\frac{2}{81}(9-1064x+2111x^2-768x^3)\ln(x) + \frac{16}{9}(4+4x+x^2)\zeta_2\ln(x) + \frac{20}{81}(101-462x+520x^2-159x^3) + \frac{16}{81}(11+93x-147x^2+43x^3)\zeta_2$$
$$+n_f\left\{\frac{32(1-x)^3}{81}\ln(1-x) - \frac{16(3-9x+12x^2-4x^3)}{81}\ln(x) - \frac{8(41-111x+111x^2-41x^3)}{243}\right\}$$

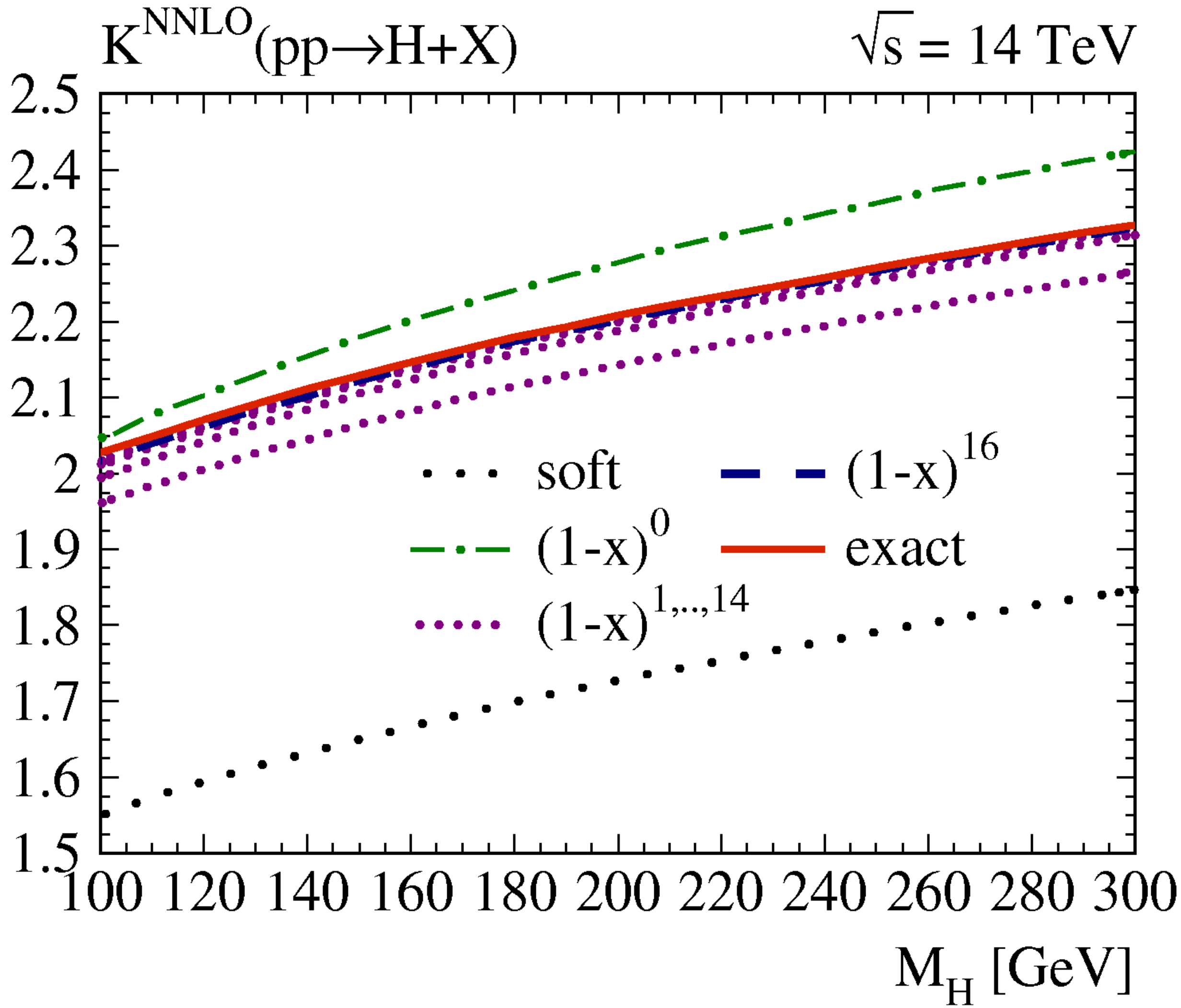
Results: Higgs at LHC



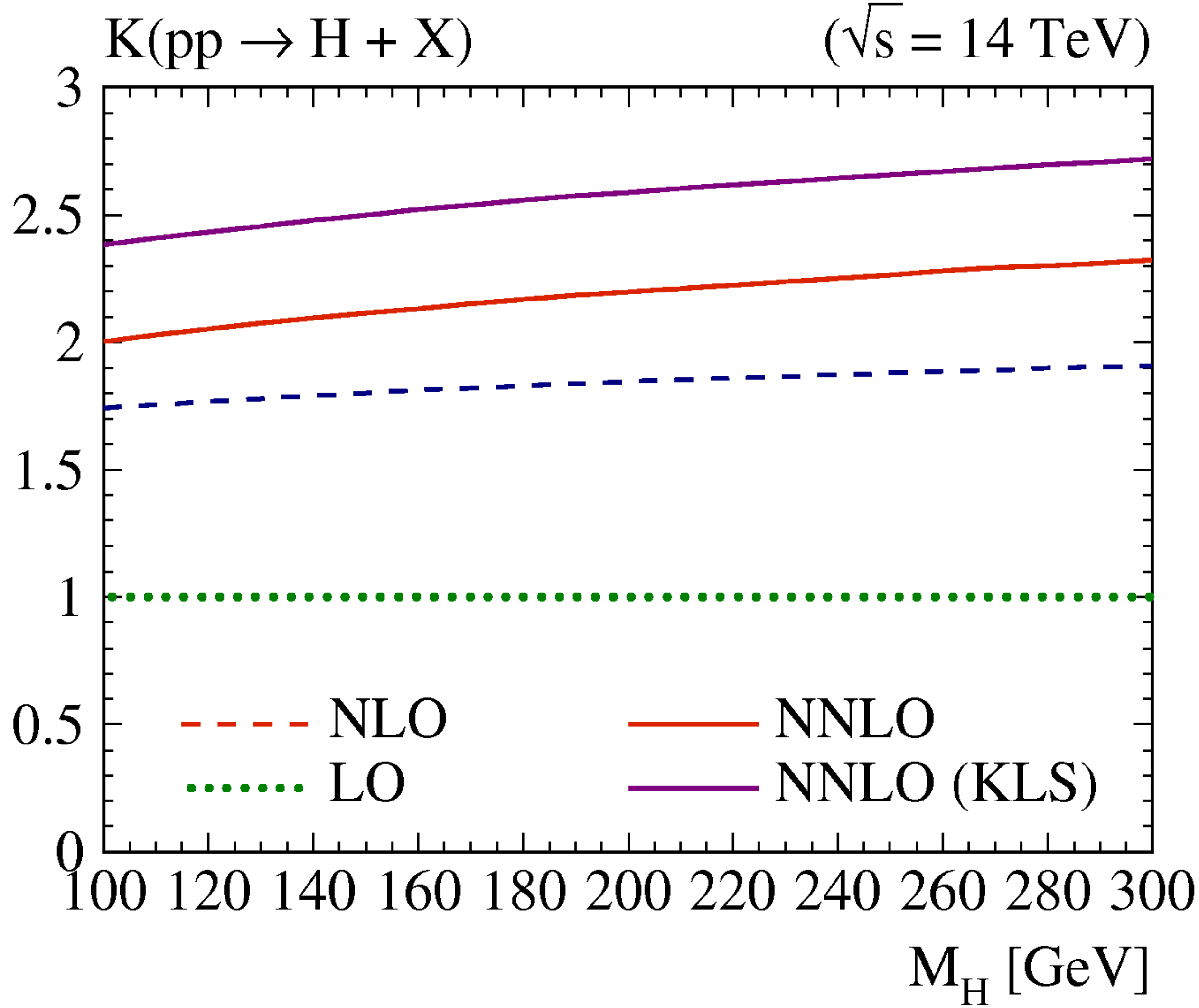
Results: Higgs at the Tevatron



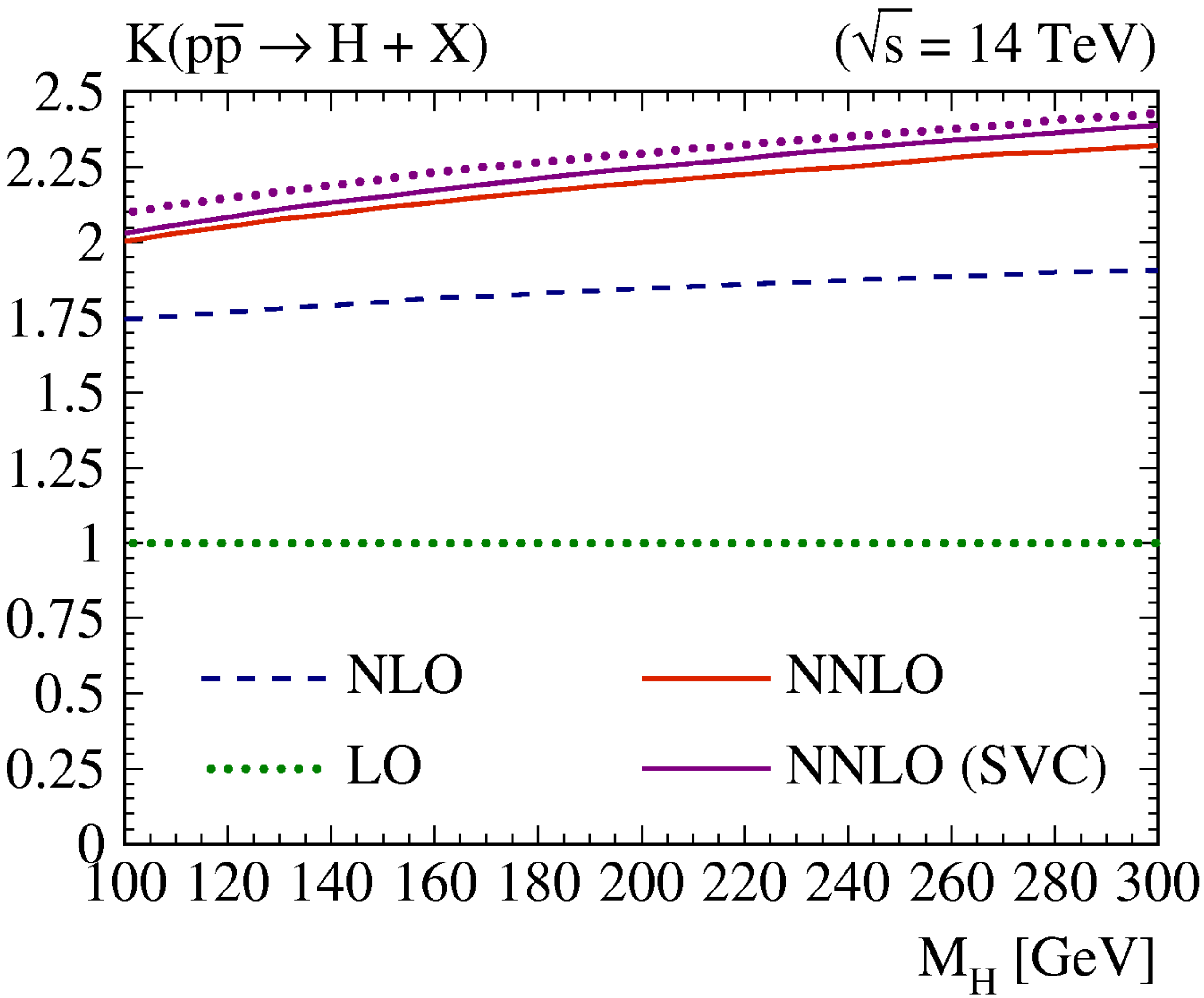
How good is the expansion?
Excellent



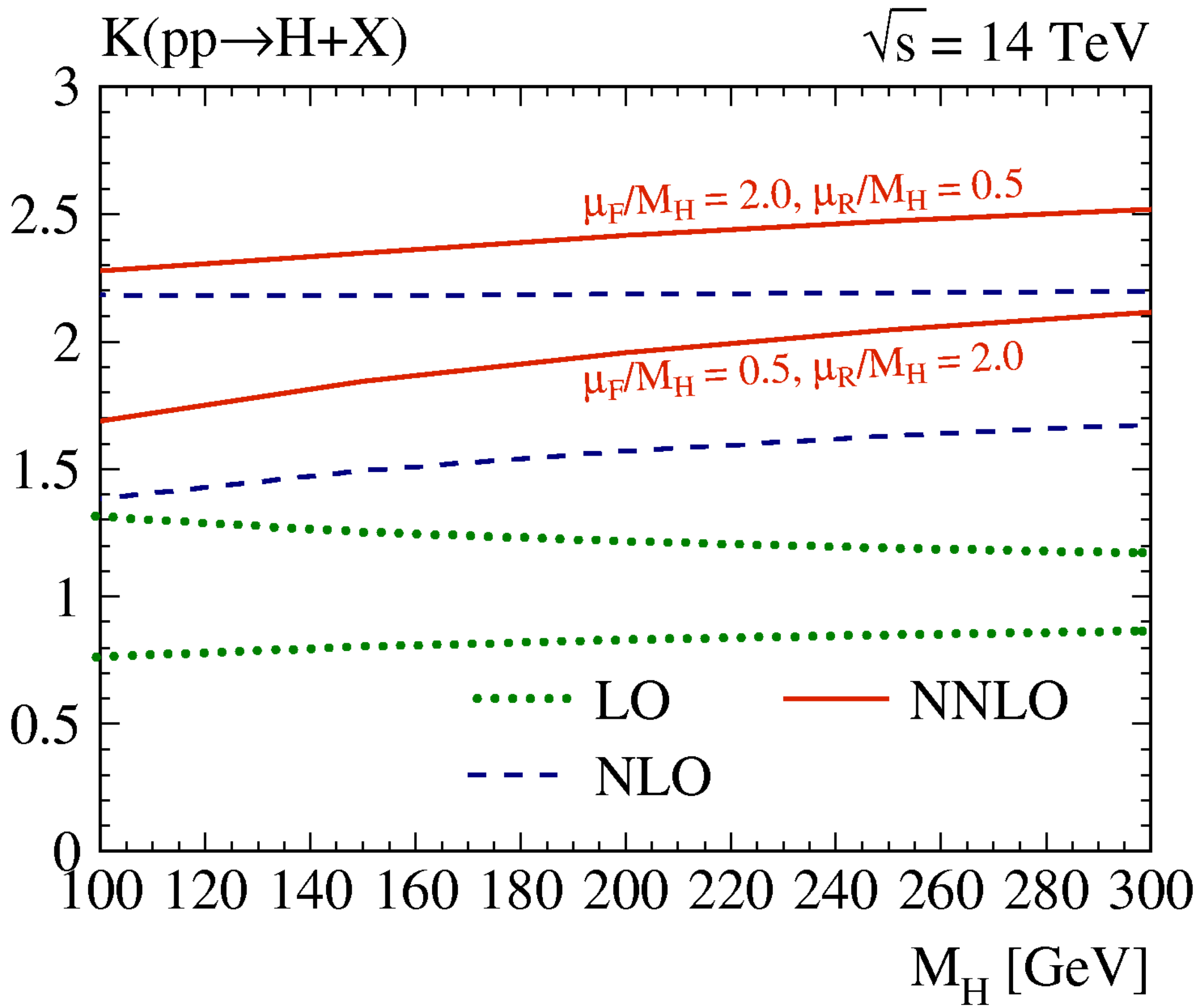
Compare to the predictions of collinear resummation:



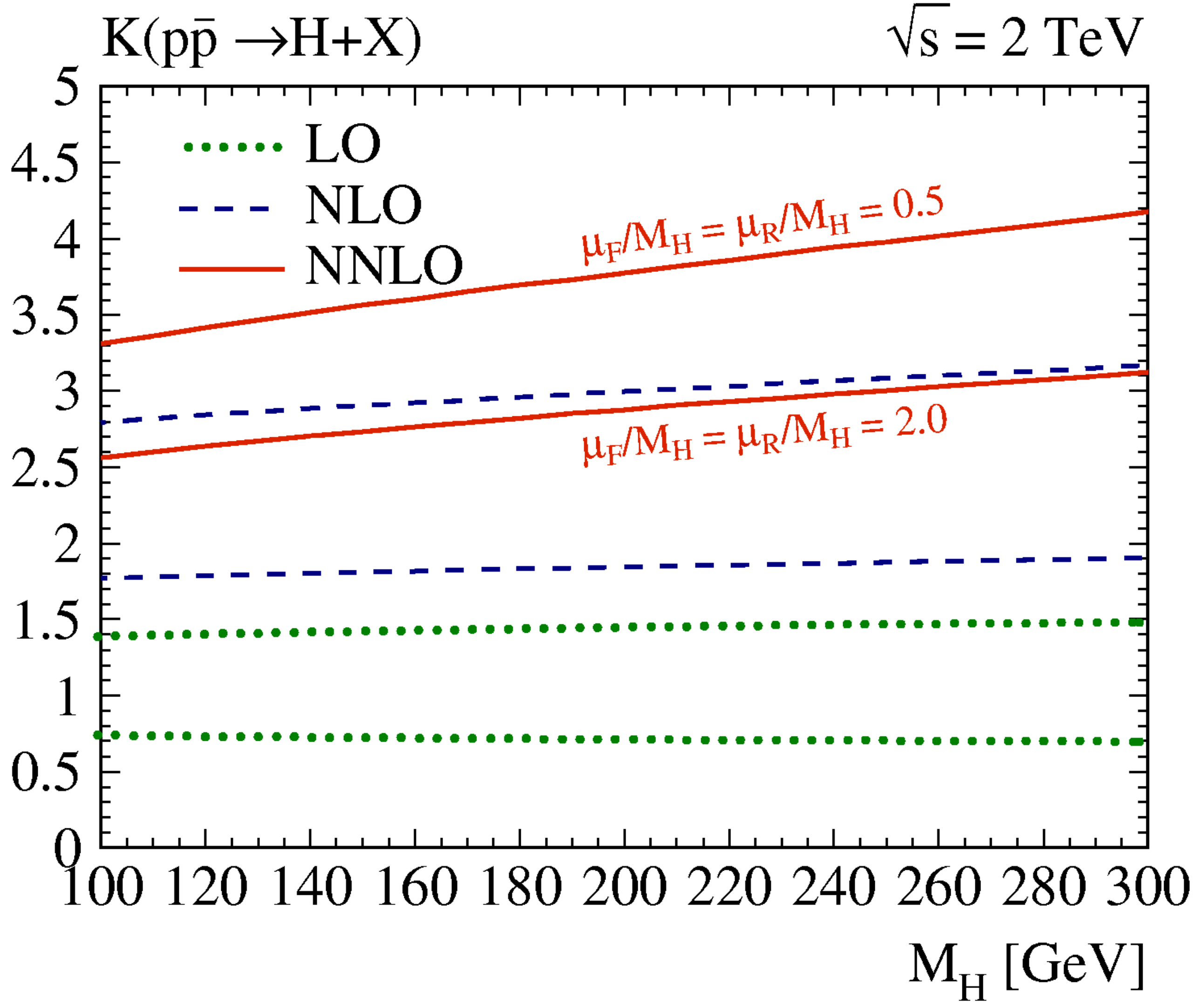
Compare to the predictions of soft + collinear resummation:



Scale Dependence at the LHC



Scale Dependence at the Tevatron



Summary: Standard Model Higgs

1. We have computed the complete NNLO correction to Inclusive Higgs Boson Production at Hadron Collider in the large M_t limit.
2. The corrections are substantial, but perturbatively well-behaved.
3. Scale Dependence is improved.

By being consistent with lower-order calculations, this is the first manifestly reliable calculation of hadronic Higgs boson production!

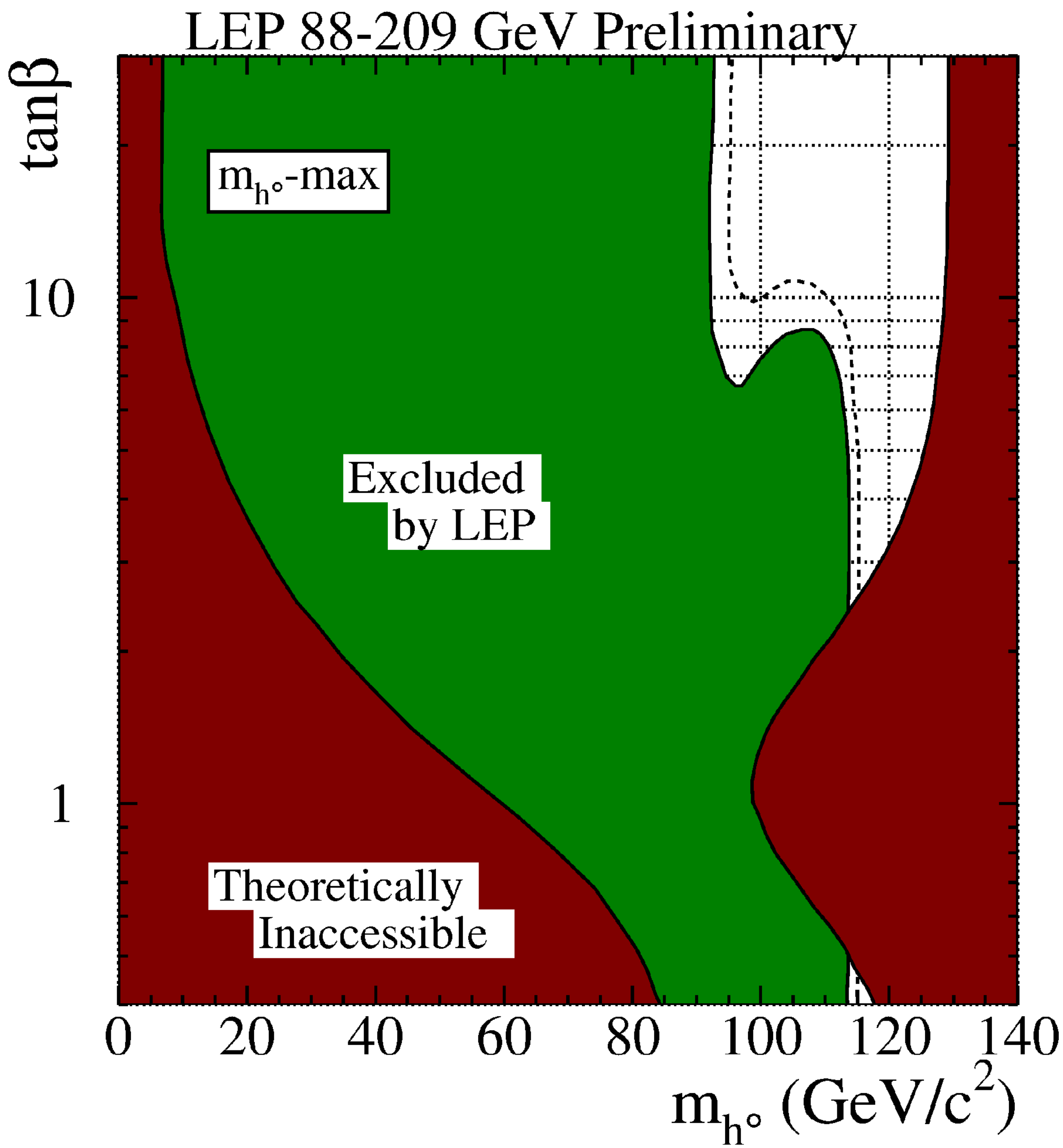
Supersymmetric Higgs Boson Production

In the Minimal Supersymmetric Standard Model (MSSM) there are two Higgs doublets, with vacuum expectation values v_u, v_d. After symmetry breaking, there are 5 physical Higgs Scalars:

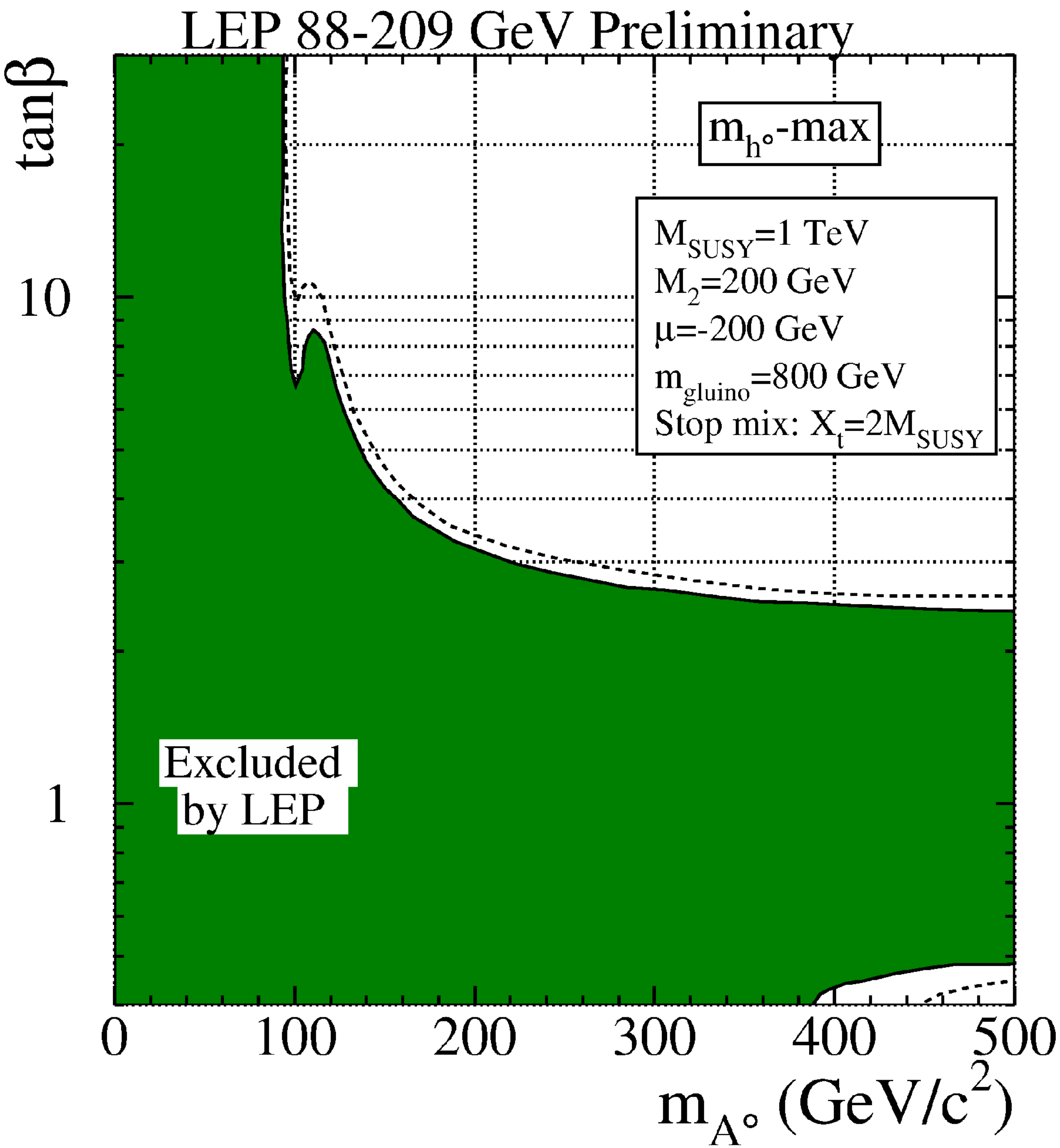
$$h^0,\ H^0,\ A,\ H^\pm$$

In the "decoupling" limit, the light neutral scalar, h^0, has properties almost identical to the Standard Model Higgs. The heavy scalar, H^0, and the pseudoscalar, A, have very different interactions.

Limits on M_h from LEP



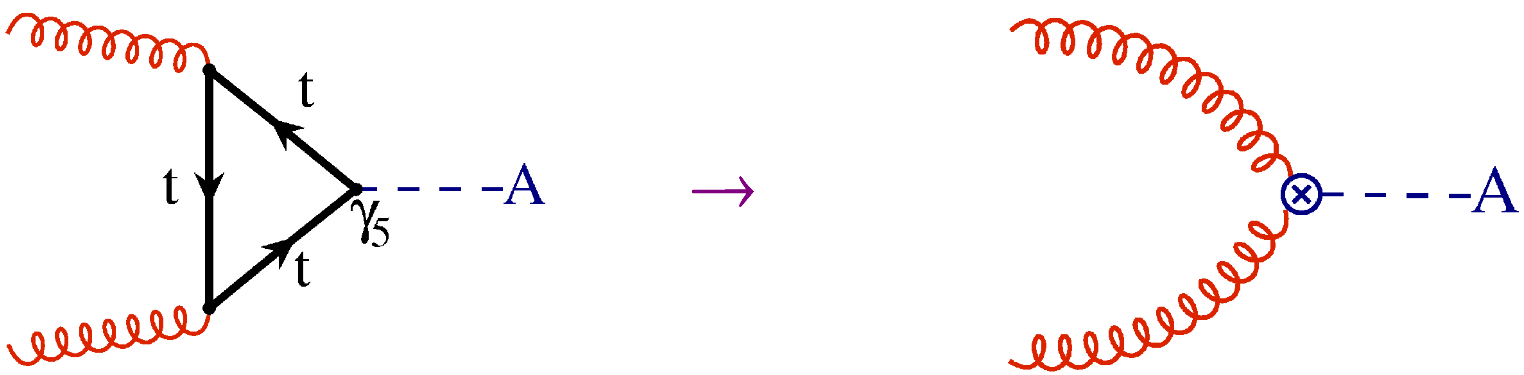
Limits on M_A from LEP



Pseudoscalar Production

Gluon fusion is also very important to pseudoscalar production and can also be described by an effective Lagrangian in which the top quark is integrated out. This effective Lagrangian coupling the pseudoscalar to gluons is:

$$\mathcal{L} = C_1 A\, \varepsilon_{\alpha\beta\mu\nu} G^{a\,\alpha\beta} G^{a\,\mu\nu} + \ldots$$



Pseudoscalar Production

Although the vertex functions are very different, the result for the scalar and pseudoscalar cross sections are very similar. The difference between the results is quite compact:

$$\Delta^{(2)}_{ggA}(x) = \Delta^{(2)}_{ggH}(x) + \left[\frac{1939}{144} - \frac{19}{8}\, l_{Ht} + 3\,\zeta_2\right] \delta(1-x) + 6\,\mathscr{D}_1(x) - (12x - 6x^2 + 6x^3)\, \ln(1-x) - 9\,x \ln^2 x$$
$$+ \frac{3}{2}\, \frac{(10 - x - 13x^2 + 4x^3 - 2x^4)}{1-x} \ln x + \frac{(154 - 189x + 24x^2 + 11x^3)}{4}$$
$$+ n_f \left[\left(-\frac{13}{16} - \frac{2}{3}\, l_{Ht} + 2\,\delta_2 \right) \delta(1-x) + \frac{2}{3}\, x \ln^2 x + x \ln x - \frac{(1 - 11x + 10x^2)}{6} \right], \quad \text{with} \quad \delta_2 = -\frac{1}{4} + \frac{1}{2}\, l_{Ht}\,,$$

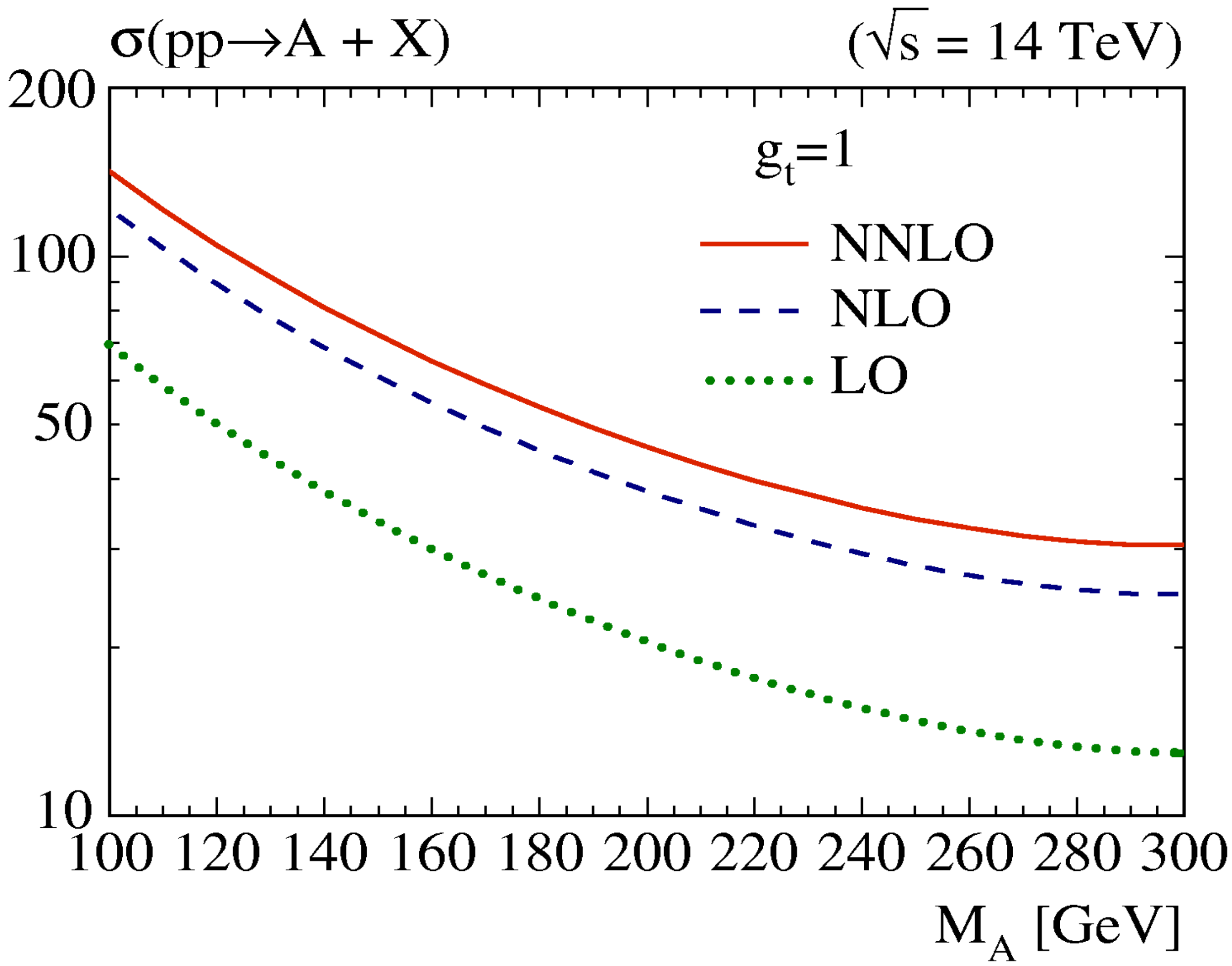
$$\Delta^{(2)}_{gqA}(x) = \Delta^{(2)}_{gqH}(x) + \frac{(4 - 4x + 2x^2)}{3} \ln(1-x) - \frac{28}{9}\, x \ln^2 x + \frac{(22 + 30x - x^2)}{3} \ln x + \frac{(337 - 382x + 51x^2)}{18}\,,$$

$$\Delta^{(2)}_{qqA}(x) = \Delta^{(2)}_{qqH}(x) - \frac{64}{27}\, x \ln^2 x + \frac{16}{27}\,(6 + 11x) \ln x + \frac{8}{27}\,(37 - 40x + 3x^2)\,,$$

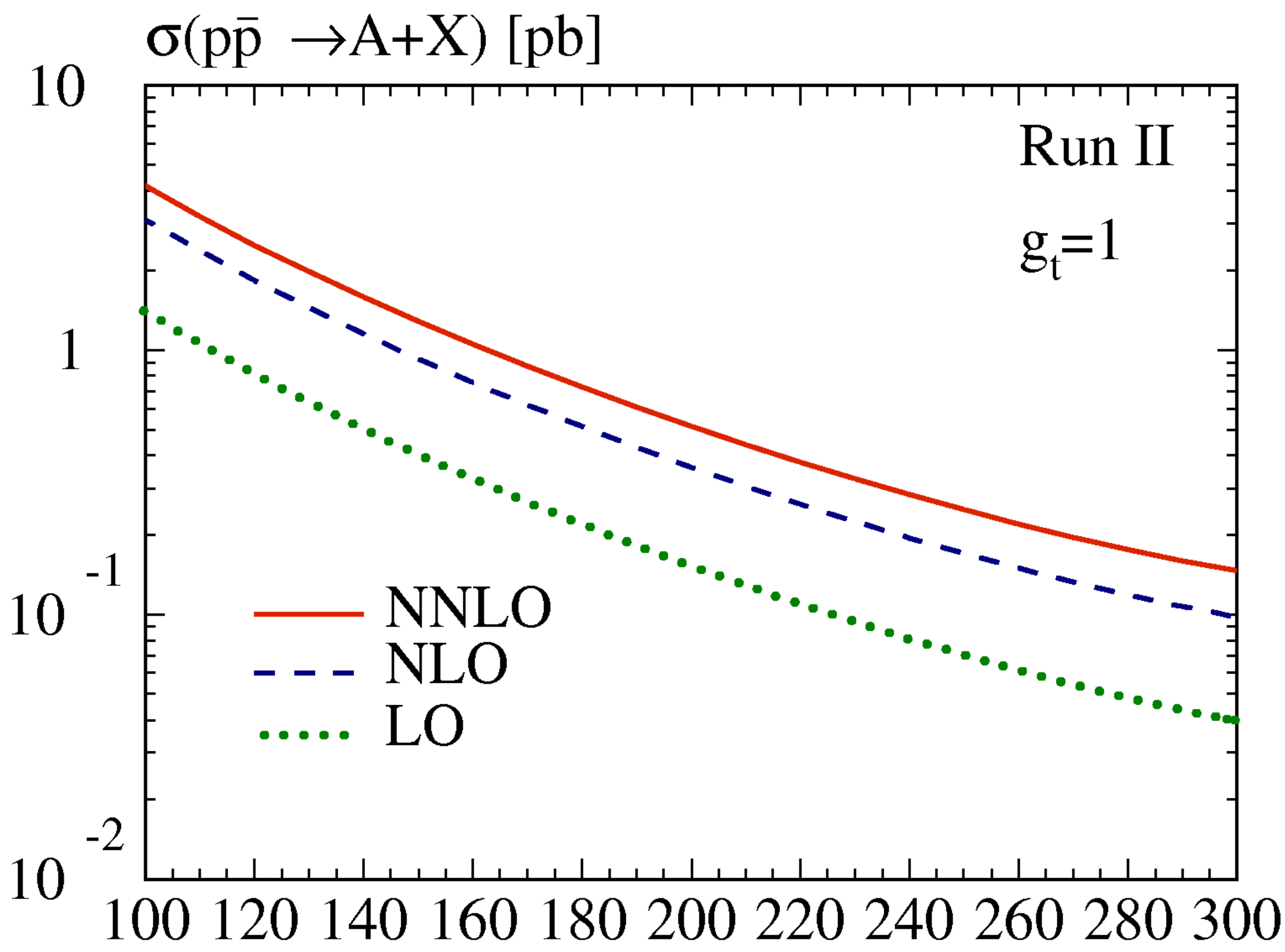
$$\Delta^{(2)}_{q\bar{q}A}(x) = \Delta^{(2)}_{q\bar{q}H}(x) + \frac{32}{27}\, x \ln^2 x + \frac{32}{27}\,(3 + 8x) \ln x + \frac{16}{27}\,(11 - x - 9x^2 - x^3) + n_f \left[-\frac{32}{27}\, x \ln x - \frac{16}{27}\,(1 - x^2) \right],$$

$$\Delta^{(2)}_{qq'A}(x) = \Delta^{(2)}_{qq'H}(x) - \frac{16}{9}\, x \ln^2 x + \frac{16}{9}\,(2 + 3x) \ln x + \frac{8}{9}\,(11 - 12x + x^2)$$

Results: Pseudoscalar Higgs at LHC



Results: Pseudoscalar Higgs at Tevatron



H/A Couplings

For the pseudoscalar (and for H^0 in the decoupling limit) the couplings to "up-type" fermions are suppressed by tan $\beta \equiv v_u/v_d$ while those to "down-type" fermions are enhanced by tan β. This presents a problem for gluon fusion calculations:

For tan β significantly larger than 1, b-quark interactions are important. But ... one cannot formulate an effective Lagrangian by integrating out the b-quark to produce ~100 GeV Higgs bosons! An NNLO calculation would require massive 3-loop diagrams.

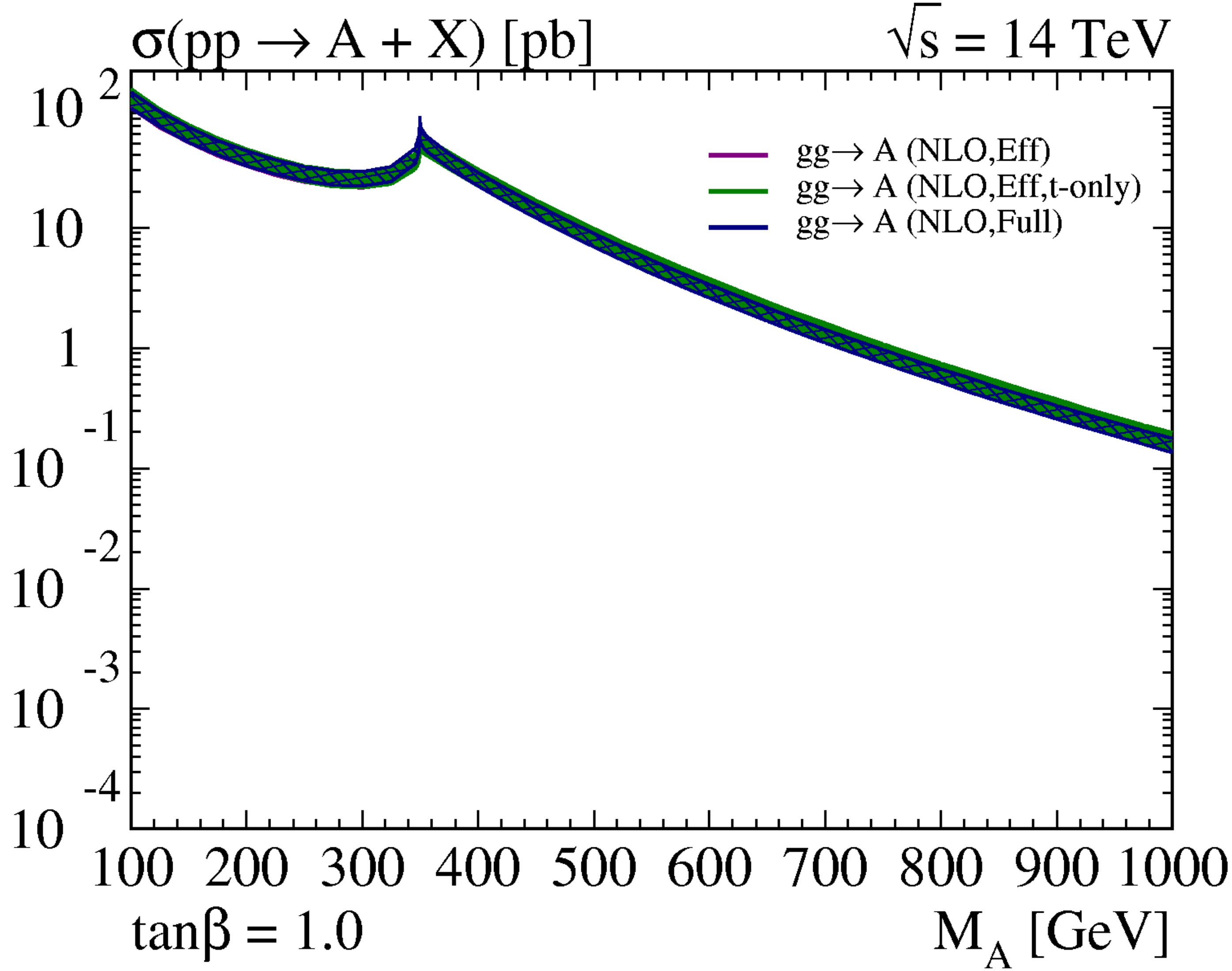
b-quark Loops

Still ..., the effective Lagrangian for t-quark loops works far beyond the top threshold. Why?

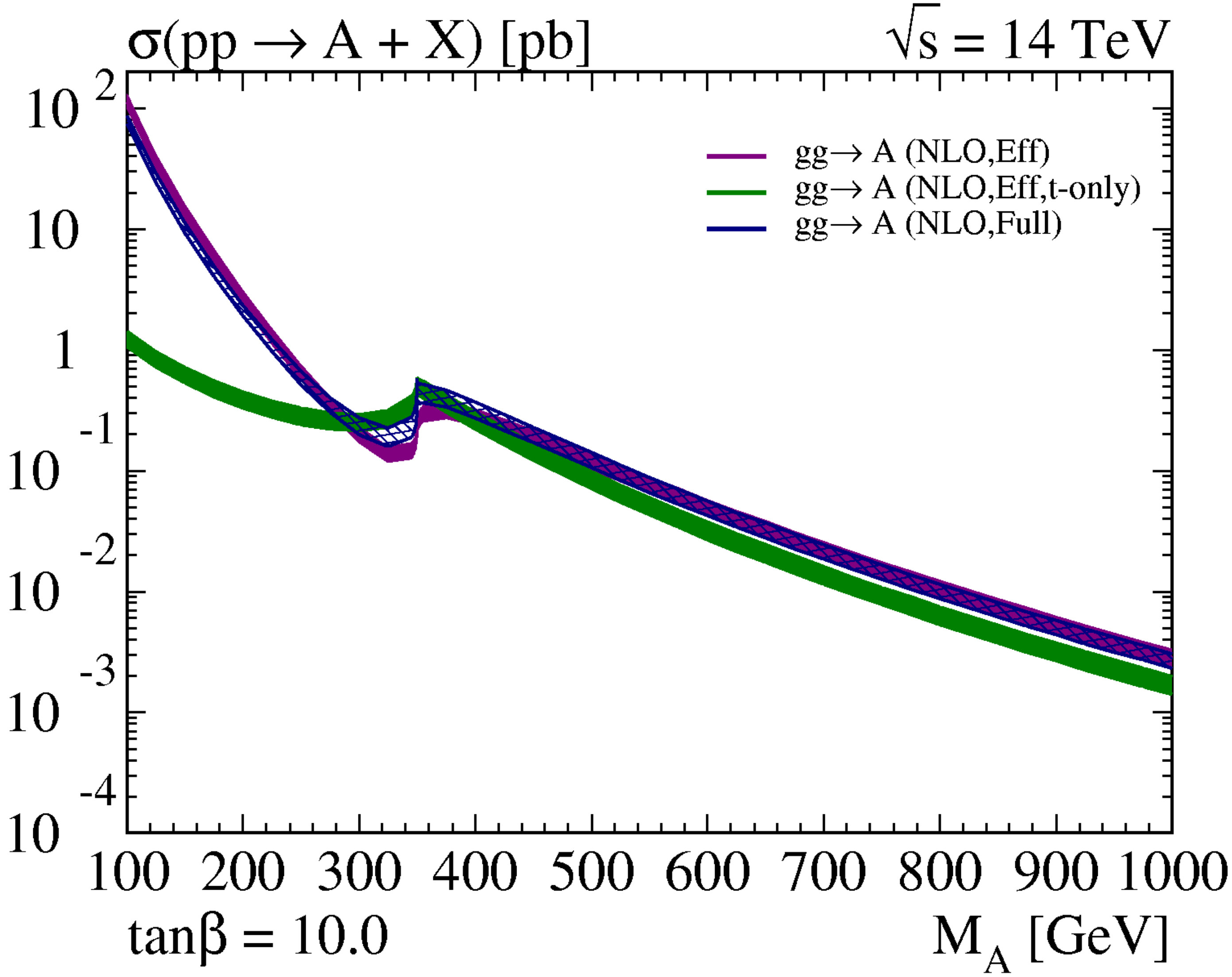
Radiative corrections are dominated by soft/collinear emission, which does not resolve the t-quark loops.

While use of the effective Lagrangian cannot be justified a priori, it can be shown, a posteriori, to be a reasonable approximation.

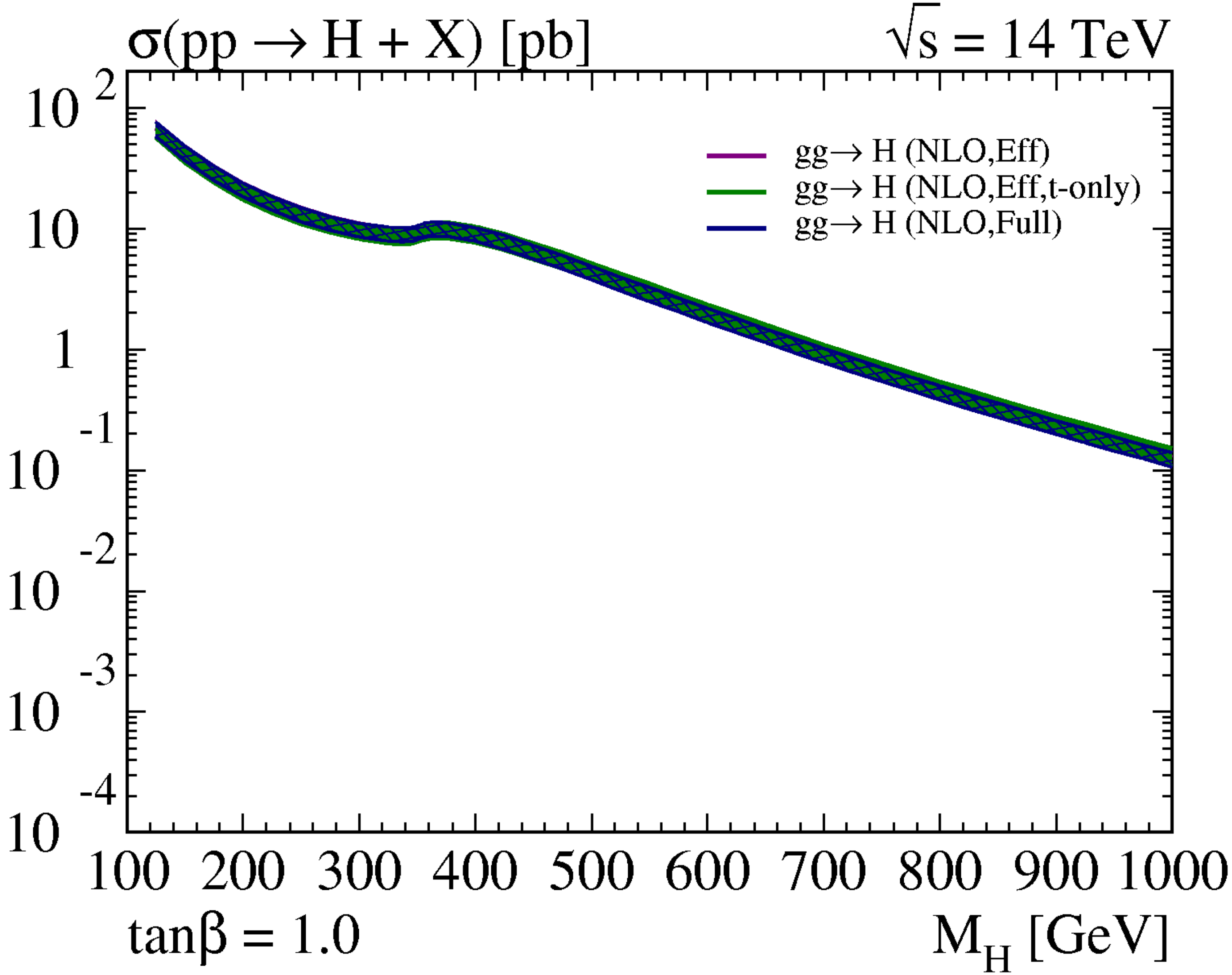
At small tan β, b-quark loops are negligible



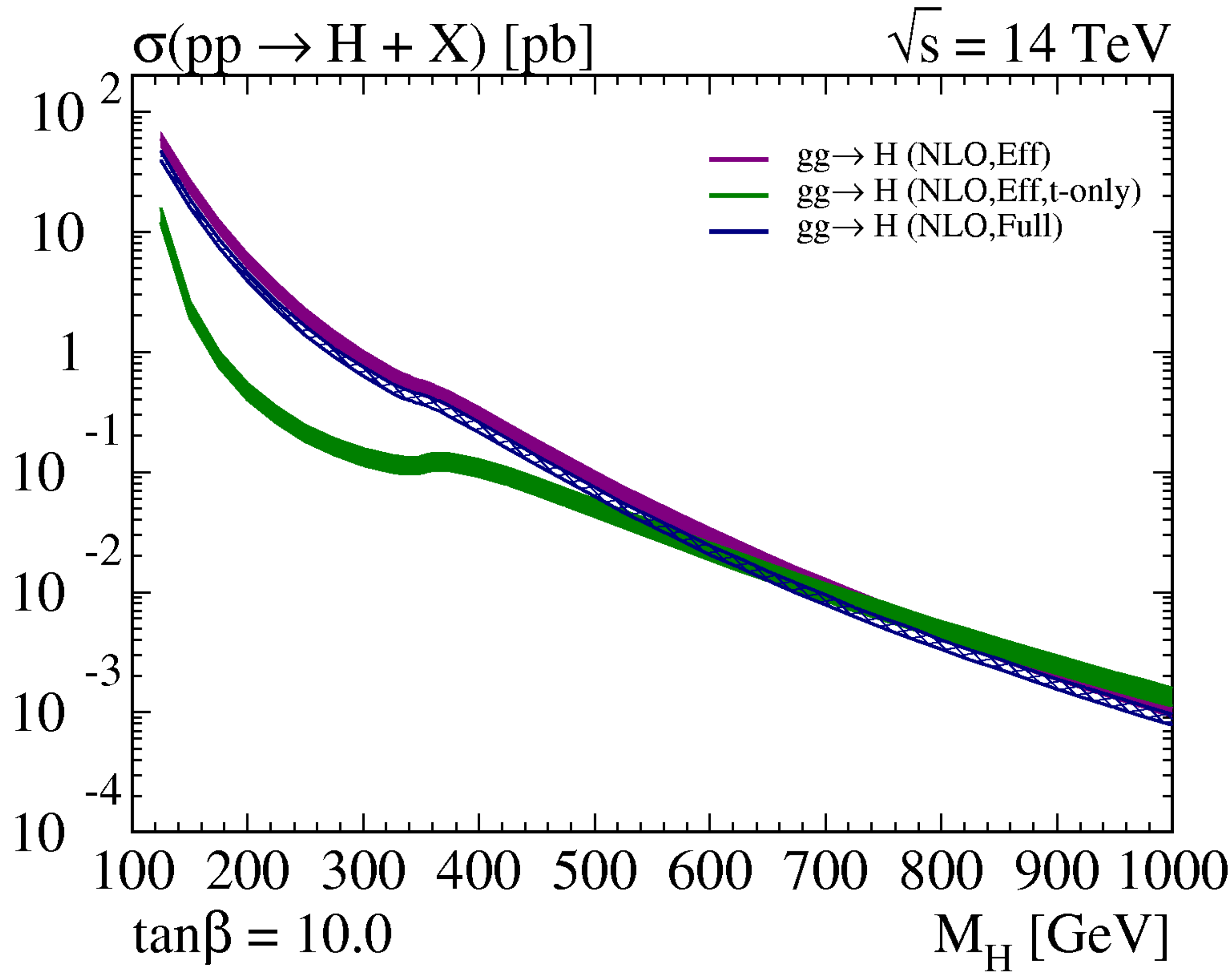
But must be included for large tan β.



At small tan β, b-quark loops are negligible

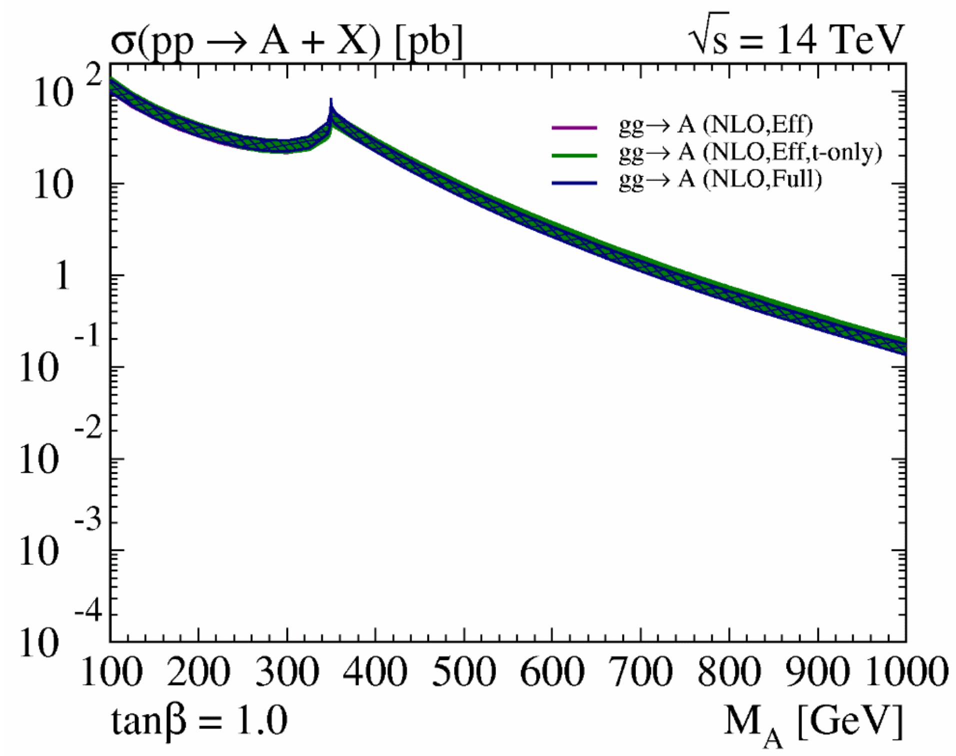


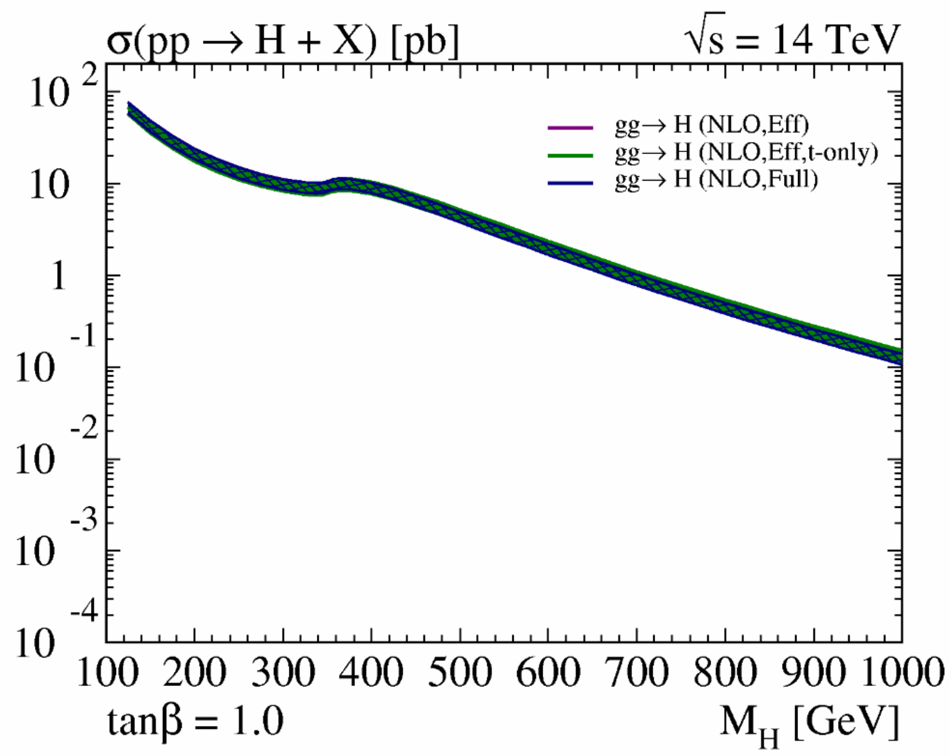
But must be included for large tan β.



b-quark Loops are Important!

and the effective Lagrangian works pretty well





A new production mode at large tan β

The importance of b-quark couplings at large tan β suggests a new inclusive production mechanism:

$$b\bar{b} \rightarrow H$$

Since b-quark distributions are generated by gluon splitting the true parent process is

$$gg \rightarrow b\bar{b}H$$

The b-quark distribution resums large logs associated with the gluon splitting, but is only truly consistent if calculated to high enough order to include the parent process.

In this case, one must compute to NNLO.

$b\bar{b} \to H/A$ at NNLO

We can ignore the b-quark mass in our calculation, except where it enters into the Yukawa couplings. Other b-quark mass effects are suppressed by factors of m_b^2/M_H^2.

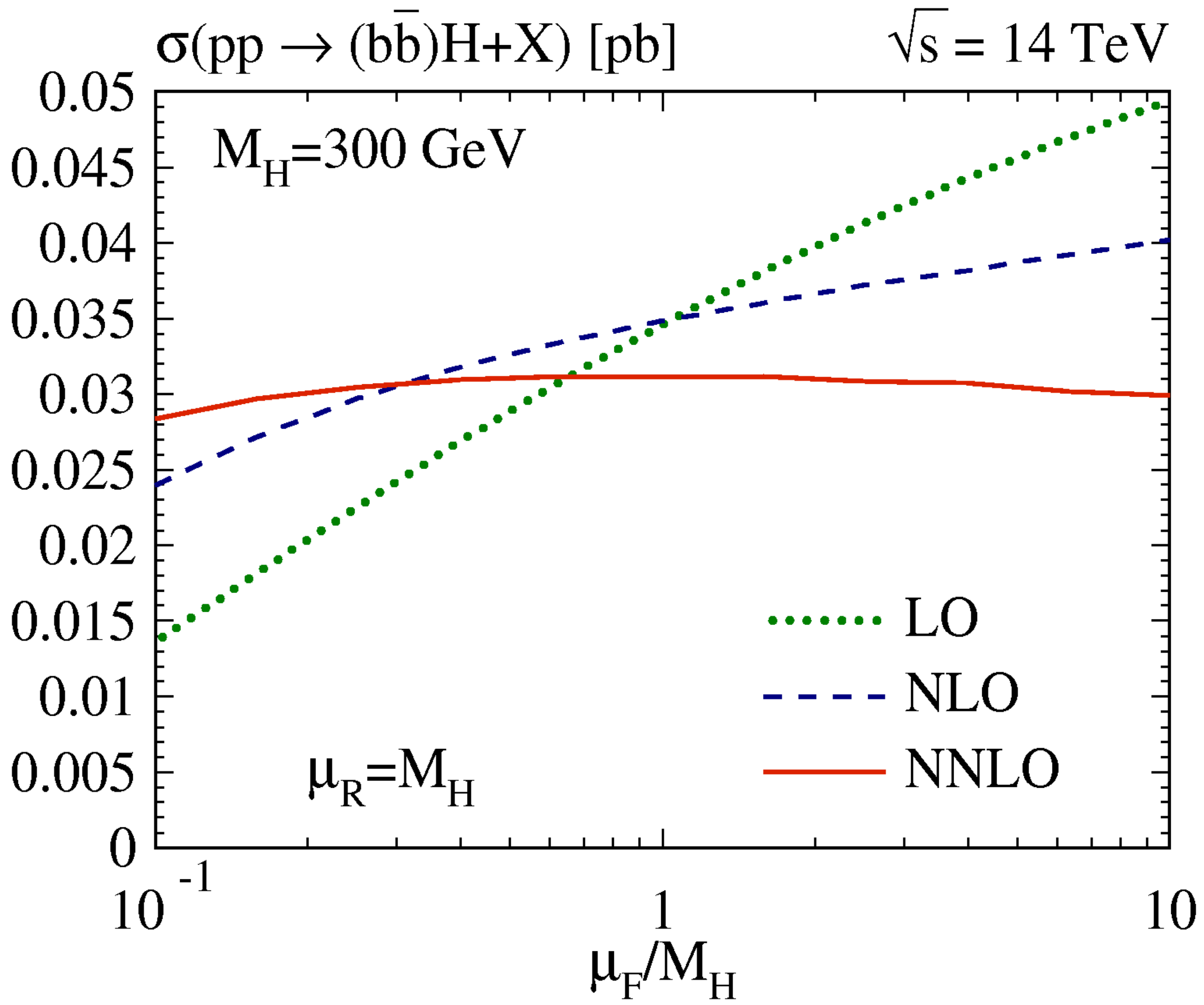
In the $m_b \to 0$ limit, the partonic cross sections for $b\bar{b} \to H$ are identically equal to those for $b\bar{b} \to A$.

$b\bar{b} \to H/A$ can dominate at large tan β because

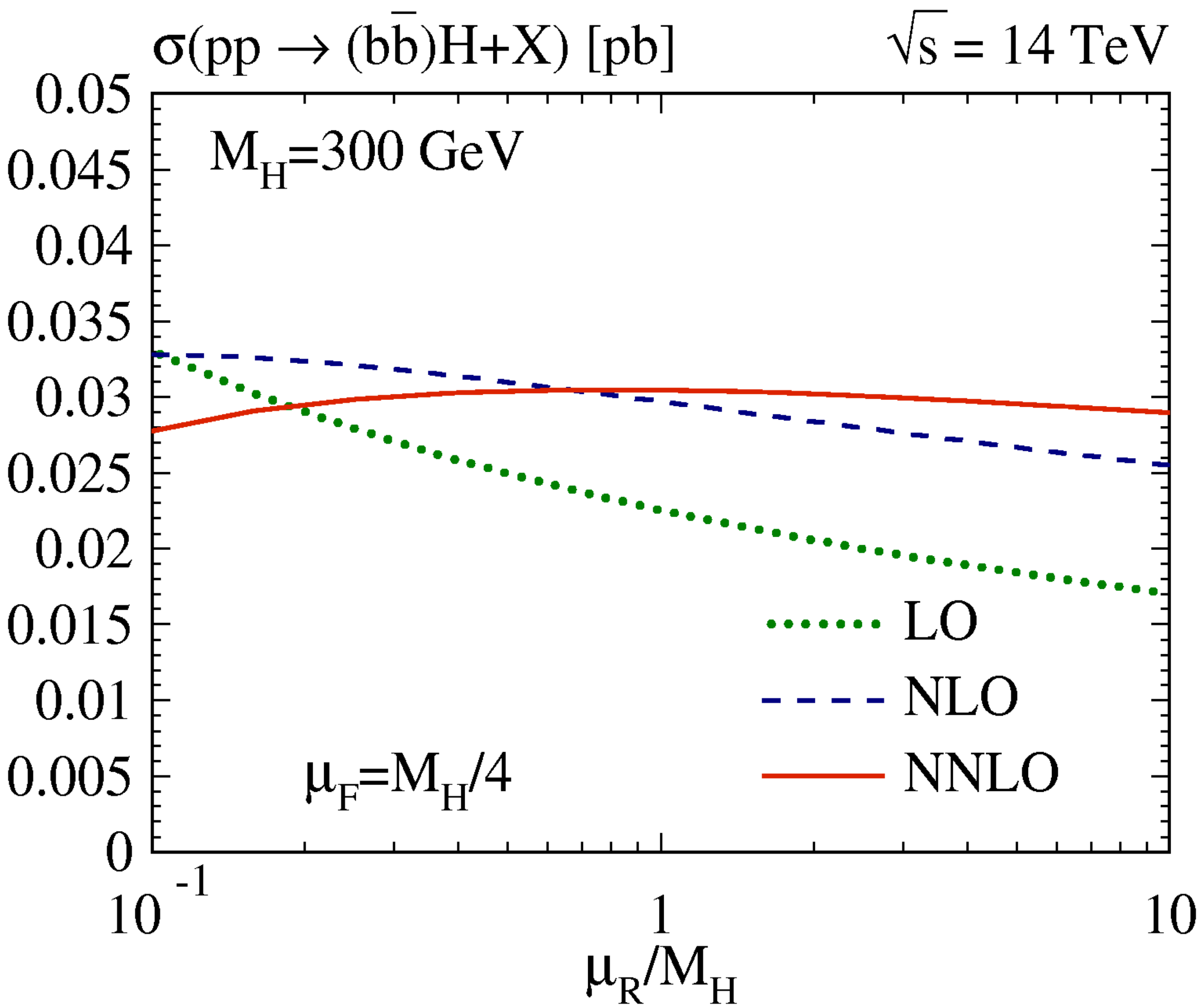
$\sigma_{bb} \sim m_b^2/M_H^2 \tan^2\beta$ while

$\sigma_{gg} \sim A \cot^2\beta + B\, m_b^2/M_H^2 + C\, m_b^4/M_H^4 \tan^2\beta$

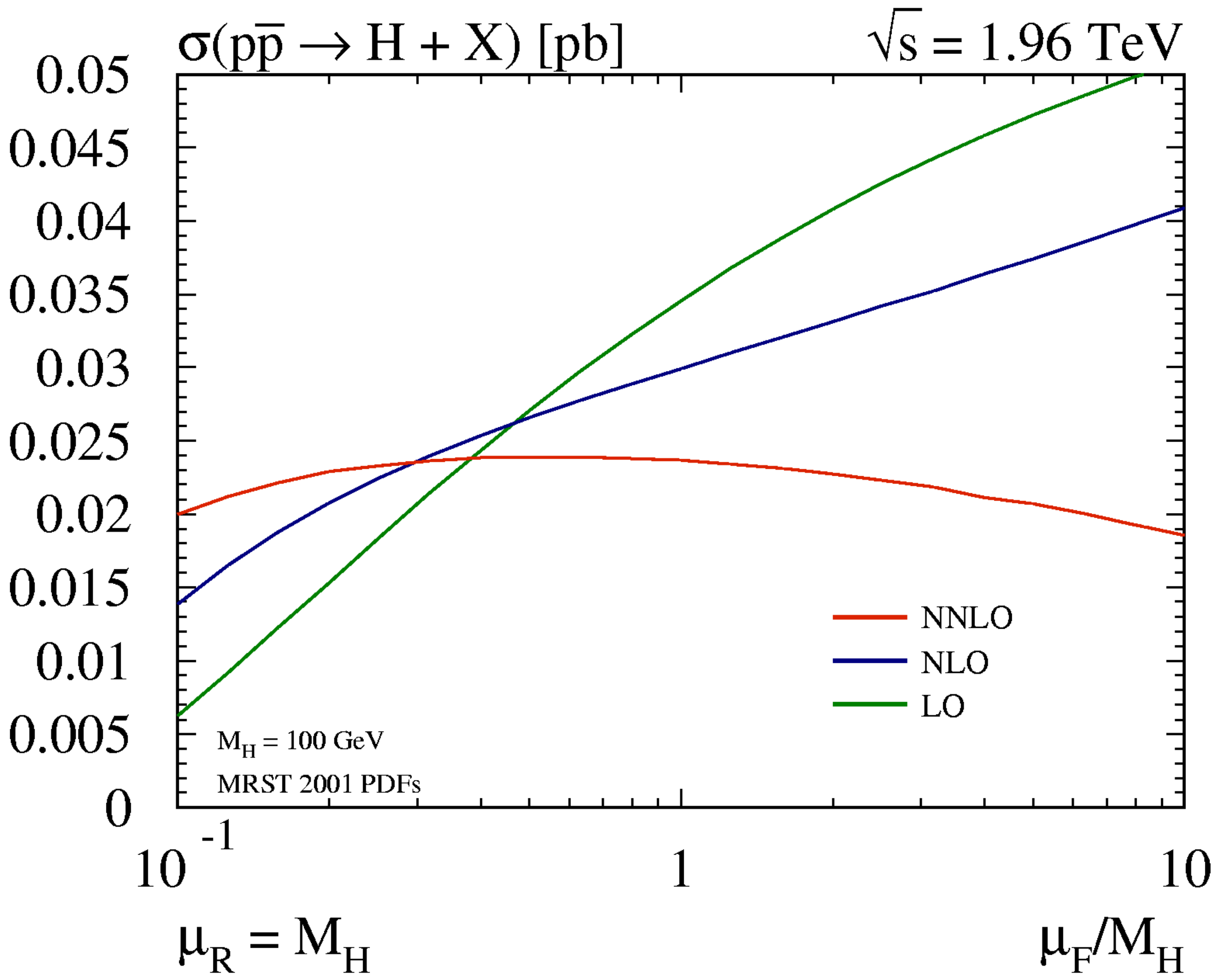
Factorization Scale Dependence of $b\bar{b} \to H/A$ at LHC



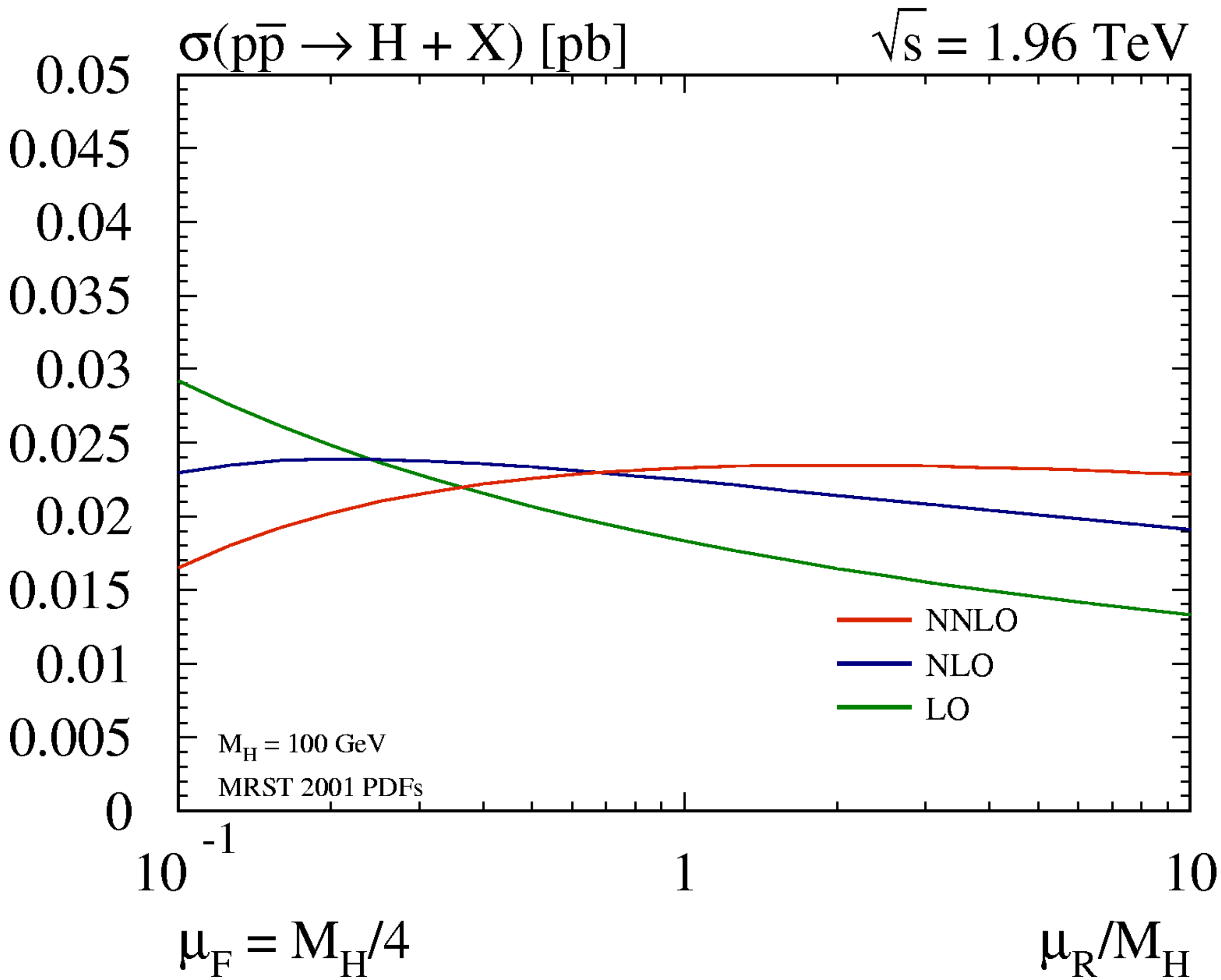
Renormalization Scale Dependence of $b\bar{b} \rightarrow H/A$ at LHC



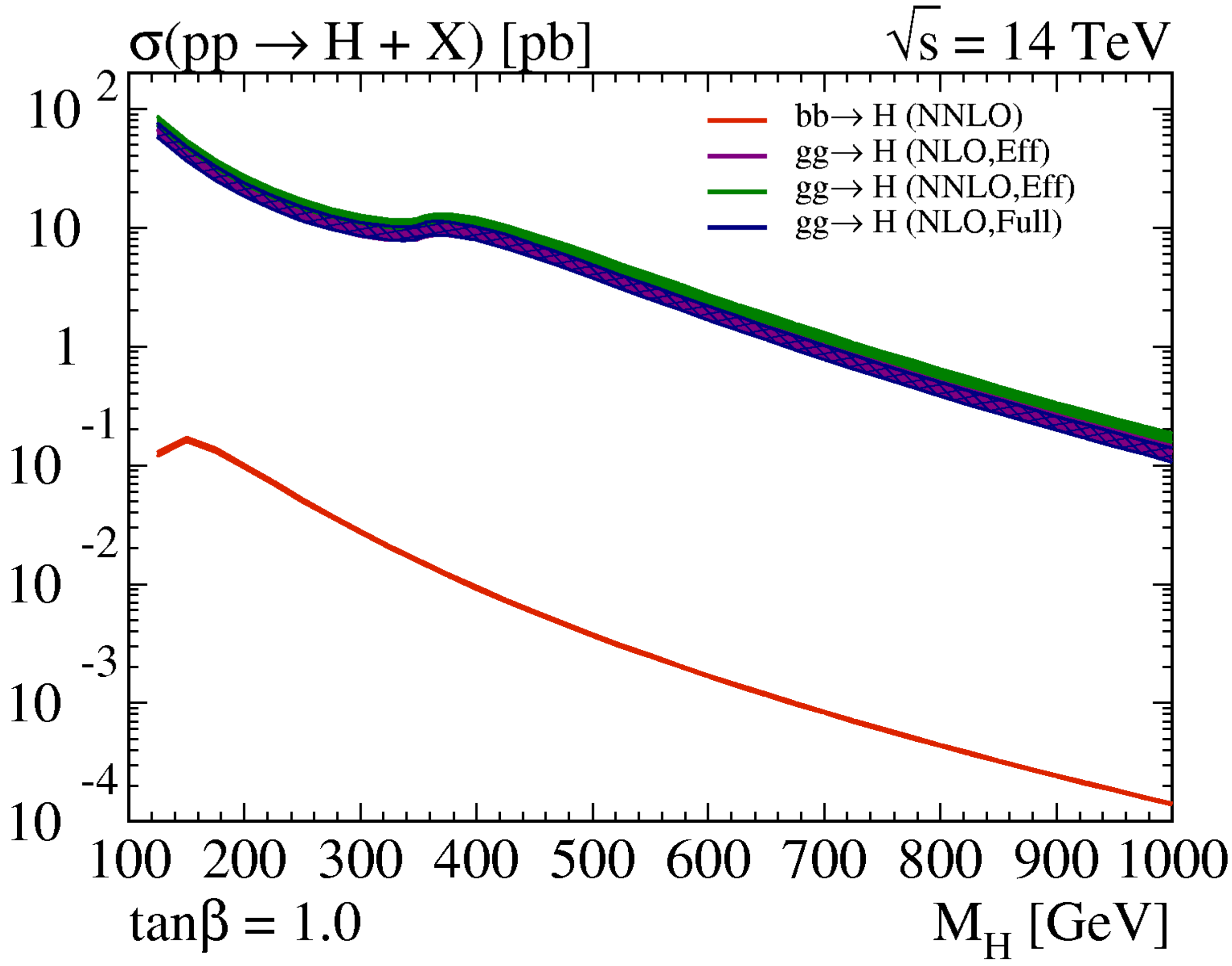
Factorization Scale Dependence of $b\bar{b} \to H/A$ at the Tevatron



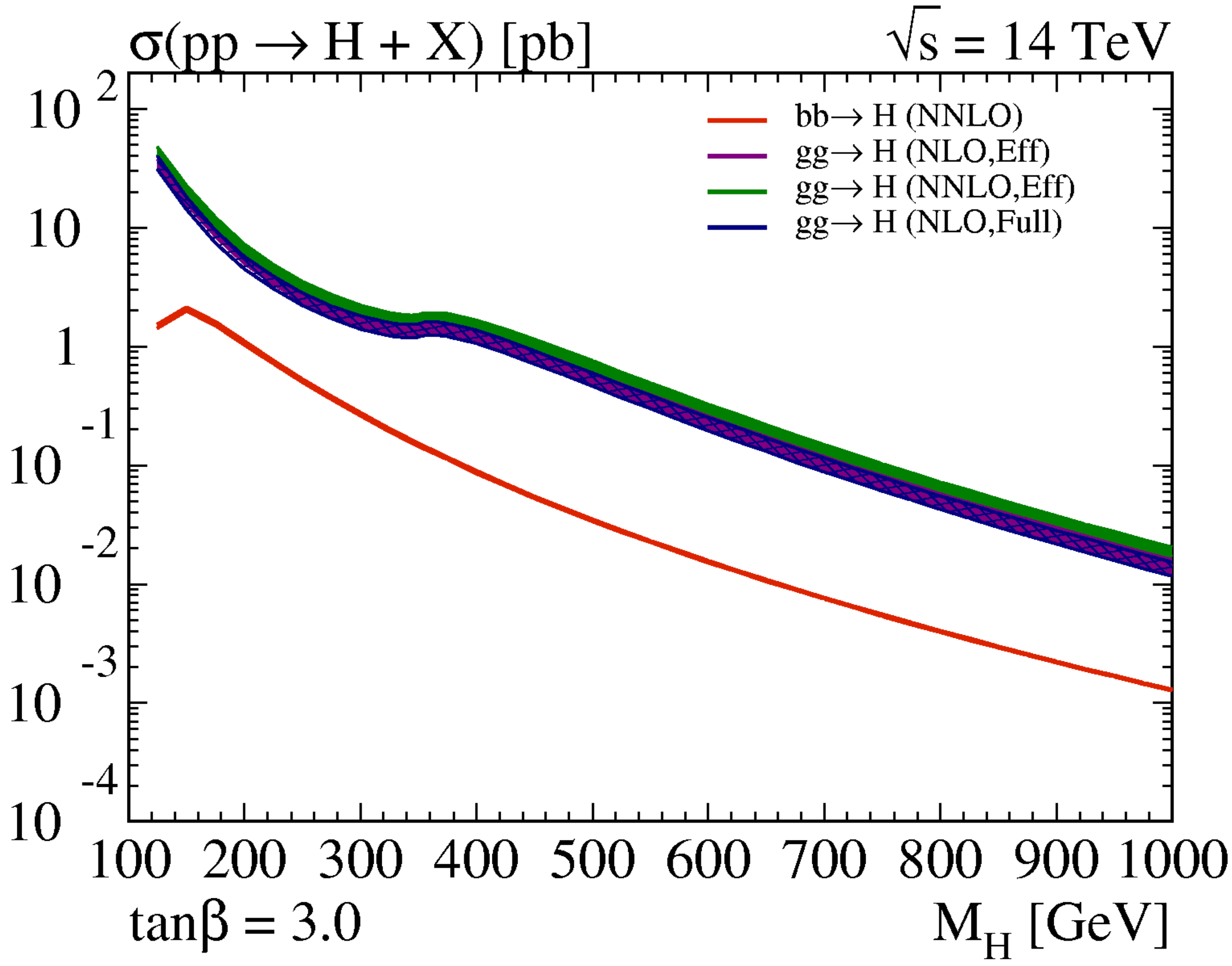
Renormalization Scale Dependence of $b\bar{b} \rightarrow H/A$ at the Tevatron



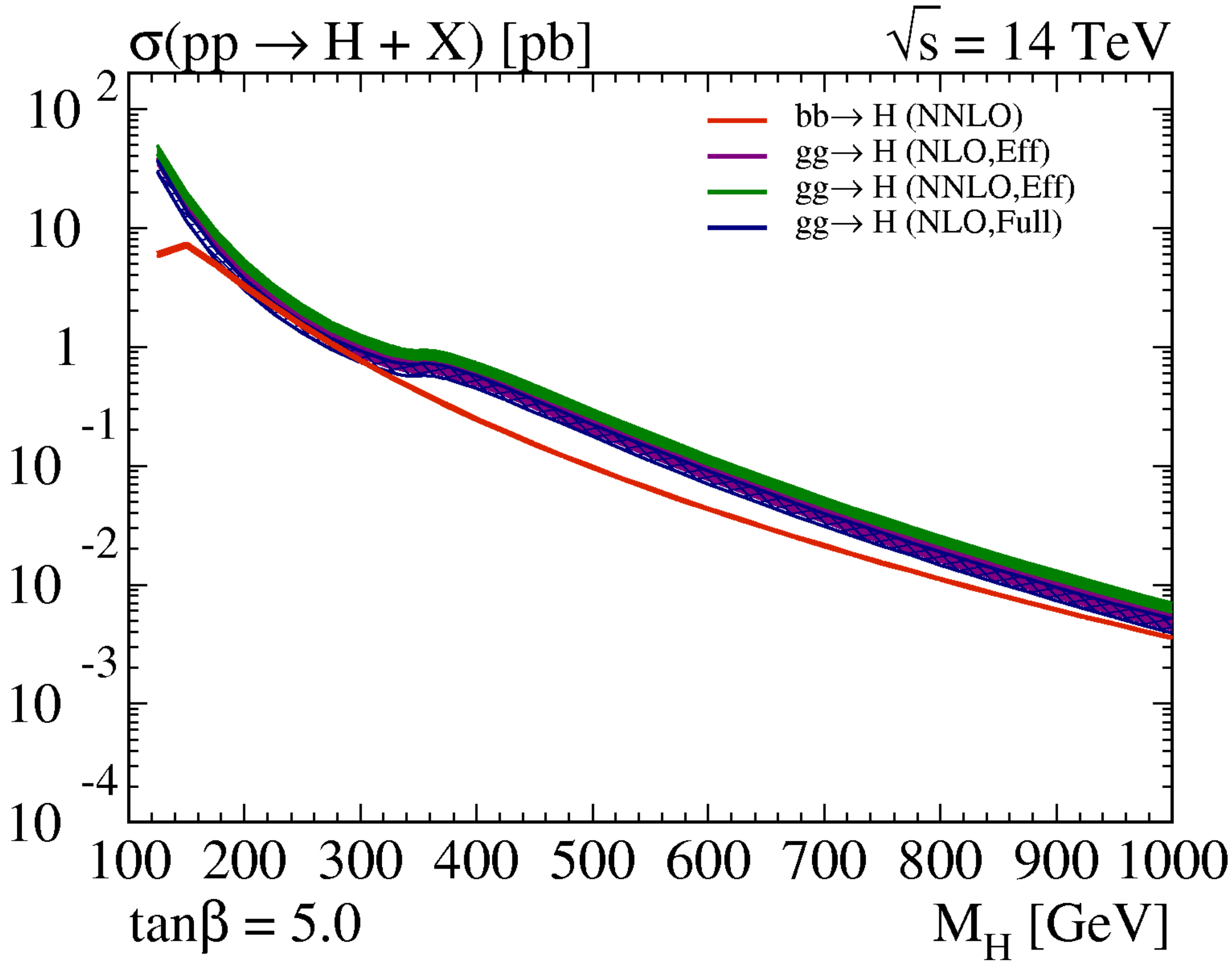
$b\bar{b} \to H$ versus $gg \to H$ ($\tan\beta = 1$)



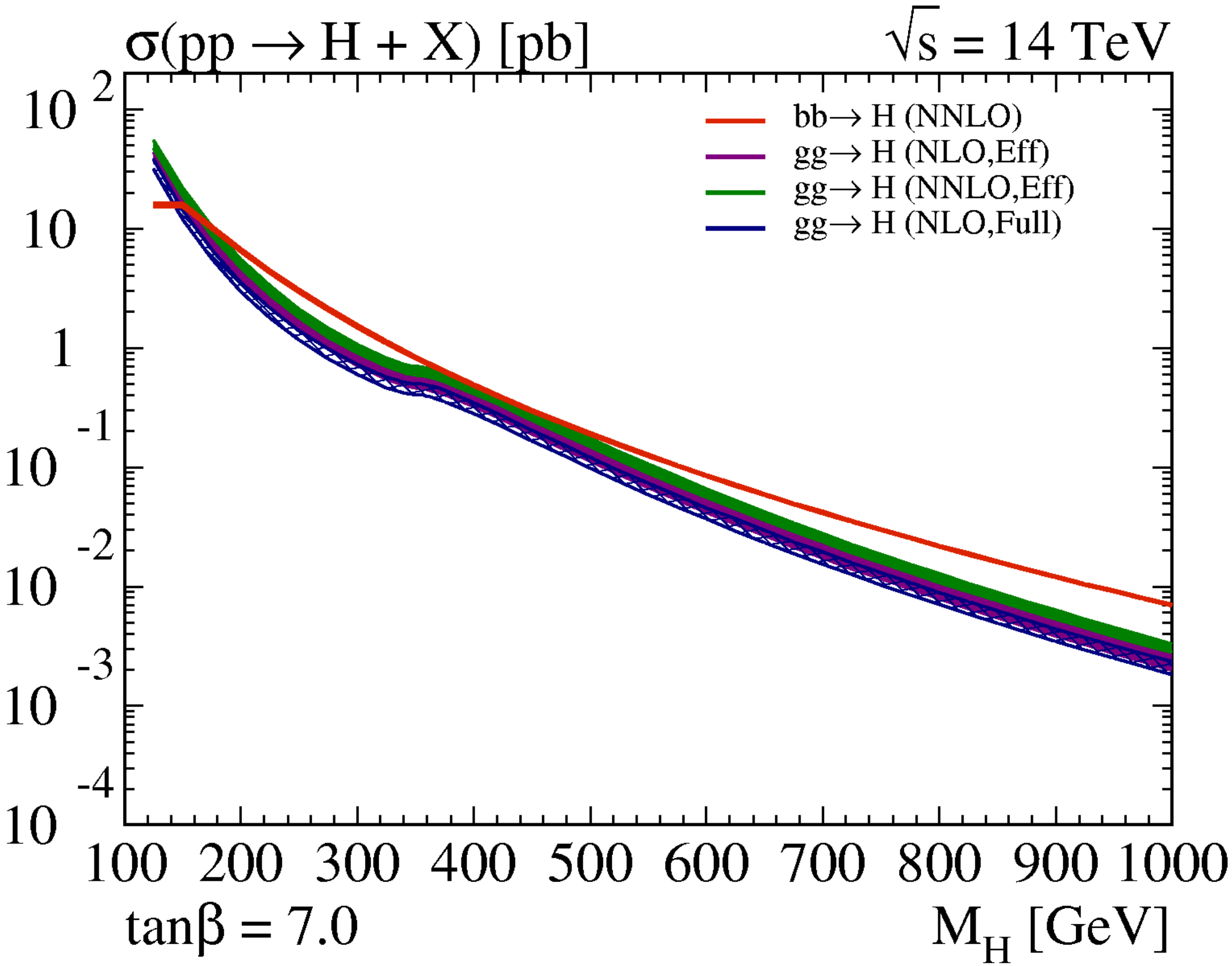
$b\bar{b} \rightarrow H$ versus $gg \rightarrow H$ ($\tan\beta = 3$)



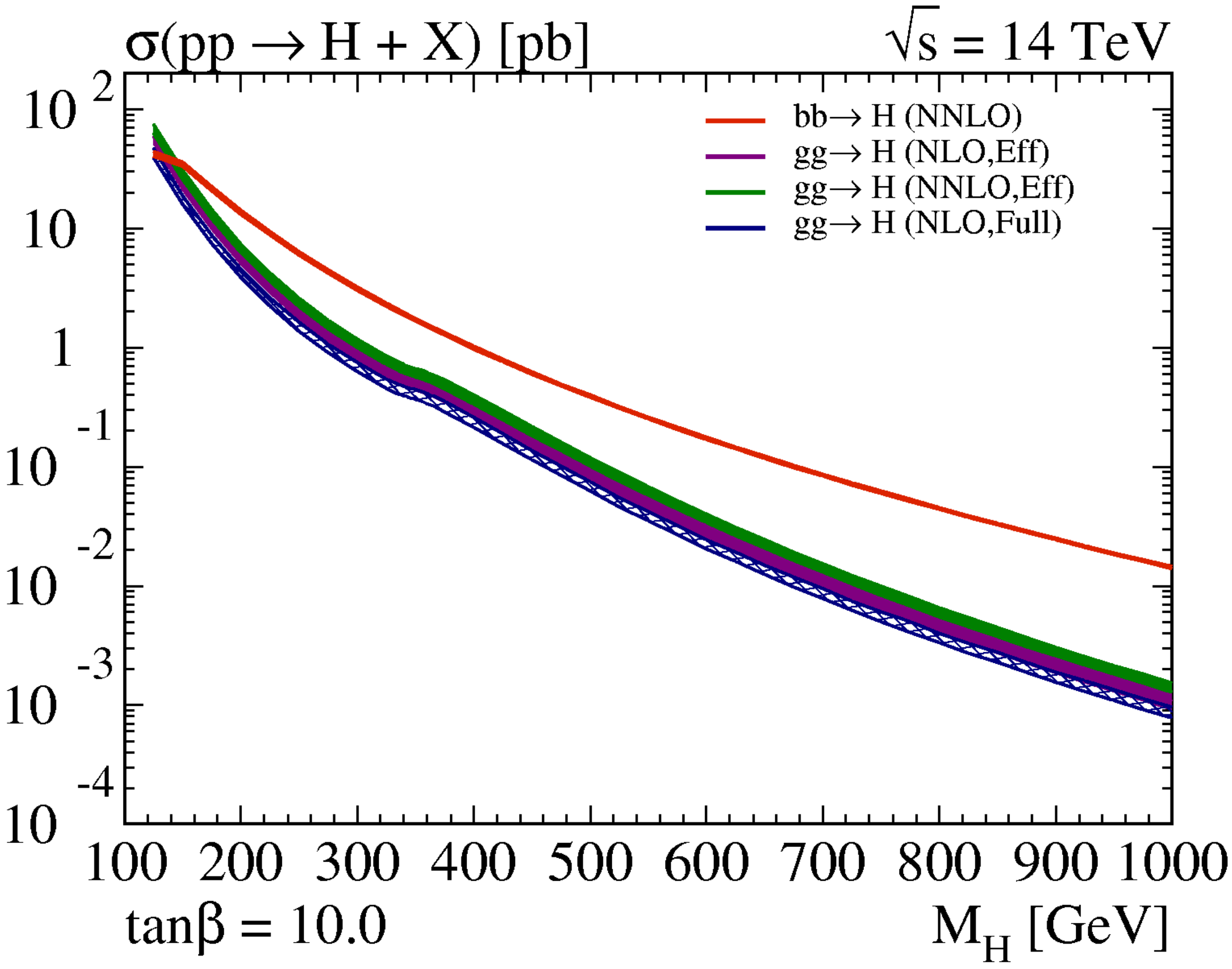
$b\bar{b} \to H$ versus $gg \to H$ ($\tan\beta = 5$)



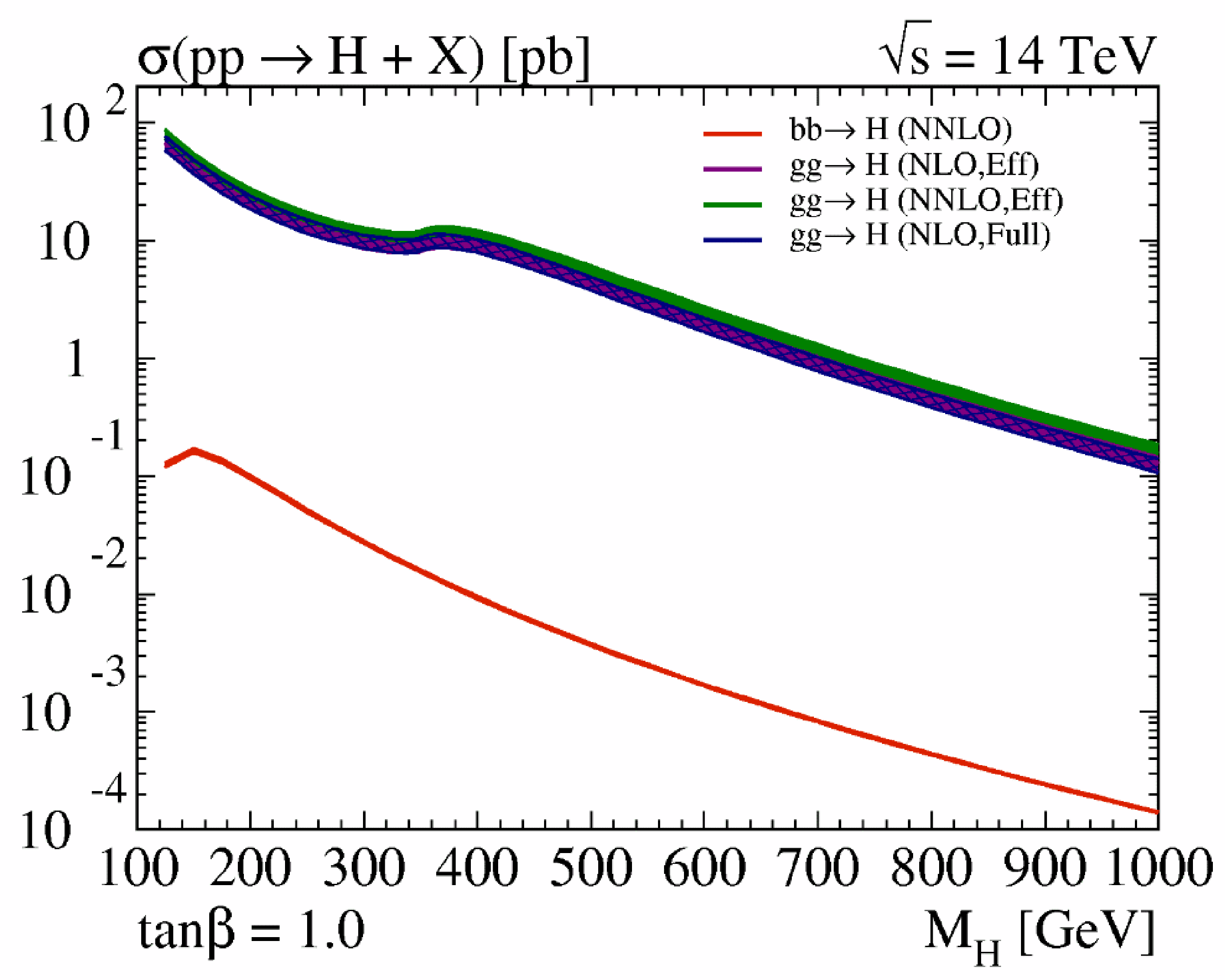
$b\bar{b} \to H$ versus $gg \to H$ ($\tan\beta = 7$)



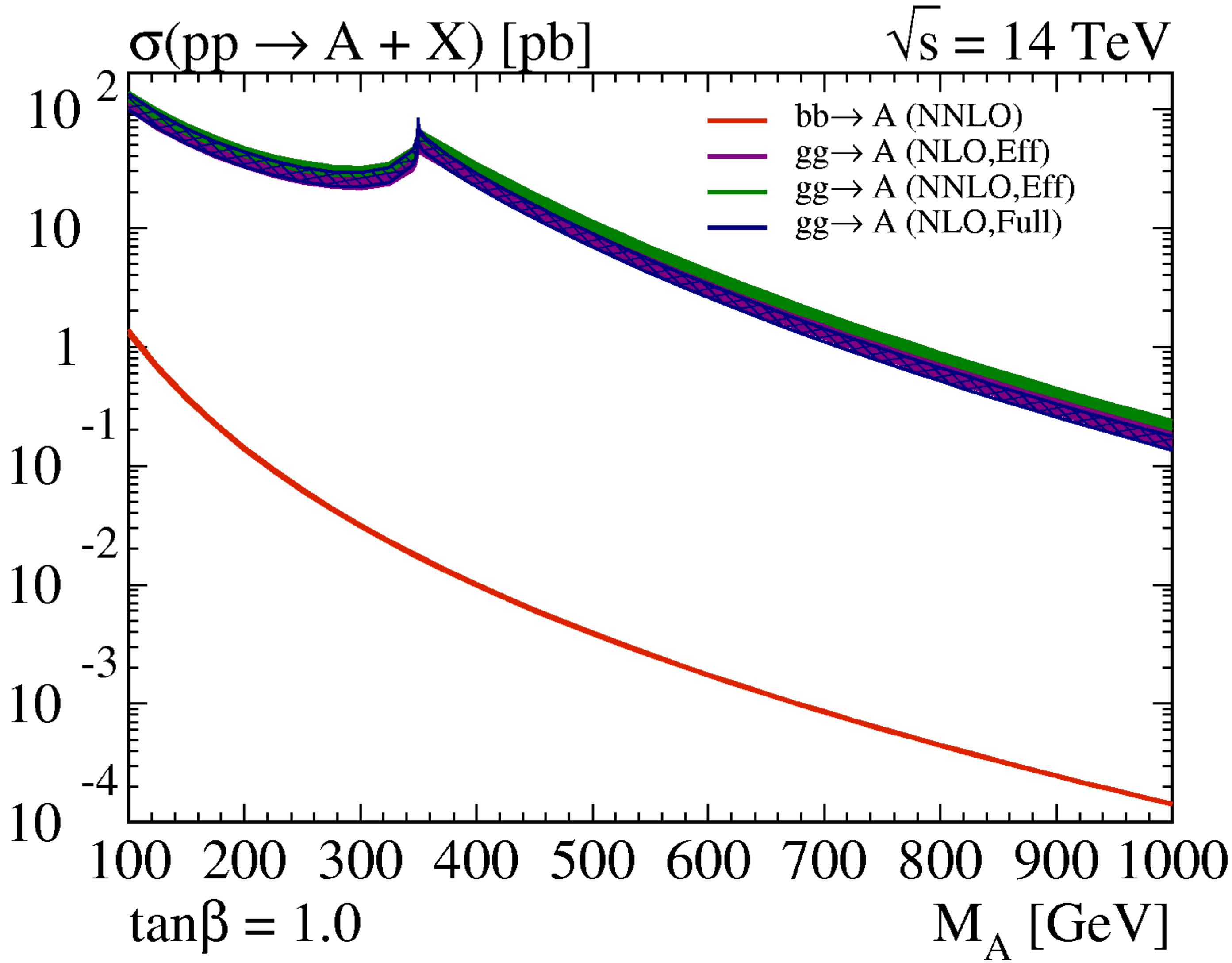
$b\bar{b} \to H$ versus $gg \to H$ ($\tan\beta = 10$)



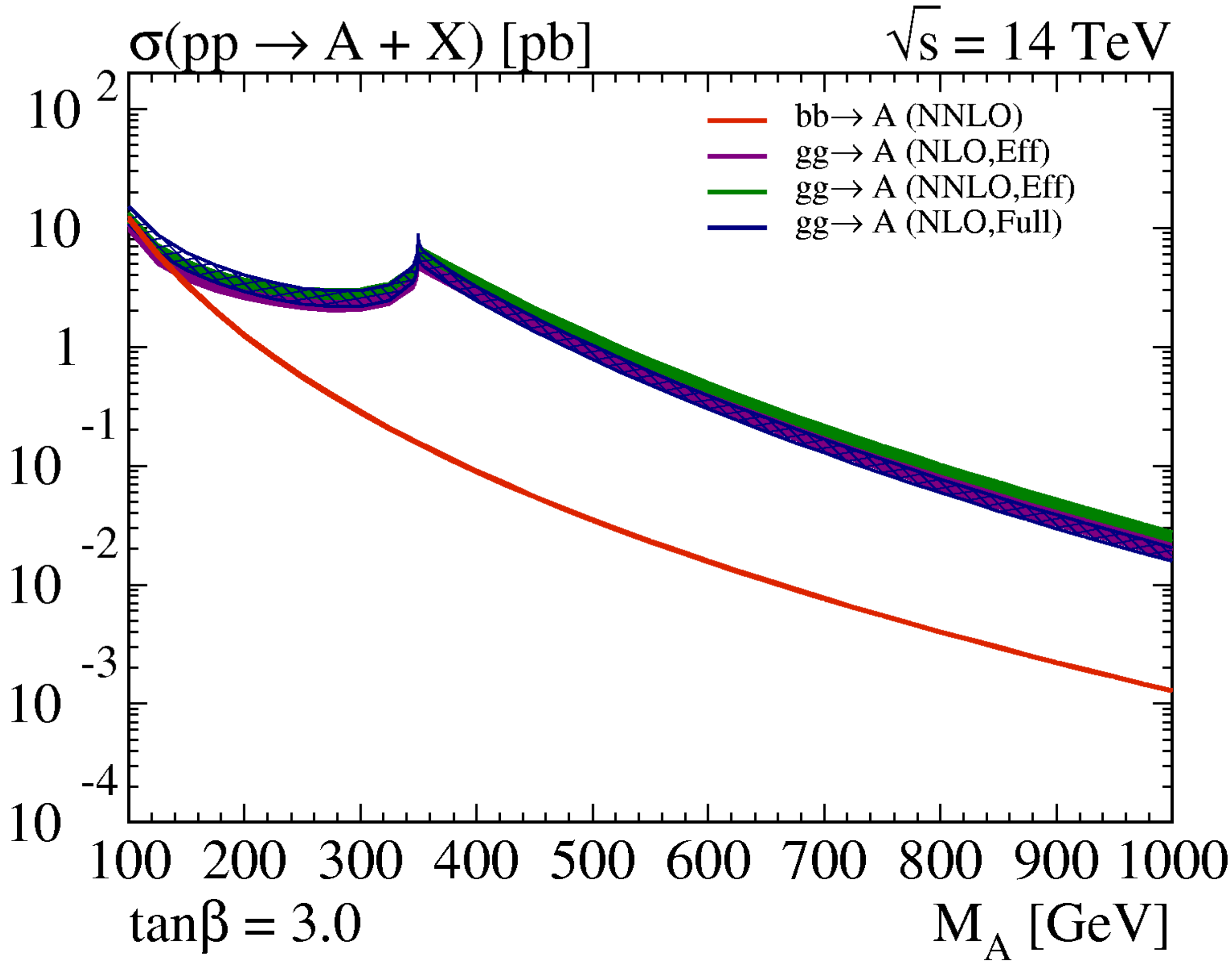
$b\bar{b} \to H$ versus $gg \to H$



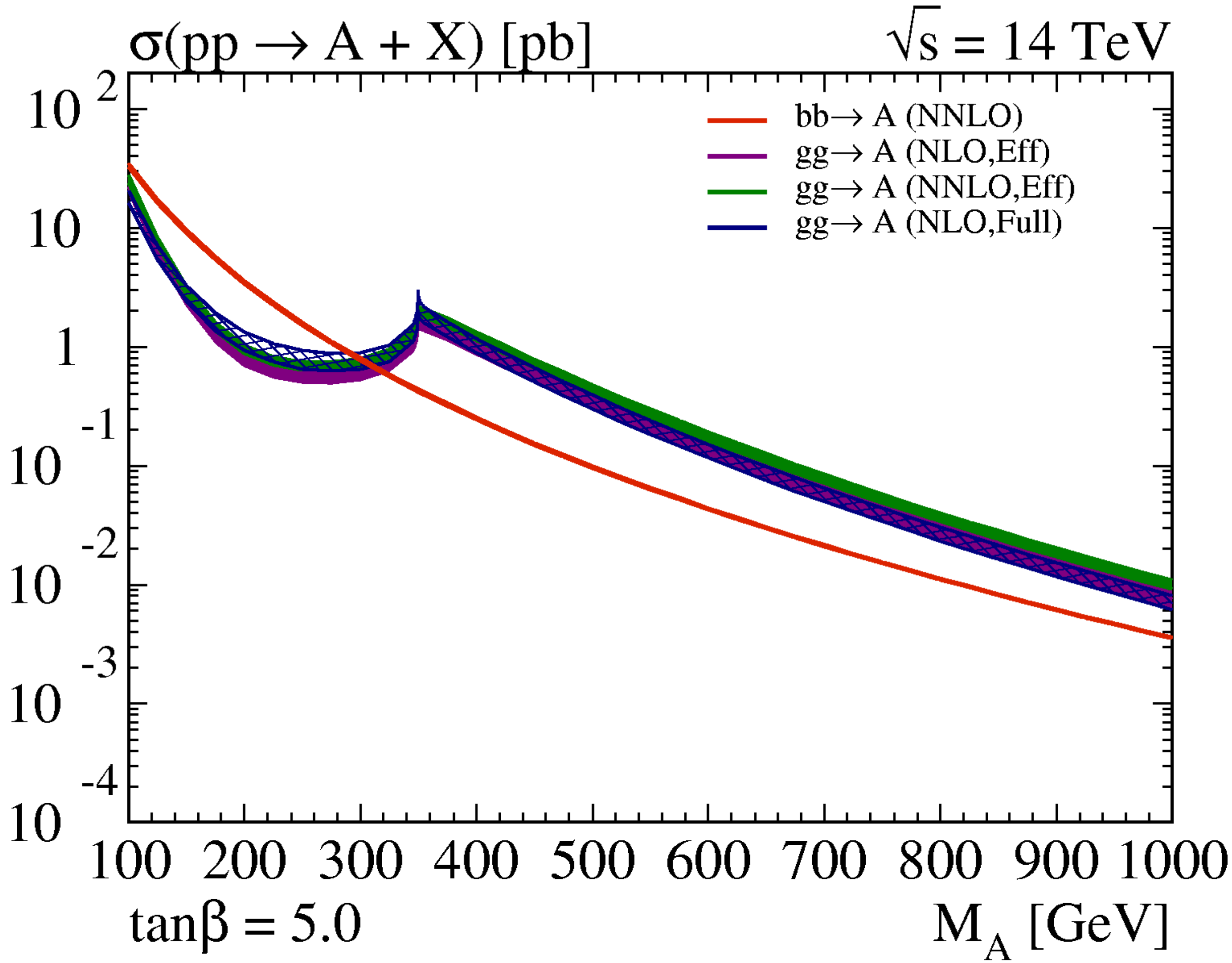
$b\bar{b} \rightarrow A$ versus $gg \rightarrow A$ ($\tan \beta = 1$)



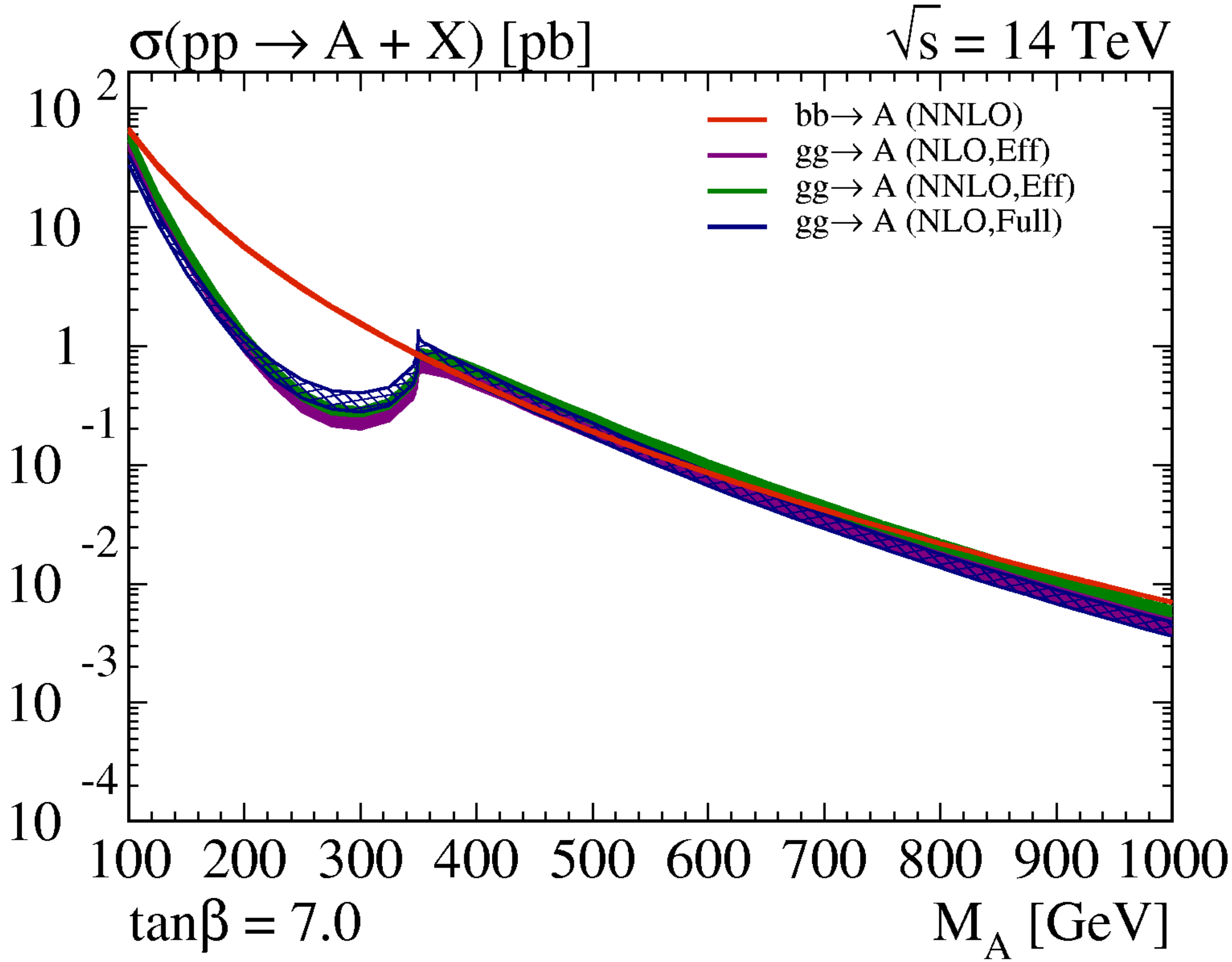
$b\bar{b} \to A$ versus $gg \to A$ ($\tan\beta = 3$)



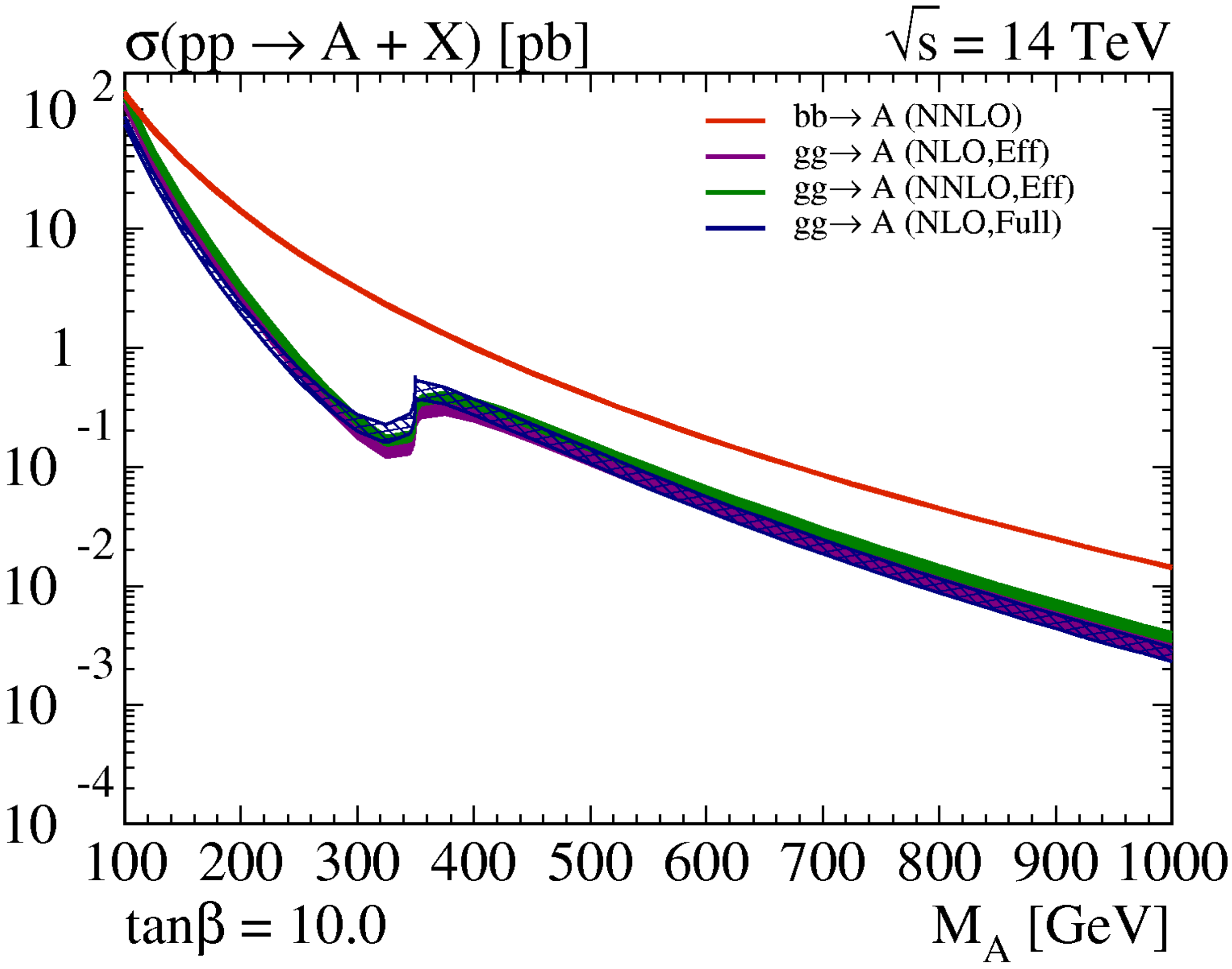
$b\bar{b} \to A$ versus $gg \to A$ ($\tan\beta = 5$)



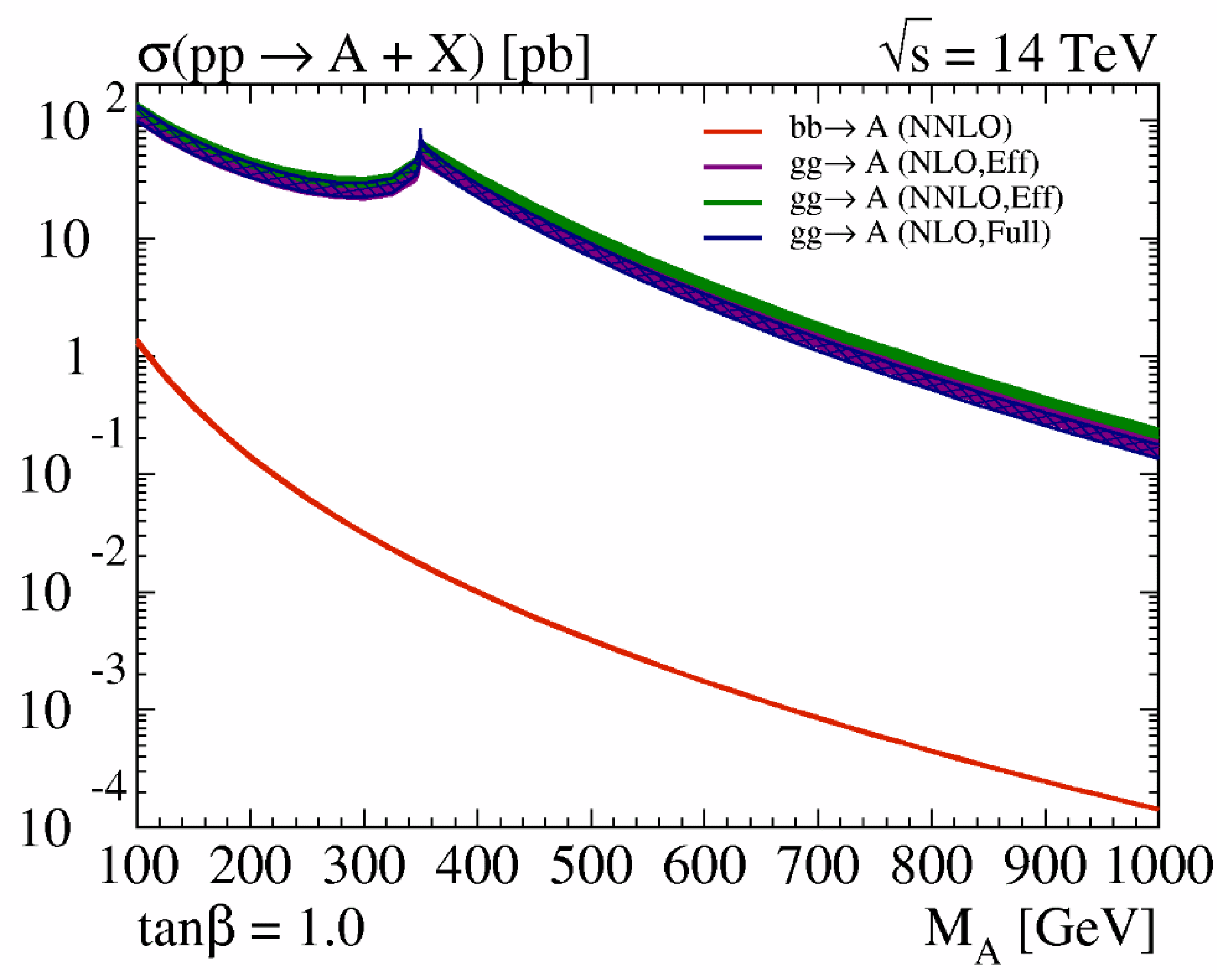
$b\bar{b}\to A$ versus $gg\to A$ (tan $\beta = 7$)



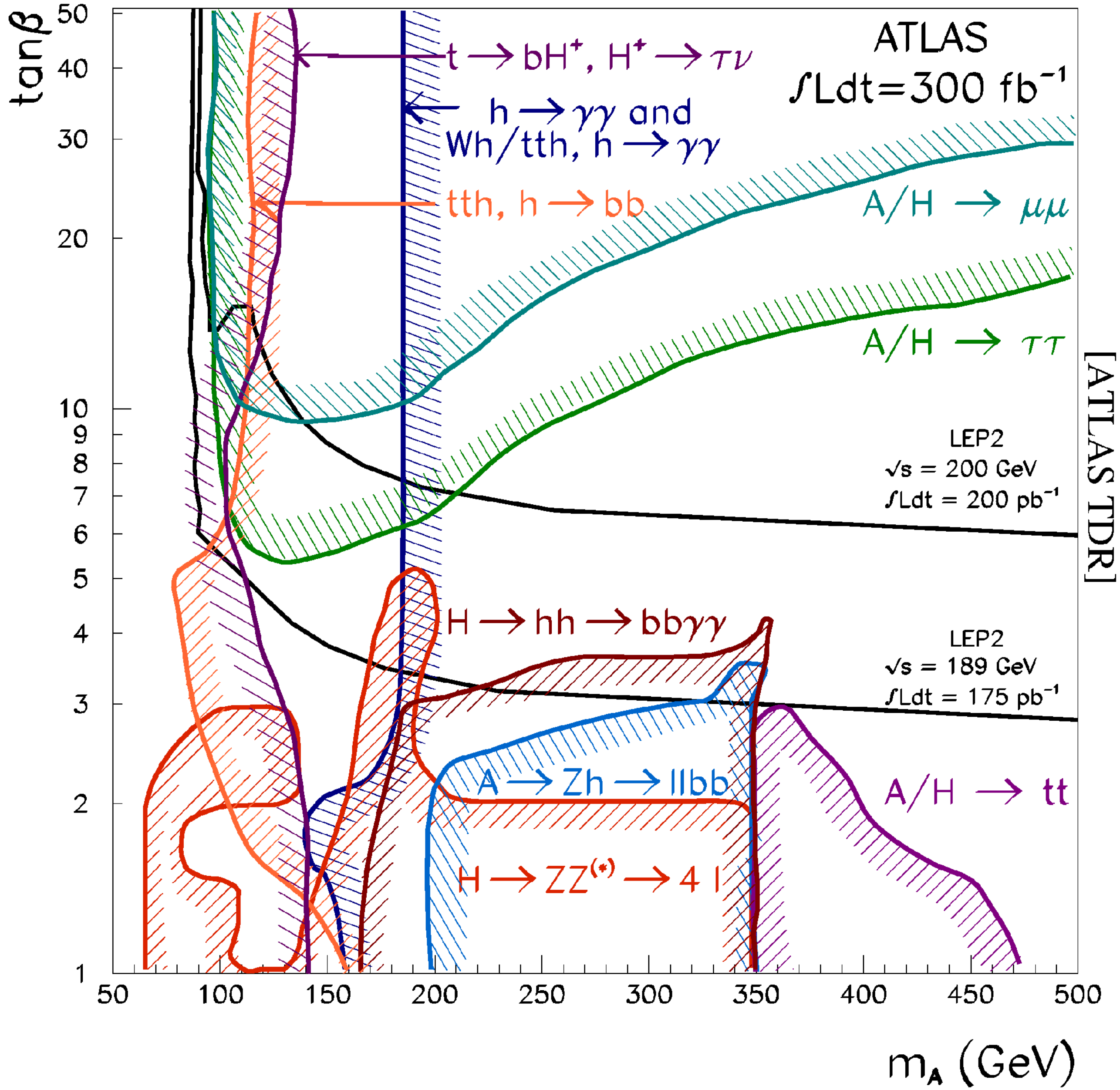
$b\bar{b} \to A$ versus $gg \to A$ ($\tan\beta = 10$)



$b\bar{b} \to A$ versus $gg \to A$



SUSY Higgs Searches at LHC



[ATLAS TDR]

Conclusions

We have computed NNLO corrections to scalar and pseudoscalar Higgs boson production via gluon fusion in the heavy top limit.

The corrections are substantial (though perturbatively well behaved) and the scale dependence is rather large.

We have also computed NNLO corrections to Higgs production via $b\bar{b}$ annihilation.

The corrections are modest and the scale dependence is quite small.

We now have reliable calculations of Higgs boson production rates at hadron colliders for both the Standard Model and Minimal Supersymmetric Standard Model.