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Constraining the ν_e flux with the measured μ^+ rate

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Abstract

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1 Calculation of the flux of $\overline{\nu}_\mu$ from μ^+ decays

Define the following terms:

- L denotes the integrated rate for 'low' energy $E < E_{\text{cut}}$,
- H denotes the integrated rate for 'high' energy $E_{\text{cut}} < E < E_{\text{hi}}$,
- E is the reconstructed neutrino energy,
- $E_{\text{cut}} = 8$ GeV, and
- $E_{\text{hi}} = 16$ GeV, nominally.

The value of E_{cut} is chosen such that the flux of $\overline{\nu}_\mu$ from $\mu^+ \to e^+\overline{\nu}_\mu\nu_e$ is negligible above E_{cut}, and E_{hi} is chosen based on comparison of data and simulation spectra above E_{cut}.

The beam flux ϕ of $\overline{\nu}_\mu$ due to $\mu^+ \to e^+\overline{\nu}_\mu\nu_e$ in units of $\overline{\nu}_\mu$ per proton-on-target (POT)

$$\phi(\mu^+ \to \overline{\nu}_\mu) \equiv \left[L_{\mu^+}/P_d - (L^{\text{MC}}_{\text{NC}} + L^{\text{MC}}_{\nu} + C \times L^{\text{MC}}_{\overline{\nu}}(\pi, K))/P_{MC}\right]/\epsilon_{\overline{\nu}} \quad (1)$$

where

- L_{μ^+} is the measured number of μ^+ candidates,

- $L_{\rm NC}^{\rm MC}$ is the estimated number of μ^+ candidates due to NC interactions,

- $L_{\nu}^{\rm MC}$ is the estimated number of μ^+ candidates due to ν_μ CC interactions,

- $L_{\overline{\nu}}^{\rm MC}(\pi, K)$ is the estimated number of μ^+ candidates due to π^+ and K^+ decays,

- C is a scale factor,

- P_d is the number of POT in the horn-on data,

- P_{MC} is the number of POT in the horn-off data, and

- $\epsilon_{\overline{\nu}}$ is the efficiency of $\overline{\nu}_\mu$ reconstruction.

All terms can have an implicit dependence on the reconstructed neutrino energy. The estimated numbers are assumed to come from simulated (MC) events.

The key to the measurement is an accurate estimate of the last term in Eqn 1. We propose two approaches that should yield at least five semi-independent estimates of the uncertainty in $\phi(\mu^+ \to \overline{\nu}_\mu)$. In the first approach, the shape of $L_{\overline{\nu}}^{\rm MC}(\pi, K)$ is taken from simulation and corrected using the scale factor C determined from a comparison of data and simulation. In the second approach, we use the SKZPII parametrization [1] to adjust the MC predictions of $L_{\overline{\nu}}^{\rm MC}(\pi, K)$ and set $C \equiv 1$.

1.1 Estimating C

In this approach, we estimate C using horn-off data and MC at either 'high' ($C_{H,\rm off}$) or 'low' ($C_{L,\rm off}$) energy, or horn-on 'high' ($C_{H,\rm on}$)energy data and MC.

1. For horn-off 'high' energy data, define

$$\begin{aligned} C_{H,\rm off} &\equiv \frac{\{H_{\mu^+,\rm off}/P_{d,\rm off} - H_{\rm NC,off}/P_{MC,\rm off} - H_{\nu,off}/P_{MC,\rm off}\}}{\{H_{\overline{\nu}}^{\rm MC}(\pi, K \text{ off})/P_{MC,\rm off}\}} \\ &= \{\frac{P_{MC,\rm off}}{P_{d,\rm off}} H_{\mu^+,\rm off} - H_{\rm NC,off} - H_{\nu,off}\}/H_{\overline{\nu}}^{\rm MC}(\pi, K \text{ off}) \quad (2) \end{aligned}$$

where

- $H_{\mu^+,\text{off}}$ is the measured number of μ^+ candidates in horn-off data,
- $H_{\text{NC,off}}$ is the estimated number of μ^+ candidates from NC interactions in horn-off MC,
- $H_{\nu,off}$ is the estimated number of μ^+ candidates from ν_μ CC interactions in horn-off MC,
- $H_{\overline{\nu}}^{\text{MC}}(\pi, K)$ is the estimated number of μ^+ candidates from $\overline{\nu}_\mu$ CC interactions from π^+ and K^+ decays in horn-off MC,
- $P_{d,\text{off}}$ is the POT in horn-off data, and
- $P_{MC,\text{off}}$ is the POT in horn-off MC.

2. For horn-off 'low' energy data, analagously define $C_{L,\text{off}}$,

$$C_{L,\text{off}} \equiv \{\frac{P_{MC,\text{off}}}{P_{d,\text{off}}} L_{\mu^+,\text{off}} - L_{\text{NC,off}} - L_{\nu,off}\}/L_{\overline{\nu}}^{\text{MC}}(\pi, K \text{ off}) \quad (3)$$

3. For horn-on 'high' energy data, define $C_{H,\text{on}}$,

$$C_{H,\text{on}} \equiv \{\frac{P_{MC,\text{on}}}{P_{d,\text{on}}} H_{\mu^+,\text{on}} - H_{\text{NC,on}} - H_{\nu,on}\}/H_{\overline{\nu}}^{\text{MC}}(\pi, K \text{ on}) \quad (4)$$

1.2 Using the SKZPII parametrization

In this case the scale factor C is defined to be unity and $L_{\overline{\nu}}^{\text{MC}}(\pi, K) \equiv L_{\overline{\nu}}^{\text{SKZP}}$ is determined by fitting data with the standard SKZP [1] parametrization but reduced number of fitted parameters:

1. $L_{\overline{\nu}}^{\text{SKZP}}(\text{off})$ is determined from horn-off data after NC and CC ν background subtraction, without any cut on the reconstructed neutrino energy, and
2. $L_{\overline{\nu}}^{\text{SKZP}}(\text{on})$ is determined from horn-on data after NC and CC ν background subtraction with the requirement that $E > E_{\text{cut}}$.

We denote the flux derived from the former by $\phi_{SKZP,\text{off}}$ and from the latter by $\phi_{SKZP,\text{on}}$.

1.3 Uncertainty in the estimate of $\phi(\mu^+ \to \overline{\nu}_\mu)$

In Sections 1.1 and 1.2, we defined five methods to estimate the flux of $\overline{\nu}_\mu$ from π and K decays. We propose to compare the results obtained for ϕ using these five methods to estimate the systematic uncertainty due to the incomplete knowledge of $L_{\overline{\nu}}(\pi, K)$. In addition, we can change $E_{\rm cut}$ and $E_{\rm hi}$ and repeat the five estimates to obtain an additional estimate of systematic uncertainty due to the incomplete knowledge of $L_{\overline{\nu}}(\pi, K)$. We then obtain a distribution of estimates of ϕ denoted by $\{\phi_i\}$ (i runs over the different approaches and changes of the energy range). We can take the mean (or median) of $\{\phi_i\}$ as the central value of A and either $\sqrt{\text{Variance}\{\phi_i\}}$ or $\frac{1}{2}(\max\{\phi_i\} - \min\{\phi_i\})$ as the estimated systematic uncertainty in ϕ due to the incomplete knowledge of $L_{\overline{\nu}}(\pi, K)$.

To estimate the systematic uncertainty due to $\epsilon_{\overline{\nu}}$, we propose to perform the five estimates of ϕ described in the previous paragraphs while varying the $\overline{\nu}_\mu$ selection criteria (varying the NuBarPID cut).

The systematic uncertainties in other quantities in Eqn. 1 are taken from the work of others and is discussed in Section 1.4.

Rewrite Eqn 1 for $C = C_{H,\rm on}$ dropping the "MC" superscript and defining $r_{\rm on} \equiv P_{d,\rm on}/P_{MC,\rm on}$ and $\epsilon \equiv \epsilon_{\overline{\nu}}$

$$\begin{aligned}
\phi_{H,\rm on} &= \frac{1}{\epsilon}\frac{1}{P_{d,\rm on}}\Big[L_{\mu^+} - r_{\rm on}(L_{\rm NC} + L_\nu) \\
&- \left\{\frac{H_{\mu^+,\rm on} - r_{\rm on}(H_{{\rm NC},\rm on} + H_{\nu,\rm on})}{H_{\overline{\nu}}(\pi, K \text{ on})}\right\} L_{\overline{\nu}}(\pi, K \text{ on})\Big] \qquad (5)
\end{aligned}$$

The variance in $\phi_{H,\rm on}$ is (dropping the "on" designation from the L, H, P_d and P_{MC} terms)

$$\begin{aligned}
\delta\phi_{H,\rm on} &\approx \frac{\partial\phi}{\partial L_{\mu^+}}\delta L_{\mu^+} \oplus \frac{\partial\phi}{\partial L_{\rm NC}}\delta L_{\rm NC} \oplus \frac{\partial\phi}{\partial L_\nu}\delta L_\nu \oplus \frac{\partial\phi}{\partial L_{\overline{\nu}}}\delta L_{\overline{\nu}} \\
&\oplus \frac{\partial\phi}{\partial H_{\mu^+}}\delta H_{\mu^+} \oplus \frac{\partial\phi}{\partial H_{\rm NC}}\delta H_{\rm NC} \oplus \frac{\partial\phi}{\partial H_\nu}\delta H_\nu \\
&\oplus \frac{\partial\phi}{\partial P_d}\delta P_d \oplus \frac{\partial\phi}{\partial P_{MC}}\delta P_{MC} \oplus \frac{\partial\phi}{\partial\epsilon}\delta\epsilon \quad . \qquad (6)
\end{aligned}$$

where $\oplus$ means "add in quadrature". The partial derivatives are

$$\begin{aligned}
\frac{\partial\phi}{\partial\epsilon} &= \frac{\phi_{H,\mathrm{on}}}{\epsilon} \qquad \frac{\partial\phi}{\partial L_{\mu^+}} = \frac{1}{\epsilon}\frac{1}{P_d} \\
\frac{\partial\phi}{\partial L_{\overline{\nu}}(\pi,K)} &= \frac{C_{H,\mathrm{on}}}{\epsilon P_d} \qquad \frac{\partial\phi}{\partial H_{\mu^+}} = -\frac{1}{\epsilon P_d}\frac{L_{\overline{\nu}}(\pi,K)}{H_{\overline{\nu}}(\pi,K)} \\
\frac{\partial\phi}{\partial L_{\nu}} &= \frac{\partial\phi}{\partial L_{\mathrm{NC}}} = -\frac{r_{\mathrm{on}}}{\epsilon}\frac{1}{P_d} \\
\frac{\partial\phi}{\partial H_{\nu}} &= \frac{\partial\phi}{\partial H_{\mathrm{NC}}} = -\frac{r_{\mathrm{on}}}{\epsilon P_d}\frac{L_{\overline{\nu}}(\pi,K)}{H_{\overline{\nu}}(\pi,K)} \\
\frac{\partial\phi}{\partial H_{\overline{\nu}}}(\pi,K) &= \frac{1}{\epsilon P_d}\Big[H_{\mu^+} - r_{\mathrm{on}}(H_{\mathrm{NC}} - H_{\nu})\Big]\frac{L_{\overline{\nu}}(\pi,K)}{H^2_{\overline{\nu}}(\pi,K)} \\
\frac{\partial\phi}{\partial P_d} &= \frac{1}{\epsilon P_d^2}\Big[\frac{H_{\mu^+}}{H_{\overline{\nu}}(\pi,K)}L_{\overline{\nu}}(\pi,K) - L_{\mu^+}\Big] \\
\frac{\partial\phi}{\partial P_{MC}} &= -\frac{1}{\epsilon P_{MC}^2}\Big[\frac{L_{\overline{\nu}}(\pi,K)}{H_{\overline{\nu}}(\pi,K)}(H_{\mathrm{NC}} + H_{\overline{\nu}}) - (L_{\mathrm{NC}} + L_{\overline{\nu}})\Big]
\end{aligned} \tag{7}$$

The $\overline{\nu}_\mu$ reconstruction efficiency for horn-on is defined as

$$\epsilon_{\overline{\nu}_\mu} = N(\mu^+)/M(\overline{\nu}_\mu) \tag{8}$$

where $N(\mu^+)$ is the number of μ^+ candidates from $\overline{\nu}_\mu$ CC interactions in horn-on MC and $M(\overline{\nu}_\mu)$ is the nubmer of $\overline{\nu}_\mu$ CC interactions in horn-on MC. The uncertainties in the quantities in Eqn. 7 are

$$\begin{aligned}
\delta\epsilon &= \sqrt{\frac{\epsilon(1-\epsilon)}{M(\overline{\nu}_\mu)}} \\
\delta P_d &= g_{P_d}P_d \\
\delta L_{\mu^+} &= \sqrt{L_{\mu^+}} \\
(\delta L_{\mathrm{NC}})^2 &= (\sqrt{L_{\mathrm{NC}}})^2 + (g_{\mathrm{NC}}L_{\mathrm{NC}})^2 \\
(\delta L_{\nu})^2 &= (\sqrt{L_{\nu}})^2 + (g_{\nu}L_{\nu})^2 \\
\delta L_{\overline{\nu}}(\pi,K) &= \sqrt{L_{\overline{\nu}}(\pi,K)} \\
\delta P_{MC} &\equiv 0
\end{aligned} \tag{9}$$

where the statistical and systematic uncertainties are given explicitly for clarity. Analogous expressions hold for the H terms. The rationale for the values of g to be used in the analysis are given in Section 1.4.

The evaluation of Eqn. 1 with $C = C_{H,\mathrm{off}}$ or $C = C_{L,\mathrm{off}}$ differs from Eqn. 5 because C is estimated from horn-off data and MC. For $C = C_{L,\mathrm{off}}$

$$\phi_{L,\mathrm{off}} \equiv \frac{1}{\epsilon}\Big[\frac{L_{\mu^+}}{P_d} - \frac{L_{\mathrm{NC}} + L_{\nu}}{P_{MC}} - C_{L,\mathrm{off}}\frac{L_{\overline{\nu}}(\pi,K)}{P_{MC}}\Big] \tag{10}$$

where $C_{L,\mathrm{off}}$ is defined in Eqn. 3.and the uncertainty in $\phi_{L,\mathrm{off}}$ is

$$\begin{aligned} \delta\phi_{L,\mathrm{off}} \quad &\approx \quad \frac{\partial\phi_{L,\mathrm{off}}}{\partial\epsilon}\delta\epsilon \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial L_{\mu^+}}\delta L_{\mu^+} \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial L_{\mathrm{NC}}}\delta L_{\mathrm{NC}} \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial L_{\nu}}\delta L_{\nu} \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial L_{\overline{\nu}}(\pi,K)}\delta L_{\overline{\nu}}(\pi,K) \\ &\oplus \quad \frac{\partial\phi_{L,\mathrm{off}}}{\partial P_d}\delta P_d \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial P_{MC}}\delta P_{MC} \oplus \frac{\partial\phi_{L,\mathrm{off}}}{\partial C_{L,\mathrm{off}}}\delta C_{L,\mathrm{off}} \quad . \end{aligned} \tag{11}$$

The evaluation of most of the partial derivatives is given in Eqn. 7. The remaining terms are

$$\frac{\partial\phi_{L,\mathrm{off}}}{\partial C_{L,\mathrm{off}}} = -\frac{L_{\overline{\nu}}(\pi,K \text{ on})}{\epsilon P_{MC,\mathrm{on}}} \tag{12}$$

and the uncertainty in $C_{L,\mathrm{off}}$ requires the evaluation of the following partial derivatives

$$\begin{aligned} \frac{\partial C_{L,\mathrm{off}}}{\partial L_{\mu^+}} \quad &= \quad -\frac{\partial C_{L,\mathrm{off}}}{\partial L_{\mathrm{NC}}} = -\frac{\partial C_{L,\mathrm{off}}}{\partial L_{\nu}} = \frac{1}{L_{\overline{\nu}}(\pi,K \text{ off})}\frac{P_{MC,\mathrm{off}}}{P_{d,\mathrm{off}}} \\ \frac{\partial C_{L,\mathrm{off}}}{\partial P_{d\ \mathrm{off}}} \quad &= \quad -\frac{L_{\mu^+}}{L_{\overline{\nu}}(\pi,K \text{ off})}\frac{P^2_{MC,\mathrm{off}}}{P_{d,\mathrm{off}}} \quad . \end{aligned} \tag{13}$$

Similar expressions hold for $\phi_{H,\mathrm{off}}$.

For the approach described in Section 1.2, $\delta C = 0$ since $C \equiv 1$ by definition, and the results in Equations 5, 6 and 7 can be used to evaluate the uncertainties in $\phi_{SKZP,\mathrm{on}}$ and $\phi_{SKZP,\mathrm{off}}$.

1.4 Other systematic uncertainties

1.4.1 Uncertainty in NC background

For the CC disappearance analysis, the uncertainty in the NC background is taken to be 50% based upon studies of μ^--removed data and simulation [2].

Using this guidance, we take $g_{\rm NC} = \pm 100\%$. The uncertainty has been doubled to take into account the possibility that the simulated NC events that remain may differ from NC events selected in the data because the $\overline{\nu}_\mu$ selection has been designed to achieve very high purity.

1.4.2 Uncertainty in ν_μ CC background

A conservative error of 15% in the total ν_μ cross section of NEUGEN3 is estimated in Ref. [3]. We double this estimate to obtain $g_\nu = \pm 30\%$ using similar reasoning to that in the preceding section. In addition, note that $g_\nu = \pm 30\%$ is consistent with the level of agreement between the NEUGEN3 quasielastic cross section ($\sigma_{\rm QE} = 0.930 \times 10^{-38}$ cm^2) and an analysis of QE ν-Carbon data [3] of $\sigma_{\rm QE} = (0.720 \pm 0.010 \pm 0.030) \times 10^{-38}$ cm^2. Given the background suppression of the NuBarPID requirements, the charm component of the ν_μ CC background is enhanced relative to the non-charm component. A visual scan of 23 ν_μ CC horn-off MC events, yielded 4 events ($17 \pm 8\%$) attributable to charm. Assuming that 25% of selected ν_μ CC events are due to charm with a production rate uncertainty of 100% and assigning a $\pm 15\%$ uncertainty to the remainder yields a total uncertainty of $\sim 27\%$ which is consistent with $g_\nu = \pm 30\%$.

1.4.3 Uncertainty in protons-on-target

We follow the guidance of Ref. [4] and assume $g_{P_d} = \pm 3\%$ for both horn-on and horn-off data. We assume that there is no uncertainty in $P_{\rm MC}$.

References

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[4] P.Adamson, M.Bishai, D.Cherdack, S.Childress, M.Dierckzsens, J.Hylen, A.Ibrahim, D.Indurthy, D.Jensen, G.Koizumi, S.Kopp, P.Lucas, A.Marino, B.Rebel, B.Viren and B.Zwaska, “Measuring the Number of Protons-on-Target (POT) in the NuMI Beamline”, MINOS-doc-1491.