For the E949 collaboration:
Ilektra A. Christidi
SUNY at Stony Brook
(now at the Aristotle University of Thessaloniki...)

New Opportunities in Kaon Physics
Birmingham, UK, Nov 27 2008
• Motivation, current status of CKM measurements, the Standard Model $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ BR
• The measurement with the E949 detector
  • Background overview
  • Detector and measurement method
  • Peculiarities of the PNN2 region
• The analysis
  • Strategy and tools
  • The 2 major backgrounds
  • Cross-check methods
  • Expected background and likelihood ratio method
• The result
  • This analysis
  • Past (E787, E949 PNN1) results
  • Combined E949-E787 result
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is the rarest process ever detected in Particle Physics. It can give a direct and independent measurement of $|V_{td}|$, the smallest and most elusive element of the CKM matrix.

The Kaon Unitarity triangle: $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$

Motivation

Processes w/ small theoretical uncertainties:

- $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$: E787/E949, FNAL-E921
- $B(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$: KOPIO, E391a
- $A(B \rightarrow J/\psi K^0_S)$: BaBar, Belle

$\Delta M_{B_s}/\Delta M_{B_d}$ ratio of mixing frequencies of $B_s$ and $B_d$ mesons
A better determination of $V_{td}$ from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will provide a sensitive test of the SM by comparing the results from the K and B sector and probe new physics.
The SM $K^+ \to \pi^+ \nu \bar{\nu}$ BR

- All processes at 2\textsuperscript{nd} order, with main contribution of $t$ in the loop
- Very clean calculation (precision < 5\%, uncertainties mainly from c sector)

\[ B R (K^+ \to \pi^+ \nu \bar{\nu}) \propto \sum_{l=e, \mu, \tau} \left[ V_{c_s}^* V_{c_d}^* X(\chi_c) + V_{s_s}^* V_{s_d}^* X(\chi_t) \right] \times (H A D R) \times (\bar{\nu}_l \nu_l)^2 \Rightarrow \]

\[ \ldots B R \propto (\sigma \bar{\eta})^2 + (\rho_o - \bar{\rho})^2 \rightarrow \text{ellipse in } \rho-\eta \text{ plane} \]

\[ \sigma = \left( \frac{1}{1 - \lambda^2 / 2} \right)^2 \]

\[ BR_{th}(K^+ \to \pi^+ \nu \bar{\nu}) = (0.84 \pm 0.7) \times 10^{-10} \]
The E949 collaboration

P. Kitching
Centre for Subatomic Research, University of Alberta
D.A. Bryman
University of British Columbia
L.S. Littenberg, G. Redlinger, R.C. Strand and B. Viren
Brookhaven National Laboratory (BNL)
P.S. Cooper, E. Ramberg and R.S. Tschirhart
Fermi National Accelerator Laboratory (FNAL)
M. Miyajima and Y. Tamagawa
Fuku University
A. Artamonov, A. Kozhevnikov, A. Kushnirenko, L. Landsberg, V. Mukhin, V. Obraztsov, D. Patalakha,
S. Petrenko and D. Vavilov
Institute for High Energy Physics (IHEP)
V.V. Anisimovsky, A.P. Ivashkin, M.M. Khabibullin, A.N. Khotjantsev, Y.G. Kudenko, O.V. Mineev and
N.V. Yershov
Institute for Nuclear Research (INR)
T. Tsunemi, Y. Yoshimura and T. Yoshioka
High Energy Accelerator Research Organization (KEK)
N. Muramatsu
Japan Atomic Energy Research Institute (JAERI)
T. Fujiwara, K. Mizouchi, T. Nomura and N. Sasao
Kyoto University
T. Shinkawa
National Defense Academy of Japan
B. Bassalleck, B. Lewis and J. Lowe
University of New Mexico (UNM)
M. Nomachi
Osaka University
T. Nakano
Research Center for Nuclear Physics (RCNP), Osaka University
I.-A. Christidi and M.D. Marx
Stony Brook University
P.C. Bergbusch, E.W. Blackmore, S. Chen, J. Hu, A. Konaka, J.A. Macdonald,
T. Numao, J.-M. Poutissou and R. Poutissou

TRIUMF

Students and post-docs in red.

~70 physicists, plus a lot of hard work from earlier E787 collaborators.
The measurement, backgrounds

• 3-body decay w/ 2 missing particles: \(0 \leq p_{\pi^+} \leq 227\) MeV/c \(\Rightarrow\)  
  Signal: \(\pi^+ + \text{nothing}\), backgrounds need to be vetoed \(~10^{-11}\)!

• Need
  • particle identification (PID)
  • all other charged particles vetoed \(< 10^{-3}\)
  • redundant precise kinematic measurements

Latest result on PNN2 will be presented

<table>
<thead>
<tr>
<th>Background</th>
<th>BR ((\times 10^{-3}))</th>
<th>Suppression method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PID</td>
<td>veto</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^0) ((K_{\pi^2}))</td>
<td>209.2</td>
<td>-</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^0\gamma) ((K_{\pi^2\gamma}))</td>
<td>0.275</td>
<td>-</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^0\mu^+\nu) ((K_{\mu^3}))</td>
<td>33.2</td>
<td>√</td>
</tr>
<tr>
<td>(K^+ \rightarrow \mu^+\nu\gamma) ((K_{\mu^3\gamma}))</td>
<td>6.2</td>
<td>√ √</td>
</tr>
<tr>
<td>(K^+ \rightarrow \pi^+\pi^-e^+\nu) ((K_{e4}))</td>
<td>0.041</td>
<td>-</td>
</tr>
</tbody>
</table>

Beam backgrounds:
  single beam
  double beam

CEX: \(K^+n \rightarrow K^0 p\)
  \(R_{KL} = 2.8 \times 10^{-5}\)
  \(K_L^0 \rightarrow \pi^+\mu^-\bar{\nu}\)
  \(K_L^0 \rightarrow \pi^+\nu\bar{\nu}\)
  \(135.0\)
  \(194.0\)
The AGS extracts \(~ 65 \times 10^{12}\) protons at 22 GeV/c over a 2.2 sec spill, every 5.4s.

They are shot on a platinum target and particles produced at \(~ 0^\circ\) are sent to the Low Energy Separated Beamline (LESB III), where \(K^+\) are electrostatically separated from \(\pi^+\) and focused.

Finally in the E949 target, \(~ 3.5 \times 10^6\) \(K^+\)/spill arrive and stop, with a ratio of \(K/\pi \sim 2.5-3\).
• Incoming 700MeV/c beam $K^+$: identified by ckov, WC, scint. hodoscope (B4). Slowed down by BeO and AD

• $K^+$ stops & decays at rest in scintillating fiber target – measure delay (2ns)

• Outgoing $\pi^+$: verified by IC, VC, T counter. Momentum measured in UTC, energy & range in RS and target (1T magnetic field parallel to beam)

• $\pi^+$ stops & decays in RS – detect $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ chain

• Photons vetoed hermetically in BV-BVL, RS, EC, CO, USPV, DSPV

• New/upgraded elements
• Incoming 700MeV/c beam K⁺ identified by ckov, WC, scint. hodoscope (B4). Slowed down by BeO and AD
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• π\(^+\) stops & decays in RS – detect π\(^+\)→μ\(^+\)→e\(^+\) chain

• Photons vetoed hermetically in BV-BVL, RS, EC, CO, USPV, DSPV
What’s new in E949?

- New/upgraded PV elements
- More protons from AGS
- Improved tracking and energy resolution
- Higher rate capability due to DAQ, electronics and trigger improvements

* Lower beam duty factor
* Lower proton energy
* Problematic separators, worse $K/\pi$ ratio
Peculiarities of the PNN2 region

- More phase space than PNN1
- Probes different part of $p_\pi$ spectrum $\rightarrow$ enhance validity of PNN1 result
- More backgrounds, difficult to disentangle from signal and from each other

Main background: Kp2 scatters in the Target

- Simultaneous shift in range AND momentum
- Photons head near beam direction, the weakest PV region of the detector

Decay product ($\pi^+$ or $\mu^+$) range in scintillator vs momentum:

- 2-body decay peaks
- multi-body decay and $\pi^+$ scatter bands
- scattering tails
• Improved Photon Veto, especially near the beam
  • Optimize at the Kp2 peak, check performance with xy-scatters
  • Measure rejection on different samples of scatters tagged by different “target quality” criteria ⇒ get a central value and the systematic error

• Improved algorithms to identify $\pi^+$ scatters in target:
  • Transverse (xy-scatters) – pattern finding, fit quality
  • Longitudinal (z-scatters) – double pulse in the fiber due to the scatter, disagreement between TG and UTC information
Tools to suppress main background

- Improved Photon Veto, especially near the beam
  - Optimize at the Kp2 peak, check performance with xy-scatters
  - Measure rejection on different samples of scatters tagged by different “target quality” criteria ⇒ get a central value and the systematic error

- Improved algorithms to identify $\pi^+$ scatters in target:
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Analysis strategy

- “Blind” analysis: don’t examine signal region until all backgrounds are verified
- To avoid bias, tune cuts using *randomly selected* 1/3 of the data, then measure background with remaining 2/3
- A priori identification of background sources
- Suppress each background source with at least two independent cuts

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<tr>
<th>Background</th>
<th>BR ($\times 10^{-3}$)</th>
<th>Suppression method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \rightarrow \pi^+\pi^0 \ (K_{\pi 2})$</td>
<td>209.2</td>
<td>PID: -, veto: $\sqrt{\sqrt{}}$, TG: $\sqrt{}$, time: -</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+\pi^0\gamma \ (K_{\pi 2\gamma})$</td>
<td>0.275</td>
<td>PID: -, veto: $\sqrt{\sqrt{\sqrt{}}}$, TG: -</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^0\mu^+\nu \ (K_{\mu 3})$</td>
<td>33.2</td>
<td>PID: $\sqrt{}$, veto: $\sqrt{}$, TG: -</td>
</tr>
<tr>
<td>$K^+ \rightarrow \mu^+\nu\gamma \ (K_{\mu 2\gamma})$</td>
<td>6.2</td>
<td>PID: $\sqrt{}$, veto: $\sqrt{}$, TG: -</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+\pi^-e^+\nu \ (K_{e 4})$</td>
<td>0.041</td>
<td>PID: -, veto: $\sqrt{}$, TG: $\sqrt{}$, time: -</td>
</tr>
<tr>
<td>Beam backgrounds:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>single beam</td>
<td>-</td>
<td>PID: $\sqrt{}$, veto: -</td>
</tr>
<tr>
<td>double beam</td>
<td>-</td>
<td>PID: -, veto: $\sqrt{\sqrt{}}$</td>
</tr>
<tr>
<td>CEX: $K^+n \rightarrow K^0p$</td>
<td>$R_{K_L} = 2.8 \times 10^{-5}$</td>
<td>PID: -</td>
</tr>
<tr>
<td>$K^0_L \rightarrow \pi^+\mu^-\bar{\nu}$</td>
<td>135.0</td>
<td>veto: $\sqrt{}$</td>
</tr>
<tr>
<td>$K^0_L \rightarrow \pi^+e^-\bar{\nu}$</td>
<td>194.0</td>
<td>TG: $\sqrt{}$, time: $\sqrt{\sqrt{}}$</td>
</tr>
</tbody>
</table>
Bifurcated analysis

• Background cannot be reliably simulated \(\Rightarrow\) measure with data by inverting cuts and measuring rejection

• For backgrounds that cannot be reliably isolated in data, use MC to measure rejection and data for normalization
Select events with photons, apply target quality cuts → region B

Select events with target scatter, apply PV → measure rejection of photon veto 
\[(C+D)/C\]
\[ \text{Ke4 can be a background if the kinetic energy of the } \pi^- \text{ and the } e^+ \text{ is low, so that they are not detected} \]

- Isolate a pure Ke4 sample from data using target cuts, for normalization
- Find the rejection of those cuts from MC, which uses the measured energy deposit of the \( \pi^- \)
- Verify background estimates and check for correlations by loosening cuts and comparing observed and predicted number of events remaining near the signal region ("Outside-The-Box" study)

- Examine events that failed only one or two (sets of) cuts, to make sure all background sources have been accounted for ("Single- and Double-Cut-Failure" study)

<table>
<thead>
<tr>
<th>Region</th>
<th>$N_{\text{exp.}}$</th>
<th>$N_{\text{obs.}}$</th>
<th>$P(N_{\text{obs.}}; N_{\text{exp.}})$</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target-scatter ID</td>
<td>$0.79^{+0.46}_{-0.51}$</td>
<td>0</td>
<td>0.45 [0.29,0.62]</td>
<td>N/A</td>
</tr>
<tr>
<td>Photon Veto 1</td>
<td>$9.09^{+1.53}_{-1.32}$</td>
<td>3</td>
<td>0.02 [0.01,0.05]</td>
<td>0.05 [0.02,0.14]</td>
</tr>
<tr>
<td>Photon Veto 2</td>
<td>$32.4^{+12.3}_{-8.1}$</td>
<td>34</td>
<td>0.61 [0.05,0.98]</td>
<td>0.14 [0.01,0.40]</td>
</tr>
</tbody>
</table>
## Expected background

<table>
<thead>
<tr>
<th>Process</th>
<th>Bkgd events (E949)</th>
<th>Bkgd events (E787)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\pi 2}$-scatter</td>
<td>$0.649 \pm 0.150^{+0.067}_{-0.100}$</td>
<td>$1.030 \pm 0.230$</td>
</tr>
<tr>
<td>$K_{\pi 2 \gamma}$</td>
<td>$0.076 \pm 0.007 \pm 0.006$</td>
<td>$0.033 \pm 0.004$</td>
</tr>
<tr>
<td>$K_{e 4}$</td>
<td>$0.176 \pm 0.072^{+0.233}_{-0.124}$</td>
<td>$0.052 \pm 0.041$</td>
</tr>
<tr>
<td>CEX</td>
<td>$0.013 \pm 0.013^{+0.010}_{-0.003}$</td>
<td>$0.024 \pm 0.017$</td>
</tr>
<tr>
<td>Muon</td>
<td>$0.011 \pm 0.011$</td>
<td>$0.016 \pm 0.011$</td>
</tr>
<tr>
<td>Beam</td>
<td>$0.001 \pm 0.001$</td>
<td>$0.066 \pm 0.045$</td>
</tr>
<tr>
<td>Total bkgd</td>
<td>$0.93 \pm 0.17^{+0.32}_{-0.24}$</td>
<td>$1.22 \pm 0.24$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>E949 pnn2</th>
<th>E787 pnn2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Kaons</td>
<td>$1.70 \times 10^{12}$</td>
<td>$1.73 \times 10^{12}$</td>
</tr>
<tr>
<td>Total Acceptance</td>
<td>$1.37 \times 10^{-3}$</td>
<td>$0.84 \times 10^{-3}$</td>
</tr>
<tr>
<td>SES</td>
<td>$4.3 \times 10^{-10}$</td>
<td>$6.9 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Compared to the E787-PNN2 analysis, the total background decreased by 24% and total acceptance increased by 63%
Division of signal region

- Signal and background are not uniformly distributed in the signal region
  ⇒ A candidate in a cleaner region is less likely to be background

- Divide signal region into 9 cells, by varying the PV, kinematic, muon ID and delayed coincidence cuts

<table>
<thead>
<tr>
<th>rel. Acc</th>
<th>bkg</th>
<th>Acc/bkg</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.314</td>
<td>0.152</td>
<td>2.065</td>
</tr>
<tr>
<td>0.073</td>
<td>0.038</td>
<td>1.921</td>
</tr>
<tr>
<td>0.031</td>
<td>0.019</td>
<td>1.653</td>
</tr>
<tr>
<td>0.007</td>
<td>0.005</td>
<td>1.559</td>
</tr>
<tr>
<td>0.287</td>
<td>0.243</td>
<td>1.183</td>
</tr>
<tr>
<td>0.066</td>
<td>0.059</td>
<td>1.135</td>
</tr>
<tr>
<td>0.028</td>
<td>0.027</td>
<td>1.036</td>
</tr>
<tr>
<td>0.006</td>
<td>0.007</td>
<td>0.998</td>
</tr>
<tr>
<td>0.188</td>
<td>0.379</td>
<td>0.496</td>
</tr>
<tr>
<td>1</td>
<td>0.93</td>
<td></td>
</tr>
</tbody>
</table>
Likelihood ratio method

Calculate the BR using $s_i/b_i$ of cells where event(s) are found, using the likelihood ratio method:

Maximize $X = \prod_{i=1}^{n} X_i$, \hspace{1cm} X_i = \frac{e^{-(s_i + b_i)} (s_i + b_i)^{d_i}}{d_i!} \cdot e^{-b_i} b_i^{d_i} / d_i!$

where $d_i$ the number of candidates in cell $i$, $n$ the total number of cells
3 candidate events found!

\[ BR = (7.89^{+9.26}_{-5.10}) \times 10^{-10} \]

Probability that all 3 are due to background: \(0.037\)

SM signal + background: \(0.056\)
## Previous results

<table>
<thead>
<tr>
<th></th>
<th>E787</th>
<th>E949</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PNN1</td>
<td>PNN2</td>
</tr>
<tr>
<td>(p(\pi)) MeV/c</td>
<td>[211,229]</td>
<td>[140,195]</td>
</tr>
<tr>
<td>Stopped (K^+)</td>
<td>5.9x10^{12}</td>
<td>1.7x10^{12}</td>
</tr>
<tr>
<td>Background</td>
<td>0.14±0.05</td>
<td>1.22±0.24</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.0020</td>
<td>0.0008</td>
</tr>
<tr>
<td>S.E.S.</td>
<td>0.8x10^{-10}</td>
<td>6.9x10^{-10}</td>
</tr>
<tr>
<td>Candidates</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>BR (x10^{-10})</td>
<td>1.57^{+1.75}_{-0.82}</td>
<td>&lt;22 (90% CL)</td>
</tr>
</tbody>
</table>

**Candidate E787A**

**Candidate E787C**

**Candidate E949A**
7 candidate events in total

\[ BR = (1.73^{+1.15}_{-1.05}) \times 10^{-10} \]

Probability that all 7 are due to background: 0.001
SM signal+background: 0.060

Central value, although still \( \sim 2 \times \text{SM} \), it is consistent with it within errors…
• Obviously, more statistics are needed → it would have been nice if E949 had all its promised running time...

• Fortunately others have taken over: NA62 at CERN, E391a at JPARC, Project X at Fermilab(?)

E949(02) = combined E787 & E949.
E949 projection with full running period.
• E787 upgrade into E949 worked as expected

• Final bit of the puzzle is now in place (PNN2 measurement), and E787/E949 have a final and definitive result

• Thanks to detector upgrades and deeper understanding of the analysis techniques, the PNN2 study was fruitful and observed 3 candidate events with an expected background of ~1

• The combined BR measured by E787/E949 in both kinematic regions is $BR(K^+\rightarrow\pi^+\nu\bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$, which is still consistent with the SM

• More experiments are running or are under preparation, set to constrain the CKM parameters from the Kaon sector – *good luck!*
The CKM matrix relates weak with mass eigenstates. In the Wolfenstein parametrization,

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
\frac{1 - \frac{\lambda^2}{2}}{2} & \lambda & A \lambda^3 (\rho - i \eta) \\
- \lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\
A \lambda^3 (1 - \bar{\rho} - i \bar{\eta}) & A \lambda^2 & 1
\end{pmatrix}
\]

where \( \bar{\rho} = \rho \left(1 - \frac{\lambda^2}{2}\right) \), \( \bar{\eta} = \eta \left(1 - \frac{\lambda^2}{2}\right) \)

CP violation arises from the irreducible phase of \( V_{CKM} \), because it’s 3x3 (3 generations)
Photon Veto improvement

~ $2 \times$ better rejection at nominal PNN1 acceptance (80%) or

~ 5% more acceptance with E787 rejection!

* Good news for PNN2 as well…

E787, E949
Analysis strategy (1)

- “Blind” analysis: don’t examine signal region until all bg are verified
- To avoid bias, tune cuts using *randomly selected* 1/3 of the data, then measure bg with remaining 2/3
- A priori identification of bg sources
  - Suppress each bg source w/ at least two independent cuts
  - Bg cannot be reliably simulated ⇒ measure w/ data by inverting cuts and measuring rejection

<table>
<thead>
<tr>
<th>Source</th>
<th>Suppresion method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kinematics</td>
</tr>
<tr>
<td>$K^+ \to \mu^+\nu(\gamma)$ ($K_{\mu2}$)</td>
<td>✓</td>
</tr>
<tr>
<td>$K^+ \to \pi^+\pi^0$ ($K_{\pi2}$)</td>
<td>✓</td>
</tr>
<tr>
<td>Scattered $\pi^+$ beam</td>
<td></td>
</tr>
<tr>
<td>CEX</td>
<td></td>
</tr>
</tbody>
</table>

$\text{CEX} \equiv K^+n \to K^0p, \ K_L^0 \to \pi^+\ell^-\nu$
Example: $K^+ \rightarrow \pi^+ \pi^0$ bg rejection

Select events with photons, measure rejection of kinematic cuts (P, R, E “box”)  

Select $K^+ \rightarrow \pi^+ \pi^0$ kinematically, measure rejection of photon veto
• Verify bg estimates & check for correlations by loosening cuts and comparing observed and predicted number of events remaining.

• Construct **background functions** by varying *one cut at a time*, keeping the other inverted. Use them to estimate bg in the signal region.

• Use MC to measure geometrical acceptance, verify by measuring BR(K⁺→π⁺π⁰)
• Verify bg estimates & check for correlations by *simultaneously* loosening both cuts and comparing observed and predicted number of events remaining. Construct **background functions** by varying *one cut at a time*, keeping the other inverted.
• Verify bg estimates & check for correlations by loosening cuts and comparing observed and predicted number of events remaining ("outside-the-box" study).

**Acceptance measurement:**

• Use special background samples (with loose trigger) to measure cuts’ and part of trigger acceptance

• Use signal MC to measure geometrical acceptance

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<thead>
<tr>
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<th>E949</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stopped K⁺ (N_K)</td>
<td>5.9 × 10^{12}</td>
<td>1.8 × 10^{12}</td>
</tr>
<tr>
<td>Total Acceptance</td>
<td>0.0020 ± 0.0002</td>
<td>0.0022 ± 0.0002</td>
</tr>
<tr>
<td>S.E.S.</td>
<td>0.8 × 10^{-10}</td>
<td>2.6 × 10^{-10}</td>
</tr>
<tr>
<td>Total Background</td>
<td>0.14 ± 0.05</td>
<td>0.30 ± 0.03</td>
</tr>
<tr>
<td>Candidate</td>
<td>E787A</td>
<td>E787C</td>
</tr>
<tr>
<td>S_i / b_i</td>
<td>50</td>
<td>7</td>
</tr>
<tr>
<td>W_i = \frac{S_i}{S_i + b_i}</td>
<td>0.98</td>
<td>0.88</td>
</tr>
</tbody>
</table>
\[ K^+ \rightarrow \pi^+ \pi^0 e^+ \nu \]

\( K^+ \rightarrow \pi^+ \pi^0 \) TG scatter event