Waiting for precise measurements of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_{\rm L} \to \pi^0 \nu\bar{\nu}$

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In view of future plans for accurate measurements of the theoretically clean branching ratios $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$, that should take place in this decade, we collect the relevant formulae for quantities of interest and analyze their theoretical and parametric uncertainties. We point out that in addition to the angle β in the unitarity triangle (UT) also the angle γ can in principle be determined from these decays with respectable precision and emphasize in this context the necessity of a calculation of the charm contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ at the NNLO level. In addition to known expressions we present several new ones that should allow transparent tests of the Standard Model (SM) and of its extensions. While our presentation is centered around the SM, we also discuss models with minimal flavour violation and scenarios with new complex phases in enhanced Z^0 penguins and/or $B_d^0 - \bar{B}_d^0$ mixing. We give a brief review of existing results within specific extensions of the SM and investigate the interplay between the $K \to \pi\nu\bar{\nu}$ complex, the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mass differences $\Delta M_{d,s}$ and the angles β and γ that can be measured precisely in two body B decays one day. We derive a new "golden" relation between B and K systems that involves (β, γ) and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ and investigate the virtues of (R_t, β), (R_b, γ), (β, γ) and $(\bar{\eta}, \gamma)$ strategies for the UT in the context of $K \to \pi\nu\bar{\nu}$ decays with the goal of testing the SM and its extensions.

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I. INTRODUCTION

The rare decays of K and B mesons play an important role in the search for the underlying flavour dynamics and in particular in the search for the origin of CP violation (Ali, 2003; Buchalla, 2003; Buchalla *et al.*, 1996a; Buras, 1998, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001). Among the many K and B decays, the rare decays $K^+ \to \pi^+\nu\bar\nu$ and $K_{\rm L} \to \pi^0\nu\bar\nu$ are very special as their branching ratios can be computed to an exceptionally high degree of precision that is not matched by any other decay of mesons. While the theoretical uncertainties in the prominent decays like $B \to X_s\gamma$, $B \to X_s\mu^+\mu^-$ and $B_s \to \mu^+\mu^-$ amount typically to $\pm 10\%$ or larger at the level of the branching ratio, the corresponding uncertainties in $K_{\rm L} \to \pi^0\nu\bar\nu$ amount to 1-2% (Buchalla and Buras, 1993a,b, 1999; Misiak and Urban, 1999). In the case of $K^+ \to \pi^+\nu\bar\nu$, the presence of the internal charm contributions in the relevant Z^0 penguin and box diagrams implies the theoretical uncertainty of $\pm 7\%$ at the NLO level (Buchalla and Buras, 1994a, 1999), but this uncertainty could be in principle reduced to $\pm 2\%$ by performing a NNLO calculation (Buras *et al.*, 2004b).

The reason for the exceptional theoretical cleanness of $K^+ \to \pi^+\nu\bar\nu$ and $K_{\rm L} \to \pi^0\nu\bar\nu$ (Littenberg, 1989) is the fact that the required hadronic matrix elements can be extracted, including isospin breaking corrections (Marciano and Parsa, 1996), from the leading semileptonic decay $K^+ \to \pi^0 e^+\nu$. Moreover, extensive studies of other long-distance contributions (Buchalla and Isidori, 1998; Ecker *et al.*, 1988; Fajfer, 1997; Falk *et al.*, 2001; Geng *et al.*, 1996; Hagelin and Littenberg, 1989; Lu and Wise, 1994; Rein and Sehgal, 1989) and of higher order electroweak effects (Buchalla and Buras, 1998) have shown that they can safely be neglected at present. On the other hand when the uncertainties in the charm contribution will be decreased to a few percent level, also long distance contributions and higher order electroweak effects have to be better quantified. The most recent reviews on $K \to \pi\nu\bar\nu$ can be found in (Buras, 2003a,b, 2004; Isidori, 2003).

We are fortunate that, while the decay $K^+ \to \pi^+\nu\bar\nu$ is CP conserving and depends sensitively on the underlying flavour dynamics, its partner $K_{\rm L} \to \pi^0\nu\bar\nu$ is purely CP violating within the Standard Model (SM) and most of its extensions and consequently depends also on the mechanism of CP violation. Moreover, the combination of these two decays allows to eliminate the parametric uncertainties due to the CKM element $|V_{cb}|$ and m_t in the determination of the angle β in the unitarity triangle (UT) or equivalently of the phase of the CKM element V_{td} (Buchalla and Buras, 1994b, 1996). The resulting theoretical uncertainty in $\sin 2\beta$ is comparable to the one present in the mixing induced CP asymmetry $a_{\psi K_S}$ and with the measurements of both branching ratios at the $\pm 10\%$ and $\pm 5\%$ level, $\sin 2\beta$ could be determined with ± 0.05 and ± 0.025 precision, respectively. This independent determination of $\sin 2\beta$ with a very small theoretical error offers a powerful test of the SM and of its simplest extensions in which the flavour and CP violation are governed by the CKM matrix, the so-called MFV (minimal flavour violation) models (Buras, 2003a,b, 2004; Buras *et al.*, 2001b; D'Ambrosio *et al.*, 2002). Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in Z^0 penguin diagrams ($\Delta S = 1$), whereas in the case of $a_{\psi K_S}$ in the $B_d^0 - \bar{B}_d^0$ box diagrams ($\Delta B = 2$). Any "non-minimal" contributions to Z^0 penguin diagrams and/or box $B_d^0 - \bar{B}_d^0$ diagrams would then be signaled by the violation of the MFV "golden" relation (Buchalla and Buras, 1994b)

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}. \tag{I.1}$$

Now, strictly speaking, according to the common classification of different types of CP violation (Ali, 2003; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001), both the asymmetry $a_{\psi K_S}$ and a non-vanishing rate for $K_{\rm L} \to \pi^0\nu\bar{\nu}$ in the SM and in most of its extensions signal the CP violation in the interference of mixing and decay. However, as the CP violation in mixing (indirect CP violation) in K decays is governed by the small parameter ε_K, one can show (Buchalla and Buras, 1996; Grossman and Nir, 1997; Littenberg, 1989) that the observation of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ at the level of 10^{-11} and higher is a manifestation of a large direct CP violation with the indirect one contributing less than $\sim 1\%$ to the branching ratio.

Additionally, this large direct CP violation can be directly measured without essentially any hadronic uncertainties, due to the presence of the $\nu\bar{\nu}$ in the final state. This should be contrasted with the very popular studies of direct CP violation in non-leptonic two–body B decays (Ali, 2003; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001), that are subject to significant hadronic uncertainties. In particular, the extraction of weak phases requires generally rather involved strategies using often certain assumptions about the strong dynamics (Anikeev *et al.*, 2001; Ball *et al.*, 2000; Harrison and Quinn, 1998). Only a handful of strategies, which we will briefly review in Section VIII, allow direct determinations of weak phases from non-leptonic B decays without practically any hadronic uncertainties.

Returning to (I.1), an important consequence of this relation is the following one (Buras and Fleischer, 2001): for a given $\sin 2\beta$ extracted from $a_{\psi K_S}$, the measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ determines up to a two-fold ambiguity the value of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$, independent of any new parameters present in the MFV models. Consequently, measuring $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ will either select one of the possible values or rule out all MFV models. A spectacular violation of this relation has been recently analyzed (Buras *et al.*, 2004c,d) in the context of a new physics scenario with enhanced Z^0 penguins carrying a new CP-violating phase (Atwood and Hiller, 2003; Buchalla *et al.*, 2001; Buras *et al.*, 2000, 1998; Buras and Silvestrini, 1999; Colangelo and Isidori, 1998; Nir and Worah, 1998).

Another important virtue of $K^+ \to \pi^+\nu\bar{\nu}$ is a theoretically clean determination of $|V_{td}|$ or equivalently of the length R_t in the unitarity triangle. This determination is only subject to theoretical uncertainties in the charm sector, that amount at present to $\pm 4\%$ and can be reduced down to 1% in the future by including NNLO corrections. The remaining parametric uncertainties in the determination of $|V_{td}|$ related to $|V_{cb}|$ and m_t should be soon reduced to the 1-2% level. Finally, the decay $K_{\rm L} \to \pi^0\nu\bar{\nu}$ offers the cleanest determination of the Jarlskog CP-invariant J_{CP} (Buchalla and Buras, 1996) or equivalently of the area of the unrescaled unitarity triangle that cannot be matched by any B decay. With the improved precision on m_t and $|V_{cb}|$, also a precise measurement of the height $\bar{\eta}$ of the unitarity triangle becomes possible.

The clean determinations of $\sin 2\beta$, $|V_{td}|$, R_t, J_{CP}, and of the UT in general, as well as the test of the MFV relation (I.1) and generally of the physics beyond the SM, put these two decays in the class of "golden decays", essentially on the same level as the determination of $\sin 2\beta$ through the asymmetry $a_{\psi K_S}$ and certain clean strategies for the determination of the angle γ in the UT (Ali, 2003; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001), that will be available at LHC (Ball *et al.*, 2000) and BTeV (Anikeev *et al.*, 2001). We will discuss briefly the latter in Section VIII. Therefore precise measurements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ are of utmost importance and should be aimed for, even when realizing that the determination of the branching ratios in question with an accuracy of 5% is extremely challenging.

Our detailed analysis results in the following predictions for the branching ratios within the SM

$$Br(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (7.8 \pm 1.2) \cdot 10^{-11}, \tag{I.2}$$

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu})_{\rm SM} = (3.0 \pm 0.6) \cdot 10^{-11}. \tag{I.3}$$

This is an accuracy of $\pm 15\%$ and $\pm 20\%$, respectively. We will demonstrate that a NNLO calculation of the charm contribution to $K^+ \to \pi^+\nu\bar{\nu}$ and further progress on the determination of the CKM parameters coming in the next few years dominantly from BaBar, Belle, Tevatron and later from LHC and BTeV, should allow eventually predictions for $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ with the uncertainties of $\pm 5\%$ or better. This accuracy cannot be matched by any other rare decay branching ratio in the field of meson decays.

On the experimental side the AGS E787 collaboration at Brookhaven was the first to observe the decay $K^+ \to \pi^+\nu\bar{\nu}$ (Adler *et al.*, 1997, 2000). The resulting branching ratio based on two events and published in 2002 was (Adler *et al.*, 2002, 2004)

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = (15.7^{+17.5}_{-8.2}) \cdot 10^{-11} \qquad (2002). \tag{I.4}$$

Very recently a new $K^+ \to \pi^+\nu\bar{\nu}$ experiment, AGS E949 (Anisimovsky *et al.*, 2004), released its first results that are based on the 2002 running. One additional event has been observed. Including the result of AGS E787 the present branching ratio reads

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = (14.7^{+13.0}_{-8.9}) \cdot 10^{-11} \qquad (2004). \tag{I.5}$$

It is not clear, at present, how this result will be improved in the coming years but AGS E949 should be able to collect in total 10 SM events. One should also hope that the efforts at Fermilab around the CKM experiment (CKM Experiment), the corresponding efforts at CERN around the NA48 collaboration (NA48 Collaboration) and at JPARC in Japan (JPARC) will provide additional 50-100 events in the next five years.

The situation is different for $K_{\rm L} \to \pi^0\nu\bar{\nu}$. While the present upper bound on its branching ratio from KTeV (Alavi-Harati *et al.*, 2000b),

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) < 5.9 \cdot 10^{-7}, \tag{I.6}$$

is about four orders of magnitude above the SM expectation, the prospects for an accurate measurement of $K_{\rm L} \to \pi^0\nu\bar{\nu}$ appear almost better than for $K^+ \to \pi^+\nu\bar{\nu}$ from the present perspective.

Indeed, a $K_{\rm L} \to \pi^0\nu\bar{\nu}$ experiment at KEK, E391a (E391 Experiment), which started taking data this year, should in its first stage improve the bound in (I.6) by three orders of magnitude. While this is insufficient to reach the SM level, a few events could be observed if $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ turned out to be by one order of magnitude larger due to new physics contributions.

Next, a very interesting experiment at Brookhaven, KOPIO (Bryman, 2002; Littenberg, 2002), should in due time provide 40-60 events of $K_{\rm L} \to \pi^0\nu\bar{\nu}$ at the SM level. Finally, the second stage of the E391 experiment could, using the high intensity 50 GeV/c proton beam from JPARC (JPARC), provide roughly 1000 SM events of $K_{\rm L} \to \pi^0\nu\bar{\nu}$, which would be truly fantastic! Perspectives of a search for $K_{\rm L} \to \pi^0\nu\bar{\nu}$ at a Φ-factory have been discussed in (Bossi *et al.*, 1999). Further reviews on experimental prospects for $K \to \pi\nu\bar{\nu}$ can be found in (Barker and Kettell, 2000; Belyaev *et al.*, 2001; Diwan, 2002).

Parallel to these efforts, during the coming years we will certainly witness unprecedented tests of the CKM picture of flavour and CP violation in B decays that will be available at SLAC, KEK, Tevatron and in the second half of this decade at CERN. The most prominent of these tests will involve the $B_s^0 - \bar{B}_s^0$ mixing mass difference ΔM_s and a number of clean strategies for the determination of the angles γ and β in the UT that will involve $B^\pm$, B_d^0 and B_s^0 two-body non-leptonic decays.

These efforts will be accompanied by the studies of CP violation in decays like $B \to \pi\pi$, $B \to \pi K$ and $B \to KK$, that in spite of being less theoretically clean than the quantities considered in the present review, will certainly contribute to the tests of the CKM paradigm (Cabibbo, 1963; Kobayashi and Maskawa, 1973). In addition, rare decays like $B \to X_s\gamma$, $B \to X_{s,d}\mu^+\mu^-$, $B_{s,d} \to \mu^+\mu^-$, $B \to X_{s,d}\nu\bar{\nu}$, $K_L \to \pi^0 e^+e^-$ and $K_L \to \pi^0\mu^+\mu^-$ will play an important role.

In 1994, two detailed analyses of $K^+ \to \pi^+\nu\bar{\nu}$, $K_{\rm L} \to \pi^0\nu\bar{\nu}$, $B_s^0 - \bar{B}_s^0$ mixing and of CP asymmetries in B decays have been presented in the anticipation of future precise measurements of several theoretically clean observables, that could be used for a determination of the CKM matrix and of the unitarity triangle within the SM (Buras, 1994; Buras *et al.*, 1994b). These analyses were very speculative as in 1994 even the top quark mass was unknown, none of the observables listed above have been measured and the CKM elements $|V_{cb}|$ and $|V_{ub}|$ were rather poorly known.

During the last ten years the situation changed significantly: the top quark mass and the angle β in the UT have been rather precisely measured and three events of $K^+ \to \pi^+\nu\bar{\nu}$ have been observed. We are still waiting for the observation of the $B_s^0 - \bar{B}_s^0$ mixing, $K_{\rm L} \to \pi^0\nu\bar{\nu}$ and a direct measurement of the angle γ in the UT, but now we are rather confident that we will be awarded already in this decade.

This progress makes it possible to considerably improve the analyses of (Buras, 1994; Buras *et al.*, 1994b) within the SM and to generalize them to its simplest extensions. This is one of the goals of our review. We will see that the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$, as in 1994, play an important role in these investigations.

In this context we would like to emphasize that new physics contributions in $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$, in essentially all extensions of the SM,[1] can be parametrized in a model-independent manner by just two parameters (Buras *et al.*, 1998), the magnitude of the short distance function X (Buras, 2003a,b, 2004) and its complex phase:

$$X = |X|e^{i\theta_X} \tag{I.7}$$

with $|X| = X(x_t)$ and $\theta_X = 0$ in the SM. The important virtues of the $K \to \pi\nu\bar{\nu}$ system here are the following ones

- $|X|$ and θ_X can be extracted from $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $Br(K^+ \to \pi^+\nu\bar{\nu})$ without any hadronic uncertainties,
- As in most extensions of the SM, the function X is governed by the Z^0 penguins with top quark and new particle exchanges[2], the determination of the function X is actually the determination of the Z^0 penguins that enter other decays.
- The theoretical cleanness of this determination cannot be matched by any other decay. For instance, the decays like $B \to X_{s,d}\mu^+\mu^-$ and $B_{s,d} \to \mu^+\mu^-$, that can also be used for this purpose, are subject to theoretical uncertainties of 10% or more.

Already at this stage we would like to emphasize that the clean theoretical character of these decays remains valid in essentially all extensions of the SM, whereas this is generally not the case for non-leptonic two-body B decays used to determine the CKM parameters through CP asymmetries and/or other strategies. While several mixing induced CP asymmetries in non-leptonic B decays within the SM are essentially free from hadronic uncertainties, as the latter cancel out due to the dominance of a single CKM amplitude, this is often not the case in extensions of the SM in which the amplitudes receive new contributions with different weak phases implying no cancellation of hadronic uncertainties in the relevant observables. A classic example of this situation, as stressed in (Ciuchini and Silvestrini, 2002), is the mixing induced CP asymmetry in $B_d^0(\bar{B}_d^0) \to \phi K_S$ decays that within the SM measures the angle β in the UT with very small hadronic uncertainties. As soon as the extensions of the SM are considered in which new operators and new weak phases are present, the mixing induced asymmetry $a_{\phi K_S}$ suffers from potential hadronic uncertainties that make the determination of the relevant parameters problematic unless the hadronic matrix elements can be calculated with sufficient precision. This is evident from the many papers on the anomaly in $B_d^0(\bar{B}_d^0) \to \phi K_S$ decays of which the subset is given in (Ciuchini and Silvestrini, 2002; Datta, 2002; Fleischer and Mannel, 2001; Grossman *et al.*, 2003; Hiller, 2002; Khalil and Kou, 2003; Raidal, 2002).

The goal of the present review is to collect the relevant formulae for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ and analyze their theoretical and parametric uncertainties. In addition to known expressions we derive new ones that should allow transparent tests of the SM and of its extensions. While our presentation is centered around the SM, we also discuss models with MFV and scenarios with new complex phases in enhanced Z^0 penguins and/or $B_d^0 - \bar{B}_d^0$ mixing. We also give a brief review of other models. Moreover, we investigate the interplay between the $K \to \pi\nu\bar{\nu}$ complex , the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mass differences $\Delta M_{d,s}$ and the angles β and γ in the unitarity triangle that can be measured precisely in two body B decays one day.

Our review is organized as follows. Sections II and III can be considered as a compendium of formulae for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ within the SM. We also give there the formulae for the CKM factors and the UT that are relevant for us. In particular in Section III we investigate the interplay between $K \to \pi\nu\bar{\nu}$, the mass differences $\Delta M_{d,s}$ and the angles β and γ. In Section IV a detailed numerical analysis of the formulae of Sections II and III is presented. In Section V we indicate how the discussion of previous sections is generalized to the class of the MFV models. In Section VI our discussion is further generalized to three scenarios involving new complex phases: a scenario with new physics entering only Z^0 penguins, a scenario with new physics entering only $B_d^0 - \bar{B}_d^0$ mixing and a hybrid scenario in which both Z^0 penguins and $B_d^0 - \bar{B}_d^0$ mixing are affected by new physics. Here we derive a number of expressions that were not presented in the literature so far and illustrate how the new phases, and other new physics parameters can be determined by means of the (R_b, γ) strategy (Buras *et al.*, 2003c) and the related reference unitarity triangle (Barenboim *et al.*, 1999; Cohen *et al.*, 1997; Goto *et al.*, 1996; Grossman *et al.*, 1997). In Section VII we give a brief review of the existing results for both decays within other extensions of the SM, like supersymmetric models, models with extra dimensions, models with lepton-flavour mixing and other selected models considered in the literature. In Section VIII we compare the $K \to \pi\nu\bar{\nu}$ decays with other K and B decays used for the determination of the CKM phases and of the UT with respect to the theoretical cleanness. In Section IX we describe briefly the long distance contributions. Finally, in Section X we summarize our results and give a brief outlook for the future.

[1] Exceptions will be briefly discussed in Section VII.
[2] Box diagrams are only relevant in the SM and can be calculated with high accuracy.

II. BASIC FORMULAE

A. Preliminaries

In this section we will collect the formulae for the branching ratios for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ that constitute the basis for the rest of our review. We will also give the values of the relevant parameters as well as recall the formulae related to the CKM matrix and the unitarity triangle that are relevant for our analysis. Clearly, many formulae listed below have been presented previously in the literature, in particular in (Battaglia *et al.*, 2003; Buchalla and Buras, 1996, 1999; Buchalla *et al.*, 1996a; Buras, 1998, 2003a,b, 2004; Buras *et al.*, 2003c). Still the collection of them at one place and the addition of new ones should be useful for future investigations.

The effective Hamiltonian relevant for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ decays can be written in the SM as follows (Buchalla and Buras, 1994a, 1999)

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_{\rm F}}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_{\rm w}}\sum_{l=e,\mu,\tau}\left(V_{cs}^* V_{cd} X_{\rm NL}^l + V_{ts}^* V_{td} X(x_t)\right)(\bar{s}d)_{V-A}(\bar{\nu}_l\nu_l)_{V-A} \tag{II.1}$$

with all symbols defined below. It is obtained from the relevant Z^0 penguin and box diagrams with the up, charm and top quark exchanges shown in Fig. 1 and includes QCD corrections at the NLO level (Buchalla and Buras, 1993a,b, 1994a, 1999; Misiak and Urban, 1999). The presence of up quark contributions is only needed for the GIM mechanism to work but otherwise only the internal charm and top contributions matter. The relevance of these contributions in each decay is spelled out below.

The index $l = e,\ \mu,\ \tau$ denotes the lepton flavour. The dependence on the charged lepton mass resulting from the box diagrams is negligible for the top contribution. In the charm sector this is the case only for the electron and the muon but not for the τ-lepton. In what follows we give the branching ratios that follow from (II.1).

B. $K^+ \to \pi^+\nu\bar{\nu}$

The branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ in the SM is dominated by Z^0 penguin diagrams with a significant contribution from the box diagrams. Summing over three neutrino flavours, it can be written as follows (Buchalla and Buras, 1999)

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \cdot \left[\left(\frac{{\rm Im}\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{{\rm Re}\lambda_c}{\lambda}P_c(X) + \frac{{\rm Re}\lambda_t}{\lambda^5}X(x_t)\right)^2\right], \tag{II.2}$$

$$\kappa_+ = r_{K^+}\frac{3\alpha^2 Br(K^+ \to \pi^0 e^+\nu)}{2\pi^2\sin^4\theta_w}\lambda^8 = (4.84 \pm 0.06)\cdot 10^{-11}\left[\frac{\lambda}{0.224}\right]^8. \tag{II.3}$$

An explicit derivation of (II.2) can be found in (Buras, 1998). Here $x_t = m_t^2/M_W^2$, $\lambda_i = V_{is}^* V_{id}$ are the CKM factors discussed below and $r_{K^+} = 0.901$ summarizes isospin breaking corrections in relating $K^+ \to \pi^+\nu\bar{\nu}$ to the well measured leading decay $K^+ \to \pi^0 e^+\nu$ (Marciano and Parsa, 1996). In obtaining the numerical value in (II.3) we have used (Hagiwara *et al.*, 2002)

$$\sin^2\theta_w = 0.231, \qquad \alpha = \frac{1}{127.9}, \qquad Br(K^+ \to \pi^0 e^+\nu) = (4.87 \pm 0.06)\cdot 10^{-2} \tag{II.4}$$

with the first two given in the $\overline{MS}$ scheme. Their errors are below 0.1% and can be neglected. There is an issue related to $\sin^2\theta_w$ that although very well measured in a given renormalization scheme, is a scheme dependent quantity with the scheme dependence only removed by considering higher order electroweak effects in $K \to \pi\nu\bar{\nu}$. An analysis of such effects in the large m_t limit (Buchalla and Buras, 1998) shows that in principle they could introduce a $\pm 5\%$ correction in the $K \to \pi\nu\bar{\nu}$ branching ratios but with the $\overline{MS}$ definition of $\sin^2\theta_w$, these higher order electroweak corrections are found below 2% and can also be safely neglected. Similar comments apply to α. This pattern of higher order electroweak corrections is also found in the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing (Gambino *et al.*, 1999).

The apparent large sensitivity of $Br(K^+ \to \pi^+\nu\bar{\nu})$ to λ is spurious as $P_c(X) \sim \lambda^{-4}$ and the dependence on λ in (II.3) cancels the one in (II.2) to a large extent. However, basically for aesthetic reasons it is useful to write first these formulae as given above. In doing this it is essential to keep track of the λ dependence as it is hidden in $P_c(X)$ (see (II.12)) and changing λ while keeping $P_c(X)$ fixed would give wrong results. For later purposes we will also introduce

$$\bar{\kappa}_+ = \frac{\kappa_+}{\lambda^8} = (7.64 \pm 0.09)\cdot 10^{-6}. \tag{II.5}$$

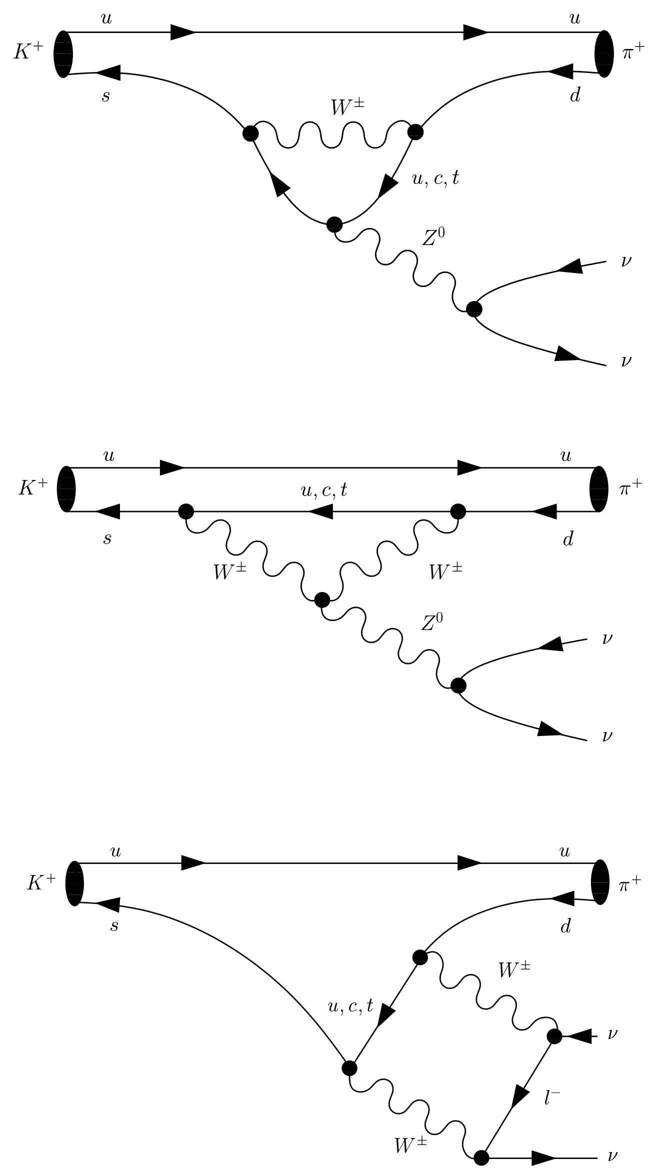

FIG. 1 The penguin and box diagrams contributing to $K^+ \to \pi^+\nu\bar{\nu}$. For $K_{\rm L} \to \pi^0\nu\bar{\nu}$ only the spectator quark is changed from u to d.

The function $X(x_t)$ relevant for the top part is given by

$$X(x_t) = X_0(x_t) + \frac{\alpha_s(m_t)}{4\pi} X_1(x_t) = \eta_X \cdot X_0(x_t), \qquad \eta_X = 0.995, \tag{II.6}$$

where

$$X_0(x_t) = \frac{x_t}{8}\left[-\frac{2+x_t}{1-x_t} + \frac{3x_t-6}{(1-x_t)^2}\ln x_t\right] \tag{II.7}$$

describes the contribution of Z^0 penguin diagrams and box diagrams without the QCD corrections (Buchalla *et al.*, 1991; Inami and Lim, 1981) and the second term stands for the QCD correction (Buchalla and Buras, 1993a,b, 1999; Misiak and Urban, 1999) with

$$X_1(x_t) = -\frac{29x_t - x_t^2 - 4x_t^3}{3(1-x_t)^2} - \frac{x_t + 9x_t^2 - x_t^3 - x_t^4}{(1-x_t)^3}\ln x_t$$

$$+ \quad \frac{8x_t + 4x_t^2 + x_t^3 - x_t^4}{2(1-x_t)^3} \ln^2 x_t - \frac{4x_t - x_t^3}{(1-x_t)^2} L_2(1-x_t)$$
$$+ \quad 8x \frac{\partial X_0(x_t)}{\partial x_t} \ln x_\mu \tag{II.8}$$

where $x_\mu = \mu_t^2/M_W^2$, $\mu_t = \mathcal{O}(m_t)$ and

$$L_2(1-x_t) = \int_1^{x_t} dt \frac{\ln t}{1-t}. \tag{II.9}$$

The μ_t-dependence in the last term in (II.8) cancels to the order considered the μ_t-dependence of the leading term $X_0(x(\mu_t))$. The leftover μ_t-dependence in $X(x_t)$ is below 1%. The factor η_X summarizes the NLO corrections represented by the second term in (II.6). With $m_{\rm t} \equiv m_{\rm t}(m_{\rm t})$ the QCD factor η_X is practically independent of m_t and $\alpha_s(M_Z)$ and is very close to unity. Varying $m_{\rm t}(m_{\rm t})$ from 150 GeV to 180 GeV changes η_X by at most 0.1%.

The uncertainty in $X(x_t)$ is then fully dominated by the experimental error in m_t. With the $\overline{MS}$ top-quark mass [3]

$$m_{\rm t}(m_{\rm t}) = (168.1 \pm 4.1)\,\mathrm{GeV}, \tag{II.10}$$

corresponding to the most recent $m_t^{\rm pole} = (178.0 \pm 4.3)\,\mathrm{GeV}$ (Azzi *et al.*, 2004), one has

$$X(x_t) = 1.529 \pm 0.042. \tag{II.11}$$

$X(x_t)$ increases with m_t roughly as $m_t^{1.15}$. After the Tevatron era the error on m_t should decrease below $\pm 2\,\mathrm{GeV}$, implying the error of ± 0.02 in $X(x_t)$ that can be neglected for all practical purposes.

In obtaining the $\overline{MS}$ top-quark mass given above we have used the relation between this mass and the pole mass of (Melnikov and Ritbergen, 2000) that includes one- two- and three-loop contributions. Taking only one-loop contributions would result in $m_{\rm t}(m_{\rm t}) = (169.8 \pm 4.1)\,\mathrm{GeV}$ and $X(x_t) = 1.55 \pm 0.04$. As the determination of $\alpha_s(M_Z)$ from various processes includes higher orders in α_s, it is more appropriate in our opinion to use the value in (II.10) even if the branching ratios for $K \to \pi\nu\bar\nu$ include only NLO effects.

The parameter $P_c(X)$ summarizes the charm contribution and is defined through

$$P_c(X) = \frac{1}{\lambda^4}\left[\frac{2}{3} X_{\rm NL}^e + \frac{1}{3} X_{\rm NL}^\tau\right] \tag{II.12}$$

where the functions $X_{\rm NL}^l$ result from the NLO calculation (Buchalla and Buras, 1994a, 1999) and are recalled for completeness in Appendix A. The index "l" distinguishes between the charged lepton flavours in the box diagrams. This distinction is irrelevant in the top contribution due to $m_t \gg m_l$ but is relevant in the charm contribution as $m_\tau > m_c$. The inclusion of NLO corrections reduced considerably the large μ_c dependence (with $\mu_c = \mathcal{O}(m_c)$) present in the leading order expressions for the charm contribution (Dib *et al.*, 1991; Ellis and Hagelin, 1983; Vainshtein *et al.*, 1977). Varying μ_c in the range $1\,\mathrm{GeV} \le \mu_c \le 3\,\mathrm{GeV}$ changes $X_{\rm NL}^l$ by roughly 24% at NLO to be compared to 56% in the leading order.

The net effect of QCD corrections is to suppress the charm contribution by roughly 30%. For our purposes we need only $P_c(X)$. In table I we give its values for different $\alpha_s(M_Z)$ and $m_c \equiv m_c(m_c)$. The chosen range for $m_c(m_c)$ is in the ballpark of the most recent estimates. For instance $m_c(m_c) = 1.304(27)$, $1.301(34)$ and $1.29(7)$ (all in GeV) have been found from $R^{e^+e^-}(s)$ (Kühn and Steinhauser, 2001), quenched lattice QCD (Rolf and Sint, 2002) and charmonium sum rules (Hoang and Jamin, 2004), respectively. Further references can be found in these papers and in (Battaglia *et al.*, 2003).

Finally, in table II we show the dependence of $P_c(X)$ on $\alpha_s(M_Z)$ and μ_c at fixed $m_c(m_c) = 1.30\,\mathrm{GeV}$. In an Appendix we give more details on how the values in tables I and II have been obtained.

Restricting the three parameters involved to the ranges

$$1.25\,\mathrm{GeV} \le m_c(m_c) \le 1.35\,\mathrm{GeV}, \qquad 1.0\,\mathrm{GeV} \le \mu_c \le 3\,\mathrm{GeV}, \tag{II.13}$$

$$0.116 \le \alpha_s(M_Z) \le 0.120 \tag{II.14}$$

[3] We thank M. Jamin for discussions on this subject.

TABLE I The parameter $P_c(X)$ for $\lambda = 0.224$ and various $\alpha_s(M_Z)$ and $m_c(m_c)$.

	$P_c(X)$		
$\alpha_s(M_Z)$ \ m_c [GeV]	1.25	1.30	1.35
0.115	0.371	0.403	0.436
0.116	0.366	0.399	0.432
0.117	0.362	0.394	0.427
0.118	0.357	0.389	0.422
0.119	0.352	0.384	0.417
0.120	0.347	0.379	0.412
0.121	0.341	0.373	0.406

and setting $\lambda = 0.224$ we arrive at

$$P_c(X) = 0.389 \pm 0.033_{m_c} \pm 0.045_{\mu_c} \pm 0.010_{\alpha_s}, \tag{II.15}$$

where the errors correspond to $m_c(m_c)$, μ_c and $\alpha_s(M_Z)$, respectively.

We observe that the error in $P_c(X)$ is dominated by the left-over scale uncertainty, implying that a calculation of $P_c(X)$ at the NNLO level is certainly desirable. The uncertainty due to m_c is smaller but still significant. On the other hand, the uncertainty due to α_s is small. In principle one could add the errors in (II.15) linearly, which would result in a total error of ± 0.088. We think that this estimate would be too conservative. Adding the errors in quadrature gives ± 0.057. This could be, on the other hand, too optimistic, since the uncertainties are not statistically distributed. Therefore, as the final result for $P_c(X)$ we quote

$$P_c(X) = 0.39 \pm 0.07 \tag{II.16}$$

that we will use in the rest of our review. This agrees for the same value of $\lambda = 0.224$ with the values used in the literature except for the increase of the error from 0.06 to 0.07. We anticipate that all long distance uncertainties, that are well below the error in (II.16), are already included in the error quoted above.

TABLE II The parameter $P_c(X)$ for $\lambda = 0.224$ and various $\alpha_s(M_Z)$ and μ_c with $m_c(m_c) = 1.30\,\mathrm{GeV}$.

	$P_c(X)$			
$\alpha_s(M_Z)$ \ μ_c [GeV]	1.0	1.3	2.0	3.0
0.115	0.423	0.403	0.371	0.343
0.116	0.420	0.399	0.365	0.337
0.117	0.418	0.394	0.359	0.331
0.118	0.415	0.389	0.353	0.324
0.119	0.412	0.384	0.346	0.317
0.120	0.409	0.379	0.339	0.309
0.121	0.405	0.373	0.332	0.302

We expect that a NNLO calculation would reduce the error in $P_c(X)$ due to μ_c by a factor of 2-3 and the reduction of the error in $\alpha_s(M_Z)$ to ± 0.001 will decrease the corresponding error to 0.005, making it negligible. Concerning the error due to $m_c(m_c)$, it should be remarked that increasing the error in $m_c(m_c)$ to $\pm 70\,\mathrm{MeV}$ would increase the first error in (II.15) to 0.047, whereas its decrease to $\pm 30\,\mathrm{MeV}$ would decrease it to 0.020. More generally we have to a good approximation

$$\sigma(P_c(X))_{m_c} = \left[\frac{0.67}{\mathrm{GeV}}\right] \sigma(m_c(m_c)). \tag{II.17}$$

This exercise shows that after a NNLO analysis has been performed, the main uncertainty in $P_c(X)$ will be due to m_c. From the present perspective, unless some important advances in the determination of $m_c(m_c)$ will be made, it will be very difficult to decrease the error on $P_c(X)$ below ± 0.03, although ± 0.02 cannot be fully excluded. We will use this information in our numerical analysis in Section IV.

C. $K_{\rm L} \to \pi^0 \nu\bar\nu$

The neutrino pair produced by $\mathcal{H}_{\rm eff}^{\rm SM}$ in (II.1) is a CP eigenstate with positive eigenvalue. Consequently, within the approximation of keeping only operators of dimension six, as done in (II.1), the decay $K_{\rm L} \to \pi^0 \nu\bar\nu$ proceeds entirely through CP violation (Littenberg, 1989). However, as pointed out in (Buchalla and Isidori, 1998), even in the SM there are CP-conserving contributions to $K_{\rm L} \to \pi^0 \nu\bar\nu$, that are generated only by local operators of $d \geq 8$ or by long distance effects. Fortunately, these effects are by a factor of 10^5 smaller than the leading CP-violating contribution and can be safely neglected (Buchalla and Isidori, 1998). As we will discuss in Section VII, the situation can be in principle very different beyond the SM.

The branching ratio for $K_{\rm L} \to \pi^0 \nu\bar\nu$ in the SM is then fully dominated by the diagrams with internal top exchanges with the charm contribution well below 1%. It can be written then as follows (Buchalla and Buras, 1996; Buchalla *et al.*, 1996a; Buras, 1998)

$$Br(K_L \to \pi^0 \nu\bar\nu) = \kappa_L \cdot \left(\frac{{\rm Im}\lambda_t}{\lambda^5} X(x_t) \right)^2 \tag{II.18}$$

$$\kappa_L = \kappa_+ \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} = (2.12 \pm 0.03) \cdot 10^{-10} \left[\frac{\lambda}{0.224} \right]^8 \tag{II.19}$$

where we used $\tau(K_L)/\tau(K^+) = 4.17 \pm 0.03$ and have summed over three neutrino flavours. An explicit derivation of (II.18) can be found in (Buras, 1998). Here $r_{K_L} = 0.944$ is the isospin breaking correction from (Marciano and Parsa, 1996) with κ_+ given in (II.3). Due to the absence of $P_c(X)$ in (II.18), $Br(K_L \to \pi^0 \nu\bar\nu)$ has essentially no theoretical uncertainties and is only affected by parametric uncertainties coming from m_t, ${\rm Im}\lambda_t$ and κ_L. They should be decreased significantly in the coming years so that a precise prediction for $Br(K_L \to \pi^0 \nu\bar\nu)$ should be available in this decade. On the other hand, as discussed below, once this branching ratio has been measured, ${\rm Im}\lambda_t$ can be in principle determined with exceptional precision not matched by any other decay (Buchalla and Buras, 1996).

D. $K_{\rm S} \to \pi^0 \nu\bar\nu$

Next, mainly for completeness, we give the expression for $Br(K_{\rm S} \to \pi^0 \nu\bar\nu)$, that, due to $\tau(K_S) \ll \tau(K_L)$, is suppressed by roughly 2 orders of magnitude relative to $Br(K_{\rm L} \to \pi^0 \nu\bar\nu)$. We have (Bossi *et al.*, 1999)

$$Br(K_{\rm S} \to \pi^0 \nu\bar\nu) = \kappa_S \cdot \left(\frac{{\rm Re}\lambda_c}{\lambda} P_c(X) + \frac{{\rm Re}\lambda_t}{\lambda^5} X(x_t) \right)^2 , \tag{II.20}$$

$$\kappa_S = \kappa_L \frac{\tau(K_S)}{\tau(K_L)} = (3.66 \pm 0.05) \cdot 10^{-13} \left[\frac{\lambda}{0.224} \right]^8 . \tag{II.21}$$

Introducing the "reduced" branching ratio

$$B_3 = \frac{Br(K_{\rm S} \to \pi^0 \nu\bar\nu)}{\kappa_S} \tag{II.22}$$

and analogous ratios B_1 and B_2 for $K^+ \to \pi^+ \nu\bar\nu$ and $K_{\rm L} \to \pi^0 \nu\bar\nu$ given in (III.24) we find a simple relation between the three $K \to \pi\nu\bar\nu$ decays

$$B_1 = B_2 + B_3. \tag{II.23}$$

We would like to emphasize that, while $Br(K_{\rm L} \to \pi^0 \nu\bar\nu)$ being only sensitive to ${\rm Im}\lambda_t$ provides a direct determination of $\bar\eta$, $Br(K_{\rm S} \to \pi^0 \nu\bar\nu)$ being only sensitive to ${\rm Re}\lambda_t$ provides a direct determination of $\bar\varrho$. The latter determination is not as clean as the one of $\bar\eta$ from $K_{\rm L} \to \pi^0 \nu\bar\nu$ due to the presence of the charm contribution in (II.20). However, it is much cleaner than the corresponding determination of $\bar\varrho$ from $K_L \to \mu^+\mu^-$. Unfortunately, the tiny branching ratio $Br(K_{\rm S} \to \pi^0 \nu\bar\nu) \approx 5 \cdot 10^{-13}$ will not allow this determination in a foreseeable future. Therefore we will not consider $K_{\rm S} \to \pi^0 \nu\bar\nu$ in the rest of our review. Still one should not forget that the presence of another theoretically clean observable would be very useful in testing the extensions of the SM. Interesting discussions of the complex $K_{\rm L} \to \pi^0 \nu\bar\nu$ and $K_{\rm S} \to \pi^0 \nu\bar\nu$ and its analogies to the studies of ε'/ε can be found in (Bossi *et al.*, 1999; D'Ambrosio *et al.*, 1994).

E. CKM Parameters

1. Unitarity Triangle, $\mathrm{Im}\lambda_t$ and $\mathrm{Re}\lambda_t$

Concerning the CKM parameters, we will use in our numerical analysis the Wolfenstein parametrization (Wolfenstein, 1983), generalized to include higher orders in $\lambda \equiv |V_{us}|$ (Buras *et al.*, 1994b). This turns out to be very useful in making the structure of various formulae transparent and gives results very close to the ones obtained by means of the exact standard parametrization (Chau and Keung, 1984; Hagiwara *et al.*, 2002). The basic parameters are then

$$\lambda, \qquad A = \frac{|V_{cb}|}{\lambda^2}, \qquad \bar\varrho = \varrho(1-\frac{\lambda^2}{2}), \qquad \bar\eta = \eta(1-\frac{\lambda^2}{2}) \tag{II.24}$$

with ϱ and η being the usual Wolfenstein parameters (Wolfenstein, 1983). The parameters $\bar\varrho$ and $\bar\eta$, introduced in (Buras *et al.*, 1994b), are particularly useful as they describe the apex of the standard UT as shown in Fig. 2. More details on the unitarity triangle and the generalized Wolfenstein parametrization can be found in (Battaglia *et al.*, 2003; Buras, 2003a,b, 2004; Buras *et al.*, 1994b). Below, we only recall certain expressions that we need in the course of our discussion.

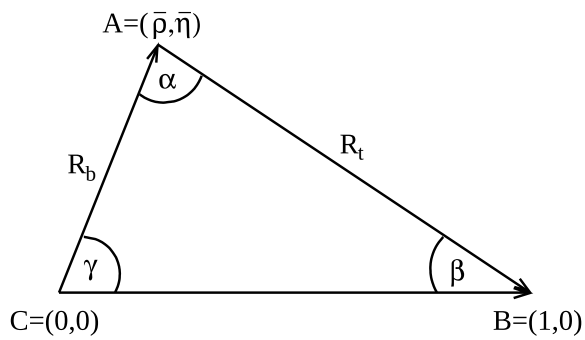

FIG. 2 Unitarity Triangle.

Parallel to the use of the parameters in (II.24) it will turn out useful to express the CKM elements V_{td} and V_{ts} as follows (Buras *et al.*, 2004c)

$$V_{td} = AR_t\lambda^3 e^{-i\beta}, \quad V_{ts} = -|V_{ts}|e^{-i\beta_s}, \tag{II.25}$$

with $\tan\beta_s \approx -\lambda^2\bar\eta$. The smallness of β_s follows from the CKM phase conventions and the unitarity of the CKM matrix. Consequently it is valid beyond the SM if three generation unitarity is assumed. R_t and β are defined in Fig. 2.

We have then

$$\lambda_t \equiv V_{ts}^* V_{td} = -\tilde r \lambda |V_{cb}|^2 R_t e^{-i\beta} e^{i\beta_s} \quad \text{with} \quad \tilde r = \left|\frac{V_{ts}}{V_{cb}}\right| = \sqrt{1+\lambda^2(2\bar\varrho-1)} \approx 0.985, \tag{II.26}$$

where in order to avoid high powers of λ we expressed the parameter A through $|V_{cb}|$. Consequently

$$\mathrm{Im}\lambda_t = \tilde r\lambda|V_{cb}|^2 R_t \sin(\beta_{\rm eff}), \qquad \mathrm{Re}\lambda_t = -\tilde r\lambda|V_{cb}|^2 R_t \cos(\beta_{\rm eff}) \tag{II.27}$$

with $\beta_{\rm eff} = \beta - \beta_s$.

Alternatively, using the parameters in (II.24), one has (Buras *et al.*, 1994b)

$$\mathrm{Im}\lambda_t = \eta\lambda|V_{cb}|^2, \qquad \mathrm{Re}\lambda_t = -(1-\frac{\lambda^2}{2})\lambda|V_{cb}|^2(1-\bar\varrho) \tag{II.28}$$

$$\mathrm{Re}\lambda_c = -\lambda(1-\frac{\lambda^2}{2})\ . \tag{II.29}$$

The expressions for $\mathrm{Im}\lambda_t$ and $\mathrm{Re}\lambda_c$ given here represent to an accuracy of 0.2% the exact formulae obtained using the standard parametrization. The expression for $\mathrm{Re}\lambda_t$ in (II.28) deviates by at most 0.5% from the exact formula in the full range of parameters considered. After inserting the expressions (II.28) and (II.29) in the exact formulae for quantities of interest a further expansion in λ should not be made.

2. Leading Strategies for $(\bar\varrho, \bar\eta)$

Next, we have the following useful relations, that correspond to the best strategies for the determination of $(\bar\varrho, \bar\eta)$ considered in (Buras *et al.*, 2003c):

(R_t, β) Strategy:

$$\bar\varrho = 1 - R_t \cos\beta, \qquad \bar\eta = R_t \sin\beta \tag{II.30}$$

with R_t determined through (II.44) below and β through $a_{\psi K_S}$. In this strategy, R_b and γ are given by

$$R_b = \sqrt{1 + R_t^2 - 2R_t \cos\beta}, \qquad \cot\gamma = \frac{1 - R_t\cos\beta}{R_t \sin\beta}. \tag{II.31}$$

(R_b, γ) Strategy:

$$\bar\varrho = R_b \cos\gamma, \qquad \bar\eta = R_b \sin\gamma \tag{II.32}$$

with γ (see Fig. 2), determined through clean strategies in tree dominated B-decays (Ali, 2003; Anikeev *et al.*, 2001; Ball *et al.*, 2000; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001). In this strategy, R_t and β are given by

$$R_t = \sqrt{1 + R_b^2 - 2R_b \cos\gamma}, \qquad \cot\beta = \frac{1 - R_b\cos\gamma}{R_b \sin\gamma}. \tag{II.33}$$

(β, γ) Strategy:

Formulae in (II.30) and

$$R_t = \frac{\sin\gamma}{\sin(\beta+\gamma)} \tag{II.34}$$

with β and γ determined through $a_{\psi K_S}$ and clean strategies for γ as in (II.32). In this strategy, the length R_b and $|V_{ub}/V_{cb}|$ can be determined through

$$R_b = \frac{\sin\beta}{\sin(\beta+\gamma)}, \qquad \left|\frac{V_{ub}}{V_{cb}}\right| = \left(\frac{\lambda}{1-\lambda^2/2}\right) R_b. \tag{II.35}$$

$(\bar\eta, \gamma)$ Strategy:

$$\bar\varrho = \frac{\bar\eta}{\tan\gamma} \tag{II.36}$$

with $\bar\eta$ determined for instance through $Br(K_{\rm L} \to \pi^0 \nu\bar\nu)$ as discussed in Section III and γ as in the two strategies above.

As demonstrated in (Buras *et al.*, 2003c), the (R_t, β) strategy will be very useful as soon as the $B_s^0 - \bar B_s^0$ mixing mass difference ΔM_s has been measured. However, the remaining three strategies turn out to be more efficient in determining $(\bar\varrho, \bar\eta)$. The strategies (β, γ) and $(\bar\eta, \gamma)$ are theoretically cleanest as β and γ can be measured precisely in two body B decays one day and $\bar\eta$ can be extracted from $Br(K_{\rm L} \to \pi^0 \nu\bar\nu)$ subject only to uncertainty in $|V_{cb}|$. Combining these two strategies offers a precise determination of the CKM matrix including $|V_{cb}|$ and $|V_{ub}|$ (Buras, 1994). On the other hand, these two strategies are subject to uncertainties coming from new physics that can enter through β and $\bar\eta$. The angle γ, the phase of V_{ub}, can be determined in principle without these uncertainties.

The strategy (R_b, γ), on the other hand, while subject to hadronic uncertainties in the determination of R_b, is not polluted by new physics contributions as, in addition to γ, also R_b can be determined from tree level decays. This strategy results in the so-called *reference unitarity triangle* as proposed and discussed in (Barenboim *et al.*, 1999; Cohen *et al.*, 1997; Goto *et al.*, 1996; Grossman *et al.*, 1997). We will return to all these strategies in the course of our presentation.

3. Constraints from the Standard Analysis of the UT

Other useful expressions that represent the constraints from the CP-violating parameter ε_K and $\Delta M_{s,d}$, that parametrize the size of $B^0_{s,d} - \bar{B}^0_{s,d}$ mixings are as follows.

First we have

$$\varepsilon_K = -C_\varepsilon \hat{B}_K \mathrm{Im}\lambda_t \left\{ \lambda^4 \mathrm{Re}\lambda_c P_c(\varepsilon) + \mathrm{Re}\lambda_t \eta_2^{QCD} S_0(x_t) \right\} e^{i\pi/4} \,, \tag{II.37}$$

where $S_0(x_t) = 2.42 \pm 0.09$ results from $\Delta S = 2$ box diagrams and the numerical constant C_ε is given by ($M_W = 80.4\,\mathrm{GeV}$)

$$C_\varepsilon = \frac{G_{\rm F}^2 F_K^2 m_K M_{\rm W}^2}{6\sqrt{2}\pi^2 \Delta M_K} = 3.837 \cdot 10^4 \,. \tag{II.38}$$

Next (Herrlich and Nierste, 1994, 1995, 1996; Jamin and Nierste, 2004),

$$P_c(\varepsilon) = \frac{\bar{P}_c(\varepsilon)}{\lambda^4} = (0.29 \pm 0.07) \left[\frac{0.224}{\lambda}\right]^4, \qquad \bar{P}_c(\varepsilon) = (7.3 \pm 1.7) \cdot 10^{-4}, \tag{II.39}$$

$\eta_2^{QCD} = 0.574 \pm 0.003$ (Buchalla *et al.*, 1996a; Buras, 1998; Buras *et al.*, 1990) and $\hat{B}_K$ is a non-perturbative parameter. In obtaining (II.37) a small term amounting to at most 5% correction to ε_K has been neglected. This is justified in view of other uncertainties, in particular those connected with $\hat{B}_K$ but in the future should be taken into account (Andriyash *et al.*, 2003).

Comparing (II.37) with the experimental value for ε_K (Hagiwara *et al.*, 2002)

$$(\varepsilon_K)_{exp} = (2.280 \pm 0.013) \cdot 10^{-3} \ \exp i\pi/4, \tag{II.40}$$

one obtains a constraint on the UT that with the help of (II.28) and (II.29) can be cast into

$$\bar{\eta} \left[(1 - \bar{\varrho}) |V_{cb}|^2 \eta_2^{QCD} S_0(x_t) + \bar{P}_c(\varepsilon) \right] |V_{cb}|^2 \hat{B}_K = 1.184 \cdot 10^{-6} \left[\frac{0.224}{\lambda}\right]^2 . \tag{II.41}$$

Next, the constraint from ΔM_d implies

$$R_t = \frac{1}{\lambda} \frac{|V_{td}|}{|V_{cb}|} = 0.834 \cdot \left[\frac{|V_{td}|}{7.75 \cdot 10^{-3}}\right] \left[\frac{0.0415}{|V_{cb}|}\right] \left[\frac{0.224}{\lambda}\right], \tag{II.42}$$

$$|V_{td}| = 7.75 \cdot 10^{-3} \left[\frac{230\,\mathrm{MeV}}{\sqrt{\hat{B}_{B_d}} F_{B_d}}\right] \sqrt{\frac{\Delta M_d}{0.50/\mathrm{ps}}} \sqrt{\frac{0.55}{\eta_B^{QCD}}} \sqrt{\frac{2.40}{S_0(x_t)}} \,. \tag{II.43}$$

Here $\sqrt{\hat{B}_{B_d}} F_{B_d}$ is a non-perturbative parameter and $\eta_B^{QCD} = 0.551 \pm 0.003$ the QCD correction (Buras *et al.*, 1990; Urban *et al.*, 1998).

Finally, the simultaneous use of ΔM_d and ΔM_s gives

$$R_t = 0.920\, \tilde{r} \left[\frac{\xi}{1.24}\right] \left[\frac{0.224}{\lambda}\right] \sqrt{\frac{18.4/ps}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.50/ps}}, \qquad \xi = \frac{\sqrt{\hat{B}_{B_s}} F_{B_s}}{\sqrt{\hat{B}_{B_d}} F_{B_d}} \tag{II.44}$$

with $\tilde{r}$ defined in (II.26) and ξ standing for a nonperturbative parameter that is subject to smaller theoretical uncertainties than the individual $\sqrt{\hat{B}_{B_d}} F_{B_d}$ and $\sqrt{\hat{B}_{B_s}} F_{B_s}$.

The main uncertainties in these constraints originate in the theoretical uncertainties in $\hat{B}_K$ and $\sqrt{\hat{B}_d} F_{B_d}$ and to a lesser extent in ξ (Battaglia *et al.*, 2003):

$$\hat{B}_K = 0.86 \pm 0.15, \quad \sqrt{\hat{B}_d} F_{B_d} = (235^{+33}_{-41})\ \mathrm{MeV}, \quad \xi = 1.24 \pm 0.08 \,. \tag{II.45}$$

The QCD sum rules results for the parameters in question are similar and can be found in (Battaglia *et al.*, 2003). Finally (Battaglia *et al.*, 2003)

$$\Delta M_d = (0.503 \pm 0.006)/\mathrm{ps}, \qquad \Delta M_s > 14.4/\mathrm{ps} \ \text{ at } \ 95\% \text{ C.L.} \tag{II.46}$$

Extensive discussion of the formulae (II.37), (II.41), (II.43) and (II.44) can be found in (Battaglia *et al.*, 2003). For our numerical analysis, we will use (Battaglia *et al.*, 2003)

$$\lambda = 0.2240 \pm 0.0036, \quad A = 0.827 \pm 0.016, \quad |V_{cb}| = (41.5 \pm 0.8) \cdot 10^{-3}, \tag{II.47}$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.092 \pm 0.014, \qquad R_b = 0.40 \pm 0.06 \tag{II.48}$$

$$\beta = (23.7 \pm 2.1)^\circ, \quad \beta_s = -1^\circ \tag{II.49}$$

with the value of β determined from measurements of the time-dependent CP asymmetry $a_{\psi K_S}(t)$ that give (Abe *et al.*, 2002; Aubert *et al.*, 2002a; Browder, 2004)

$$(\sin 2\beta)_{\psi K_S} = 0.736 \pm 0.049 \ . \tag{II.50}$$

It should be emphasized that the inputs in (II.47)–(II.49) are independent from each other, except for β_s and the upper bound (Buras *et al.*, 1994b)

$$(\sin 2\beta)_{\max} = 2R_b^{\max}\sqrt{1-(R_b^{\max})^2} \qquad \text{or} \qquad (\sin\beta)_{\max} = R_b^{\max} \tag{II.51}$$

that both follow from the unitarity of the CKM matrix.

III. PHENOMENOLOGICAL APPLICATIONS IN THE SM

A. Preliminaries

During the last ten years several analyses of $K \to \pi\nu\bar{\nu}$ decays within the SM were presented, in particular in (Buchalla and Buras, 1999; Buras, 2003a,b, 2004; D'Ambrosio and Isidori, 2002; Kettell *et al.*, 2002). Moreover, correlations with other decays have been pointed out (Bergmann and Perez, 2000, 2001; Buras *et al.*, 2000; Buras and Silvestrini, 1999). In this section we collect and update many of these formulae and derive a number of useful expressions that are new. In the next section a detailed numerical analysis of these formulae will be presented. Unless explicitely stated all the formulae below are given for $\lambda = 0.224$. The dependence on λ can easily be found from the formulae of the previous section. When it is introduced, it is often useful to replace $\lambda^2 A$ by $|V_{cb}|$ to avoid high powers of λ. On the whole, the issue of the error in λ in $K \to \pi\nu\bar{\nu}$ decays is really not an issue if changes are made consistently in all places as emphasized before.

B. Unitarity Triangle and $K^+ \to \pi^+\nu\bar{\nu}$

1. Basic Formulae

Using (II.27) in (II.2) we obtain (Buras *et al.*, 2004c)

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \left[\tilde{r}^2 A^4 R_t^2 X^2(x_t) + 2\tilde{r}\bar{P}_c(X) A^2 R_t X(x_t)\cos\beta_{\rm eff} + \bar{P}_c(X)^2\right] \tag{III.1}$$

with $\beta_{\rm eff} = \beta - \beta_s$, $\tilde{r}$ given in (II.26) and

$$\bar{P}_c(X) = \left(1 - \frac{\lambda^2}{2}\right) P_c(X). \tag{III.2}$$

In the context of the unitarity triangle also the expression following from (II.2) and (II.28) is useful (Buras *et al.*, 1994b)

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2(x_t) \frac{1}{\sigma}\left[(\sigma\bar{\eta})^2 + (\varrho_c - \bar{\varrho})^2\right] , \tag{III.3}$$

where

$$\sigma = \left(\frac{1}{1-\frac{\lambda^2}{2}}\right)^2 . \tag{III.4}$$

The measured value of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ then determines an ellipse in the $(\bar{\varrho}, \bar{\eta})$ plane centered at $(\varrho_c, 0)$ (see Fig. 3) with

$$\varrho_c = 1 + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X(x_t)} \tag{III.5}$$

and having the squared axes

$$\bar{\varrho}_1^2 = r_0^2, \qquad \bar{\eta}_1^2 = \left(\frac{r_0}{\sigma}\right)^2 , \tag{III.6}$$

where

$$r_0^2 = \left[\frac{\sigma \cdot Br(K^+ \to \pi^+ \nu\bar{\nu})}{\bar{\kappa}_+ |V_{cb}|^4 X^2(x_t)}\right] . \tag{III.7}$$

Note that r_0 depends only on the top contribution. The departure of ϱ_c from unity measures the relative importance of the internal charm contributions. $\varrho_c \approx 1.37$.

Imposing then the constraint from $|V_{ub}/V_{cb}|$ allows to determine $\bar{\varrho}$ and $\bar{\eta}$ with

$$\bar{\varrho} = \frac{1}{1-\sigma^2}\left(\varrho_c - \sqrt{\sigma^2 \varrho_c^2 + (1-\sigma^2)(r_0^2 - \sigma^2 R_b^2)}\right), \qquad \bar{\eta} = \sqrt{R_b^2 - \bar{\varrho}^2} \tag{III.8}$$

where $\bar{\eta}$ is assumed to be positive. Consequently

$$R_t^2 = 1 + R_b^2 - 2\bar{\varrho}, \qquad V_{td} = A\lambda^3(1 - \bar{\varrho} - i\bar{\eta}), \qquad |V_{td}| = A\lambda^3 R_t. \tag{III.9}$$

The determination of $|V_{td}|$ and of the unitarity triangle in this way requires the knowledge of $|V_{cb}|$ (or A) and of $|V_{ub}/V_{cb}|$. Both values are subject to theoretical uncertainties present in the existing analyses of tree level decays (Battaglia *et al.*, 2003). Whereas the dependence on $|V_{ub}/V_{cb}|$ is rather weak, the very strong dependence of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ on A or $|V_{cb}|$, as seen in (III.1) and (III.3), made in the past a precise prediction for this branching ratio and the construction of the UT difficult. With the more accurate value of $|V_{cb}|$ obtained recently (Battaglia *et al.*, 2003) and given in (II.47), the situation improved significantly. We will return to this in Section IV. The dependence of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ on $m_{\rm t}$ is also strong. However, $m_{\rm t}$ is known already within $\pm 2.3\%$ and consequently the related uncertainty in $Br(K^+ \to \pi^+ \nu\bar{\nu})$ is substantially smaller than the corresponding uncertainty due to $|V_{cb}|$. Moreover, in this decade the error on $m_{\rm t}$ should be decreased down to $\pm 1\,\mathrm{GeV}$ making this uncertainty negligible.

As $|V_{ub}/V_{cb}|$ is subject to theoretical uncertainties, a cleaner strategy is to use $Br(K^+ \to \pi^+ \nu\bar{\nu})$ in conjunction with β determined through the mixing induced CP asymmetry $a_{\psi K_S}$. We will investigate this strategy in the next section.

2. $Br(K^+ \to \pi^+ \nu\bar{\nu})$, β, $\Delta M_d/\Delta M_s$ or γ.

In (Buchalla and Buras, 1999) an upper bound on $Br(K^+ \to \pi^+ \nu\bar{\nu})$ has been derived within the SM. This bound depends only on $|V_{cb}|$, X, ξ and $\Delta M_d/\Delta M_s$. With the precise value for the angle β now available this bound can be turned into a useful formula for $Br(K^+ \to \pi^+ \nu\bar{\nu})$ (D'Ambrosio and Isidori, 2002) that expresses this branching ratio in terms of theoretically clean observables. In the SM and any MFV model this formula reads:

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) = \bar{\kappa}_+ \, |V_{cb}|^4 X^2 \left[\sigma R_t^2 \sin^2\beta + \frac{1}{\sigma}\left(R_t \cos\beta + \frac{\lambda^4 P_c(X)}{|V_{cb}|^2 X}\right)^2\right], \tag{III.10}$$

with σ defined in (III.4) and $\bar{\kappa}_+$ given in (II.5). It can be considered as the fundamental formula for a correlation between $Br(K^+ \to \pi^+ \nu\bar{\nu})$, β and any observable used to determine R_t. This formula is theoretically very clean with the uncertainties residing only in $|V_{cb}|$ and $P_c(X)$. However, when one relates R_t to some observable new uncertainties could enter. In (Buchalla and Buras, 1999) and (D'Ambrosio and Isidori, 2002) it has been proposed to express R_t through $\Delta M_d/\Delta M_s$ by means of (II.44). This implies an additional uncertainty due to the value of ξ in (II.45).

Here we would like to point out that if the strategy (β,γ) is used to determine R_t by means of (II.34), the resulting formula that relates $Br(K^+\to\pi^+\nu\bar\nu)$, β and γ is even cleaner than the one that relates $Br(K^+\to\pi^+\nu\bar\nu)$, β and $\Delta M_d/\Delta M_s$. We have then

$$Br(K^+\to\pi^+\nu\bar\nu)=\bar\kappa_+\,|V_{cb}|^4X^2\left[\sigma T_1^2+\frac{1}{\sigma}\left(T_2+\frac{\lambda^4P_c(X)}{|V_{cb}|^2X}\right)^2\right], \tag{III.11}$$

where

$$T_1=\frac{\sin\beta\sin\gamma}{\sin(\beta+\gamma)},\qquad T_2=\frac{\cos\beta\sin\gamma}{\sin(\beta+\gamma)}. \tag{III.12}$$

Similarly, the following formulae for R_t could be used in conjunction with (III.10)

$$R_t=\frac{\tilde r}{\lambda}\sqrt{\frac{Br(B\to X_d\nu\bar\nu)}{Br(B\to X_s\nu\bar\nu)}}, \tag{III.13}$$

$$R_t=\frac{\tilde r}{\lambda}\sqrt{\frac{\tau(B_s)}{\tau(B_d)}\frac{m_{B_s}}{m_{B_d}}}\left[\frac{F_{B_s}}{F_{B_d}}\right]\sqrt{\frac{Br(B_d\to\mu^+\mu^-)}{Br(B_s\to\mu^+\mu^-)}}. \tag{III.14}$$

In particular, (III.13) is essentially free of hadronic uncertainties (Buchalla and Isidori, 1998) and (III.14), not involving $\hat B_{B_s}/\hat B_{B_d}$, is a bit cleaner than (II.44).

C. $K_{\rm L}\to\pi^0\nu\bar\nu$, $\bar\eta$, ${\rm Im}\lambda_t$ and the (β,γ) Strategy

1. $\bar\eta$ and ${\rm Im}\lambda_t$

Using (II.18) and (II.27) we find

$$Br(K_{\rm L}\to\pi^0\nu\bar\nu)=\kappa_L\tilde r^2A^4R_t^2X^2(x_t)\sin^2\beta_{\rm eff}. \tag{III.15}$$

In the context of the unitarity triangle the expression following from (II.18) and (II.28) is useful:

$$Br(K_{\rm L}\to\pi^0\nu\bar\nu)=\bar\kappa_L\eta^2|V_{cb}|^4X^2(x_t),\qquad \bar\kappa_L=\frac{\kappa_L}{\lambda^8}=(3.34\pm0.05)\cdot10^{-5} \tag{III.16}$$

from which $\bar\eta$ can be determined

$$\bar\eta=0.351\sqrt{\frac{3.34\cdot10^{-5}}{\bar\kappa_L}}\left[\frac{1.53}{X(x_t)}\right]\left[\frac{0.0415}{|V_{cb}|}\right]^2\sqrt{\frac{Br(K_{\rm L}\to\pi^0\nu\bar\nu)}{3\cdot10^{-11}}}. \tag{III.17}$$

The determination of $\bar\eta$ in this manner requires the knowledge of $|V_{cb}|$ and $m_{\rm t}$. With the improved determination of these two parameters a useful determination of $\bar\eta$ should be possible.

On the other hand, the uncertainty due to $|V_{cb}|$ is not present in the determination of ${\rm Im}\lambda_t$ as (Buchalla and Buras, 1996):

$${\rm Im}\lambda_t=1.39\cdot10^{-4}\left[\frac{\lambda}{0.224}\right]\sqrt{\frac{3.34\cdot10^{-5}}{\bar\kappa_L}}\left[\frac{1.53}{X(x_t)}\right]\sqrt{\frac{Br(K_{\rm L}\to\pi^0\nu\bar\nu)}{3\cdot10^{-11}}}. \tag{III.18}$$

This formula offers the cleanest method to measure ${\rm Im}\lambda_t$ in the SM and all MFV models in which the function X takes generally different values than $X(x_t)$. This determination is even better than the one with the help of the CP asymmetries in B decays that require the knowledge of $|V_{cb}|$ to determine ${\rm Im}\lambda_t$. Measuring $Br(K_{\rm L}\to\pi^0\nu\bar\nu)$ with 10% accuracy allows to determine ${\rm Im}\lambda_t$ with an error of 5% (Buchalla and Buras, 1996; Buchalla *et al.*, 1996a; Buras, 1998).

The importance of the precise measurement of ${\rm Im}\lambda_t$ is clear: the areas A_Δ of all unitarity triangles are equal and related to the measure of CP violation $J_{\rm CP}$ (Jarlskog, 1985a,b):

$$|J_{\rm CP}|=2A_\Delta=\lambda\left(1-\frac{\lambda^2}{2}\right)|{\rm Im}\lambda_t|. \tag{III.19}$$

2. A New "Golden Relation"

Next, in the spirit of the analysis in (Buras, 1994) we can use the clean CP asymmetries in B decays and determine $\bar{\eta}$ through the (β,γ) strategy. Using (II.30) and (II.34) in (III.17) we obtain a new "golden relation"

$$\frac{\sin\beta\sin\gamma}{\sin(\beta+\gamma)}=0.351\sqrt{\frac{3.34\cdot 10^{-5}}{\bar{\kappa}_L}}\left[\frac{1.53}{X(x_t)}\right]\left[\frac{0.0415}{|V_{cb}|}\right]^2\sqrt{\frac{Br(K_{\rm L}\to\pi^0\nu\bar{\nu})}{3\cdot 10^{-11}}}. \tag{III.20}$$

This relation between β, γ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, is very clean and offers an excellent test of the SM and of its extensions. Similarly to the "golden relation" in (I.1) it connects the observables in B decays with those in K decays. Moreover, it has the following two important virtues:

- It allows to determine $|X|$;

$$|X|=F_1(\beta,\gamma,|V_{cb}|,Br(K_L)) \tag{III.21}$$

  with $Br(K_L)=Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$. The analytic expression for the function F_1 can easily be extracted from (III.20).

- As $X(x_t)$ should be known with high precision once the error on m_t has been decreased, the relation (III.20) allows to determine $|V_{cb}|$ with a remarkable precision (Buras, 1994)

$$|V_{cb}|=F_2(\beta,\gamma,X,Br(K_L)). \tag{III.22}$$

  The analytic formula for F_2 can easily be obtained from (III.20).

At first sight one could question the usefulness of the determination of $|V_{cb}|$ in this manner, since it is usually determined from tree level B decays. On the other hand, one should realize that one determines here actually the parameter A in the Wolfenstein parametrization that enters the elements V_{ub}, V_{cb}, V_{ts} and V_{td} of the CKM matrix. Moreover this determination of A benefits from the very weak dependence on $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, which is only with a power of 0.25. The weak point of this determination of $|V_{cb}|$ is the pollution from new physics that could enter through the function X, whereas the standard determination of $|V_{cb}|$ through tree level B decays is free from this dependence. Still, a determination of $|V_{cb}|$ that in precision can almost compete with the usual tree diagrams determinations and is theoretically cleaner, is clearly of interest within the SM. We will give some numerical examples in Section IV.

D. Unitarity Triangle from $K^+\to\pi^+\nu\bar{\nu}$ and $K_{\rm L}\to\pi^0\nu\bar{\nu}$

The measurement of $Br(K^+\to\pi^+\nu\bar{\nu})$ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ can determine the unitarity triangle completely (see Fig. 3), provided $m_{\rm t}$ and $|V_{cb}|$ are known (Buchalla and Buras, 1994b). Using these two branching ratios simultaneously allows to eliminate $|V_{ub}/V_{cb}|$ from the analysis which removes a considerable uncertainty in the determination of the UT, even if it is less important for $|V_{td}|$. Indeed it is evident from (II.2) and (II.18) that, given $Br(K^+\to\pi^+\nu\bar{\nu})$ and $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$, one can extract both ${\rm Im}\lambda_t$ and ${\rm Re}\lambda_t$. One finds (Buchalla and Buras, 1994b; Buchalla *et al.*, 1996a; Buras, 1998)

$${\rm Im}\lambda_t=\lambda^5\frac{\sqrt{B_2}}{X(x_t)},\qquad {\rm Re}\lambda_t=-\lambda^5\frac{\frac{{\rm Re}\lambda_c}{\lambda}P_c(X)+\sqrt{B_1-B_2}}{X(x_t)}, \tag{III.23}$$

where we have defined the "reduced" branching ratios

$$B_1=\frac{Br(K^+\to\pi^+\nu\bar{\nu})}{\kappa_+},\qquad B_2=\frac{Br(K_{\rm L}\to\pi^0\nu\bar{\nu})}{\kappa_L}. \tag{III.24}$$

Using next the expressions for ${\rm Im}\lambda_t$, ${\rm Re}\lambda_t$ and ${\rm Re}\lambda_c$ given in (II.28) and (II.29) one finds

$$\bar{\varrho}=1+\frac{P_c(X)-\sqrt{\sigma(B_1-B_2)}}{A^2X(x_t)},\qquad \bar{\eta}=\frac{\sqrt{B_2}}{\sqrt{\sigma}A^2X(x_t)} \tag{III.25}$$

with σ defined in (III.4). An exact treatment of the CKM matrix shows that the formulae (III.25), in particular the one for $\bar{\eta}$, are rather precise (Buchalla and Buras, 1994b).

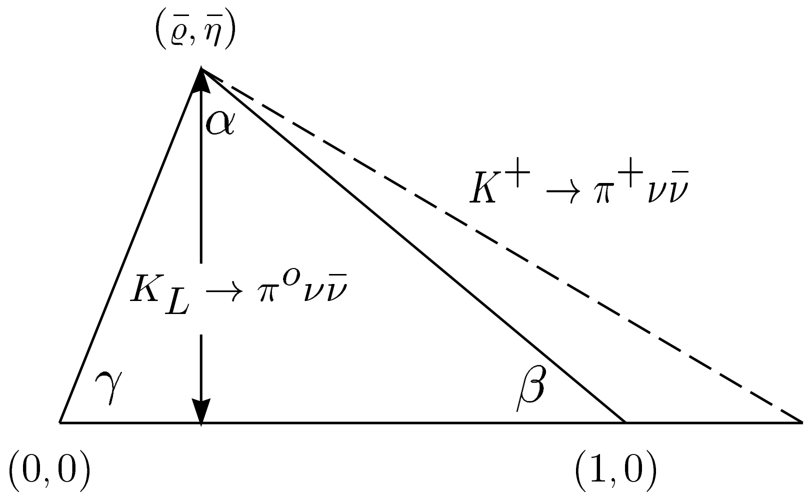

FIG. 3 Unitarity triangle from $K \to \pi\nu\bar{\nu}$.

E. $\sin 2\beta$ from $K \to \pi\nu\bar{\nu}$

Using (III.25) one finds subsequently (Buchalla and Buras, 1994b)

$$\sin 2\beta = \frac{2r_s}{1+r_s^2}, \qquad r_s = \sqrt{\sigma}\frac{\sqrt{\sigma(B_1 - B_2)} - P_c(X)}{\sqrt{B_2}} = \cot\beta. \tag{III.26}$$

Thus, within the approximation of (III.25), $\sin 2\beta$ is independent of V_{cb} (or A) and m_t and as we will see in Section IV these dependences are fully negligible.

It should be stressed that $\sin 2\beta$ determined this way depends only on two measurable branching ratios and on the parameter $P_c(X)$ which is completely calculable in perturbation theory as discussed in the previous section. Consequently this determination is free from any hadronic uncertainties and its accuracy can be estimated with a high degree of confidence. The calculation of NNLO QCD corrections to $P_c(X)$ would certainly improve the accuracy of the determination of $\sin 2\beta$ from the $K \to \pi\nu\bar{\nu}$ complex.

Alternatively, combining (III.1) and (III.15), one finds (Buras *et al.*, 2004c)

$$\sin 2\beta_{\rm eff} = \frac{2\bar{r}_s}{1+\bar{r}_s^2}, \qquad \bar{r}_s = \frac{\sqrt{B_1 - B_2} - \bar{P}_c(X)}{\sqrt{B_2}} = \cot\beta_{\rm eff} \tag{III.27}$$

where $\beta_{\rm eff} = \beta - \beta_s$. As $\beta_s = \mathcal{O}(\lambda^2)$, we have

$$\cot\beta = \sigma\cot\beta_{\rm eff} + \mathcal{O}(\lambda^2) \tag{III.28}$$

and consequently one can verify that (III.27), while being slightly more accurate, is numerically very close to (III.26). This formula turns out to be more useful than (III.26) when SM extensions with new complex phases in X are considered. We will return to it in Section VI.

Finally, as in the SM and more generally in all MFV models there are no phases beyond the CKM phase, the MFV relation (I.1) should be satisfied. The confirmation of this relation would be a very important test for the MFV idea. Indeed, in $K \to \pi\nu\bar{\nu}$ the phase β originates in the Z^0 penguin diagram, whereas in the case of $a_{\psi K_S}$ in the $B_d^0 - \bar{B}_d^0$ box diagram. We will discuss the violation of this relation in particular new physics scenarios in Sections VI and VII.

F. The Angle γ from $K \to \pi\nu\bar{\nu}$

We have seen that a precise value of β can be obtained both from the CP asymmetry $a_{\psi K_S}$ and from the $K \to \pi\nu\bar{\nu}$ complex in a theoretically clean manner. The determination of the angle γ is much harder. As briefly discussed in Section VIII and in great detail in (Ali, 2003; Buchalla, 2003; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001), there are several strategies for γ in B decays but only few of them can be considered as theoretically clean. They all are experimentally very challenging and a determination of γ with a precision of better than $\pm 5^\circ$ from these strategies alone will only be possible at LHCB and BTeV after a few years of running (Anikeev *et al.*, 2001; Ball *et al.*, 2000).

Here, we would like to point out that the $K \to \pi\nu\bar{\nu}$ decays offer a clean determination of γ that in accuracy can compete with the strategies in B decays, provided the uncertainties present in $|V_{cb}|$ and in the m_t can be further reduced and the two branching ratios measured with an accuracy of 5%.

The relevant formula, that has not been presented in the literature so far, can be directly obtained from (III.25). It reads

$$\cot\gamma = \sqrt{\frac{\sigma}{B_2}}\left(A^2 X(x_t) - \sqrt{\sigma(B_1 - B_2)} + P_c(X)\right). \tag{III.29}$$

We will investigate it numerically in Section IV.

G. A Second Route to UT from $K \to \pi\nu\bar{\nu}$

Instead of using the formulae for $\mathrm{Im}\lambda_t$ and $\mathrm{Re}\lambda_t$ in (III.23), it is instructive to construct the UT by using (III.27) to find β and subsequently determine R_t from (III.1) with the result

$$R_t = \frac{\sqrt{B_1 - \bar{P}_c^2 \sin^2\beta_{\rm eff}} - \bar{P}_c\cos\beta_{\rm eff}}{\tilde{r}A^2 X(x_t)}. \tag{III.30}$$

This (R_t, β) strategy by means of $K \to \pi\nu\bar{\nu}$ decays gives then $(\bar{\varrho}, \bar{\eta})$ as given in (II.30) and in particular

$$\cot\gamma = \frac{1 - R_t\cos\beta}{R_t\sin\beta}. \tag{III.31}$$

IV. NUMERICAL ANALYSIS IN THE SM

A. Introducing Scenarios

In our numerical analysis we will consider various scenarios for the CKM elements and the values of the branching ratios $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ that should be measured in the future. In choosing the values of these branching ratios we will be guided in this Section by their values predicted in the SM. We will consider then

- Scenario A for the present elements of the CKM matrix and future scenarios B and C with improved elements of the CKM matrix and the improved value of P_c through the inclusion of NNLO QCD corrections that should be available already this year (Buras *et al.*, 2004b). They are summarized in table III. The accuracy on β in table III corresponds to the error in $\sin 2\beta$ of ± 0.025 and ± 0.012 for scenarios B and C, respectively. It should be achieved respectively at B factories, and LHCB (BTeV). As discussed recently (Boos *et al.*, 2004), even at this level of experimental precision, theoretical uncertainties in the determination of β through $a_{\psi K_S}$ can be neglected. The accuracy on γ given in table III in scenarios B and C can presumably be achieved through the clean tree diagrams strategies in B decays that will only become effective at LHC and BTeV. We will briefly discuss them in Section VIII.
- Scenarios I and II for the measurements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ that together with future values of $|V_{cb}|$, m_t and P_c should allow the determination of the UT, that is of the angles β and γ and of the sides R_b and R_t, from $K \to \pi\nu\bar{\nu}$ alone. These scenarios are summarized in table IV. Scenario I corresponds to the end of this decade, while Scenario II is more futuristic.

A comment on the values of $\hat{B}_K$ and $\sqrt{\hat{B}_d}F_{B_d}$ in Scenario C in table III is in order. Keeping the present central values of these two parameters but decreasing the errors results in a poor fit. The fit is considerably improved by lowering the values of these two parameters. This shift is consistent with the exercise made in (Battaglia *et al.*, 2003; Stocchi, 2004) in which $\hat{B}_K$ and $\sqrt{\hat{B}_d}F_{B_d}$ were considered separately as free parameters and their values have been obtained from the UT fit, albeit with larger errors than assumed by us in Scenario C.

In the rest of the review we will frequently refer to tables III and IV indicating which observables listed there are used at a given time in our numerical calculations.

TABLE III Input for the determination of the branching ratios $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ in three scenarios. The corresponding $(\bar{\varrho}, \bar{\eta})$ are given too.

	Scenario A	Scenario B	Scenario C
β	$(23.7 \pm 2.1)^\circ$	$(23.5 \pm 1.0)^\circ$	$(23.5 \pm 0.5)^\circ$
γ	$(63.0 \pm 6.0)^\circ$	$(63.0 \pm 5.0)^\circ$	$(63.0 \pm 2.0)^\circ$
$\|V_{cb}\|/10^{-3}$	41.5 ± 0.8	41.5 ± 0.6	41.5 ± 0.4
R_b	0.40 ± 0.06	0.40 ± 0.03	0.40 ± 0.01
m_t[GeV]	168.1 ± 4.1	168 ± 3	168 ± 1
$P_c(X)$	0.39 ± 0.07	0.39 ± 0.03	0.39 ± 0.02
$\hat{B}_K$	0.86 ± 0.15	0.86 ± 0.08	0.75 ± 0.04
$\sqrt{\hat{B}_d} F_{B_d}$ [MeV]	235^{+33}_{-41}	235 ± 20	215 ± 10
ξ	1.24 ± 0.08	1.24 ± 0.04	1.24 ± 0.02
$\Delta M_s [ps^{-1}]$	≥ 14.4	18.5 ± 1.0	18.5 ± 0.2
$\bar{\eta}$	0.354 ± 0.027	0.340 ± 0.009	0.358 ± 0.007
$\bar{\varrho}$	0.187 ± 0.059	0.209 ± 0.017	0.182 ± 0.011

TABLE IV Input for the determination of CKM parameters from $K \to \pi\nu\bar{\nu}$ in two scenarios.

	Scenario I	Scenario II
$Br(K^+ \to \pi^+\nu\bar{\nu})/10^{-11}$	8.0 ± 0.8	8.0 ± 0.4
$Br(K_{\rm L} \to \pi^0\nu\bar{\nu})/10^{-11}$	3.0 ± 0.3	3.0 ± 0.15
m_t[GeV]	168 ± 3	168 ± 1
$P_c(X)$	0.39 ± 0.03	0.39 ± 0.02
$\|V_{cb}\|/10^{-3}$	41.5 ± 0.6	41.5 ± 0.4

B. The UT Fit

In order to predict branching ratios in the SM and to investigate various strategies later on, it will be useful to have at hand the results of a standard analysis of the UT that uses the available constraints from $|V_{ub}/V_{cb}|$, $|V_{cb}|$, ε_K, $\Delta M_{s,d}$ and $(\sin 2\beta)_{\psi K_S}$. The relevant expressions and input parameters have been collected in Section II.E.3. The latter ones are represented by the scenario A in table III.

The best fit values of $\bar{\rho}$ and $\bar{\eta}$ were obtained by minimizing the χ^2 function:

$$\chi^2 = \chi_1^2 + \chi_2^2, \tag{IV.1}$$

where

$$\chi_1^2 = \sum_i \frac{(x_i^{theo} - x_i^{exp})^2}{\sigma_i^2} \tag{IV.2}$$

and

$$\chi_2^2 = 2 \cdot \left[\mathrm{Erfc}^{-1} \left(\frac{1}{2} \mathrm{Erfc} \left(\frac{1 - \mathrm{A}}{\sqrt{2}\sigma_{\mathrm{A}}} \right) \right) \right]^2 . \tag{IV.3}$$

χ_1^2 includes the constraints from $\sin 2\beta$, ΔM_d, ε_K and R_b. The σ_i in χ_1^2 are the gaussian errors. The smallest value of χ^2 was found by also taking into account the flat errors of $\hat{B}_K$, $\sqrt{\hat{B}_d} F_{B_d}$ and ξ, which were scanned in their ranges. χ_2^2 corresponds to $\Delta M_d / \Delta M_s$, which is implemented using the amplitude method (Moser and Roussarie, 1997), where A is the amplitude that describes the bound on ΔM_s and $\sigma_{\mathbf{A}}$ is its uncertainty. For scenario A the minimal χ^2 of our fit was $\chi^2_{min} = 0.427$.

In the SM column of table V, we show the results for various quantities of interest that we obtained from the present analysis. They are close to the ones found in (Buras *et al.*, 2003c) and recently in (Stocchi, 2004), although the errors found by us are slightly larger than in these two papers due to a more conservative error on R_b used here. Similarly our central values for all observables are very close to the ones given in table 4 of (Ali, 2003).

The 68% C.L. region for $(\bar{\varrho}, \bar{\eta})$, the area inside the ellipse, is also shown in the upper part of Fig. 5 to which we will return later in this section. The UUT column will be discussed in the next section. The SM results in table V follow from all constraints of Section II.E, including the direct measurement of $\sin 2\beta$ in (II.50). The resulting value for $\sin 2\beta$ remains essentially unchanged from the direct measurement in (II.50) mainly because with a conservative error on R_b in scenario A this constraint has essentially no impact on the final value of $\sin 2\beta$.

Figure 5 demonstrates an excellent agreement between the direct measurement in (II.50) and the standard analysis of the UT within the SM that uses the ε_K, R_b and $\Delta M_d/\Delta M_s$ constraints, shown in the plot, and the ΔM_d constraint that we do not show to avoid too many lines. This gives a strong indication that the CKM matrix is very likely the dominant source of CP violation in flavour violating decays but as we will see in Section VI, a significant impact on the UT from new physics contributions is still possible. Certainly, the future measurements of various observables will shed light on the dominance of the CKM scenario. We refer to (Ali, 2003; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001) for reviews of other methods relevant for the determination of the UT. We will briefly review them in Section VIII.

TABLE V Values for different quantities from the UT fit. $\lambda_t = V_{ts}^* V_{td}$.

Strategy	SM	UUT
$\bar{\eta}$	0.354 ± 0.027	0.360 ± 0.031
$\bar{\varrho}$	0.187 ± 0.059	0.174 ± 0.068
$\sin 2\beta$	0.732 ± 0.049	0.735 ± 0.049
β	$(23.5 \pm 2.1)^\circ$	$(23.7 \pm 2.1)^\circ$
γ	$(62.2 \pm 8.2)^\circ$	$(64.2 \pm 9.6)^\circ$
R_b	0.400 ± 0.039	0.40 ± 0.044
R_t	0.887 ± 0.056	0.901 ± 0.064
$\lvert V_{td}\rvert/10^{-3}$	8.24 ± 0.54	8.38 ± 0.62
$\mathrm{Im}\lambda_t\ /10^{-4}$	1.40 ± 0.12	1.43 ± 0.14
$\mathrm{Re}\lambda_t\ /10^{-4}$	$-(3.06 \pm 0.25)$	$-(3.11 \pm 0.28)$

At this stage we would like to make only the following observations:

- The region $\bar{\varrho} < 0$ is disfavoured by both the value of ΔM_d and the lower bound on ΔM_s that through (II.44) puts an upper bound on R_t. Clearly the measurement of ΔM_s giving R_t through (II.44) will have a large impact on the results in table V provided the error in ξ can be reduced significantly.
- With $R_b^{\max} = 0.46$ from (II.48) we find, using (II.51)

$$\beta \leq 27.4^\circ \tag{IV.4}$$

  that should be compared with $\beta = (23.7 \pm 2.1)^\circ$ following from the direct measurement of $\sin 2\beta$. This means that the direct measurement implies a value of β that is rather close to the upper bound (II.51) that follows from the unitarity of the CKM matrix.
- This fact has the following implication: In extensions of the SM in which a new complex phase θ_d is present in the $B_d^0 - \bar{B}_d^0$ mixing and in which the CP asymmetry $a_{\psi K_S}$ measures $\beta + \theta_d$ and not β, the true value of β is likely to be lower than its value in the SM with $\theta_d > 0$ and not necessarily small. In this case the relation (II.44) between R_t and $\Delta M_d/\Delta M_s$ is likely to be simultaneously modified.

We will see in Section VI that these observations will have interesting consequences for $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ within models containing new complex phases in the $B_d^0 - \bar{B}_d^0$ mixing.

C. Branching Ratios in the SM

With the CKM parameters obtained from the UT fit we find using (II.2) and (II.18)

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM} = (7.77 \pm 0.82_{P_c} \pm 0.91) \cdot 10^{-11} = (7.8 \pm 1.2) \cdot 10^{-11}, \tag{IV.5}$$

$$Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})_{\rm SM} = (3.0 \pm 0.6) \cdot 10^{-11}. \tag{IV.6}$$

TABLE VI Values of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ in the SM in units of 10^{-11} obtained through various strategies described in the text.

Strategy	$Br(K^+ \to \pi^+\nu\bar{\nu})$ $[10^{-11}]$	$Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ $[10^{-11}]$
Scenario A	7.77 ± 1.23	3.05 ± 0.56
	$7.77 \pm 0.82_{P_c} \pm 0.91$	
Scenario B	7.46 ± 0.55	2.82 ± 0.25
	$7.46 \pm 0.35_{P_c} \pm 0.43$	
Scenario C	7.85 ± 0.35	3.12 ± 0.17
	$7.85 \pm 0.23_{P_c} \pm 0.27$	
(R_b, γ) (B)	7.85 ± 0.69	3.10 ± 0.60
	$7.85 \pm 0.35_{P_c} \pm 0.60$	
(R_b, γ) (C)	7.85 ± 0.38	3.10 ± 0.23
	$7.85 \pm 0.23_{P_c} \pm 0.30$	

TABLE VII The anatomy of parametric uncertainties in $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ corresponding to the results of table VI.

Strategy	$\sigma Br(K^+ \to \pi^+\nu\bar{\nu})$ $[10^{-11}]$	$\sigma Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ $[10^{-11}]$
Scenario A	$\pm 0.72_{\bar\varrho} \pm 0.11_{\bar\eta} \pm 0.44_{\lvert V_{cb}\rvert} \pm 0.31_{m_t}$	$\pm 0.48_{\bar\eta} \pm 0.24_{\lvert V_{cb}\rvert} \pm 0.17_{m_t}$
Scenario B	$\pm 0.20_{\bar\varrho} \pm 0.03_{\bar\eta} \pm 0.31_{\lvert V_{cb}\rvert} \pm 0.21_{m_t}$	$\pm 0.15_{\bar\eta} \pm 0.17_{\lvert V_{cb}\rvert} \pm 0.11_{m_t}$
Scenario C	$\pm 0.13_{\bar\varrho} \pm 0.03_{\bar\eta} \pm 0.22_{\lvert V_{cb}\rvert} \pm 0.08_{m_t}$	$\pm 0.12_{\bar\eta} \pm 0.12_{\lvert V_{cb}\rvert} \pm 0.04_{m_t}$
(R_b, γ) (B)	$\pm 0.06_{R_b} \pm 0.44_{\gamma} \pm 0.33_{\lvert V_{cb}\rvert} \pm 0.23_{m_t}$	$\pm 0.48_{R_b} \pm 0.29_{\gamma} \pm 0.18_{\lvert V_{cb}\rvert} \pm 0.13_{m_t}$
(R_b, γ) (C)	$\pm 0.02_{R_b} \pm 0.18_{\gamma} \pm 0.22_{\lvert V_{cb}\rvert} \pm 0.08_{m_t}$	$\pm 0.16_{R_b} \pm 0.11_{\gamma} \pm 0.12_{\lvert V_{cb}\rvert} \pm 0.04_{m_t}$

In (IV.5) we have separated the error due to $P_c(X)$, given first, from the parametric error coming from the CKM parameters and the value of m_t. Adding these errors in quadrature we find the final result for $Br(K^+ \to \pi^+\nu\bar{\nu})$ in the SM. In the case of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ only parametric uncertainties matter. Our results are rather close to the ones obtained in (Isidori, 2003; Kettell *et al.*, 2002) although the central values of both branching ratios found by us here and in (Buras *et al.*, 2004c) are slightly larger than found in these papers.

The central value of $Br(K^+ \to \pi^+\nu\bar{\nu})$ in (IV.5) is below the central experimental value in (I.5), but within theoretical, parametric and experimental uncertainties, the SM result is fully consistent with the data. We also observe that the error in $P_c(X)$ constitutes still a significant portion of the full error.

One of the main origins of the parametric uncertainties in both branching ratios is the value of $|V_{cb}|$. As pointed out in (Kettell *et al.*, 2002) with the help of ε_K the dependence on $|V_{cb}|$ can be eliminated. Indeed, from the expression for ε_K in (II.37) and the relation

$$\frac{{\rm Im}\lambda_t}{{\rm Re}\lambda_t} = -\tan\beta_{\rm eff}, \qquad \beta_{\rm eff} = \beta - \beta_s, \tag{IV.7}$$

that follows from (II.27), ${\rm Im}\lambda_t$ and ${\rm Re}\lambda_t$ can be determined subject mainly to the uncertainty in $\hat{B}_K$ that should be decreased through lattice simulations in the future. Note that β will soon be determined with high precision from the $a_{\psi K_S}$ asymmetry. While we find this point interesting, the fact that the ε_K constraint has been already used in the UT fit presented above, makes it clear that the idea of (Kettell *et al.*, 2002) is automatically taken into account in this fit.

We can next investigate what kind of predictions one will get in a few years when β and γ will be measured with high precision through theoretically clean strategies at LHCB (Ball *et al.*, 2000) and BTeV (Anikeev *et al.*, 2001). As pointed out in (Buras *et al.*, 2003c), the use of β and γ is the most powerful strategy to get $(\bar\varrho, \bar\eta)$. Performing then the UT fit in scenarios B and C of table III with all constraints taken into account, we find

$${\rm Im}\lambda_t = (1.35 \pm 0.05)\cdot 10^{-4}, \qquad {\rm Re}\lambda_t = -(2.97 \pm 0.11)\cdot 10^{-4} \quad \text{(Scenario B)} \tag{IV.8}$$

$${\rm Im}\lambda_t = (1.42 \pm \pm 0.04)\cdot 10^{-4}, \qquad {\rm Re}\lambda_t = -(3.08 \pm 0.07)\cdot 10^{-4} \quad \text{(Scenario C).} \tag{IV.9}$$

The results for the branching ratios in these two scenarios are given in table VI, where we separated the error due to P_c from the parametric uncertainties.

It is also instructive to use only the (R_b, γ) strategy to fix the CKM parameters as this construction of the UT is unpolluted by new physics contributions. Working then with the values of $|V_{cb}|$, R_b and γ in scenarios B and C we obtain the values for the branching ratios in the last two rows of table VI. We observe that this strategy has significantly larger uncertainties than the full fit in Scenario B that is governed by the combination of β and γ. On the other hand due to small errors assumed on R_b and γ in Scenario C, the (R_b, γ) strategy is competitive in this case with the full fit. Still one should keep in mind that it will take some time before the errors on R_b and γ in Scenario C can be a realized.

In table VII we present the anatomy of parametric uncertainties given in table VI. Adding these uncertainties in quadrature gives the values in table VI. We observe that $|V_{cb}|$ plays a prominent role in these uncertainties.

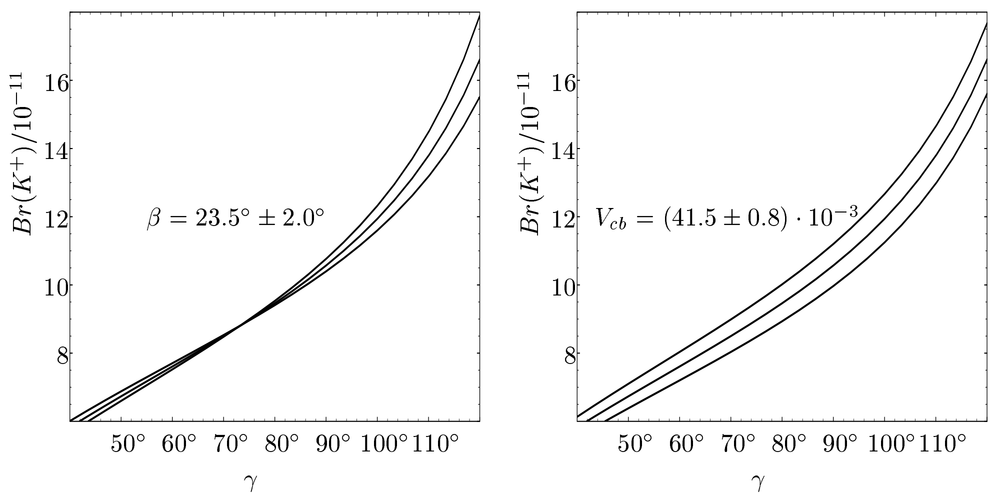

FIG. 4 $Br(K^+ \to \pi^+ \nu\bar{\nu})$ as a function of γ for different values of β and $|V_{cb}|$.

The improvement on the prediction for $Br(K^+ \to \pi^+ \nu\bar{\nu})$ will certainly come from the measurement of ΔM_s that hopefully will be available at the end of this year from Tevatron. To this end one can use in the case of $K^+ \to \pi^+ \nu\bar{\nu}$, the elegant formula (III.10) or just incorporate this new constraint through (II.44) in our numerical analysis that led to (IV.5) and (IV.6). We find that, unless the error on ξ will be decreased and ΔM_s is measured with an accuracy of better than $0.5/ps$ its influence will be insignificant. The impact of the measurement of ΔM_s on $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ is generally much smaller than on $Br(K^+ \to \pi^+ \nu\bar{\nu})$, as $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ is sensitive directly only to $\bar{\eta}$ and only indirectly to R_t.

Finally in Fig. 4 we show $Br(K^+ \to \pi^+ \nu\bar{\nu})$ as a function of γ for different values of β and $|V_{cb}|$. To this end we have used the formula (III.11) with $m_t = 168\,\mathrm{GeV}$ and $P_c = 0.39$. We observe that the dependence on β is rather weak, while the dependence on γ is very strong. Also the dependence on $|V_{cb}|$ is significant. This implies that a precise measurement of γ one day will also have a large impact on the prediction for $Br(K^+ \to \pi^+ \nu\bar{\nu})$.

D. Impact of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ on the UT

1. Preliminaries

Let us then reverse the analysis and investigate the impact of present and future measurements of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ on $|V_{td}|$ and on the UT. To this end we take as additional inputs the present values of $|V_{cb}|$ and β given in scenario A in table III. We observe immediately that now a precise value of $|V_{cb}|$ is required in order to obtain a satisfactory result for $(\bar{\varrho}, \bar{\eta})$. Indeed $K \to \pi\nu\bar{\nu}$ decays are excellent means to determine ${\rm Im}\lambda_t$ and ${\rm Re}\lambda_t$ or equivalently the "*sd*" unitarity triangle and in this respect have no competition from any B decay, but in order to construct the standard "*bd*" triangle of Fig. 2 from these decays, $|V_{cb}|$ is required. Here the CP-asymmetries in B decays measuring directly angles of the UT are superior as the value of $|V_{cb}|$ is not required. Consequently the precise value of $|V_{cb}|$ is of utmost

TABLE VIII The values for R_t and $|V_{td}|/10^{-3}$ (in parentheses) from $K^+ \to \pi^+\nu\bar{\nu}$ for various cases considered in the text.

	Scenario I	Scenario II
Scenario B (β)	0.903 ± 0.078 (8.39 ± 0.69)	0.903 ± 0.041 (8.39 ± 0.36)
Scenario B (R_b)	0.905 ± 0.078 (8.41 ± 0.69)	0.905 ± 0.041 (8.41 ± 0.36)

importance if we want to make useful comparisons between various observables in K and B decays. On the other hand, in some relations such as (I.1), the $|V_{cb}|$ dependence is absent to an excellent accuracy.

2. $|V_{td}|$ from $K^+ \to \pi^+\nu\bar{\nu}$

Taking the present experimental value of $Br(K^+ \to \pi^+\nu\bar{\nu})$ in (I.5), we determine first the UT side R_t and next the CKM element $|V_{td}|$. As discussed in Sect 3.2.1, either β or R_b can provide the last necessary CKM input. Using then the accurate expression for $Br(K^+ \to \pi^+\nu\bar{\nu})$ in (III.10) and the values of $|V_{cb}|$ and β in the present scenario A of table III, we find

$$R_t = 1.35 \pm 0.64, \qquad |V_{td}| = (12.5 \pm 5.9) \cdot 10^{-3} \ , \tag{IV.10}$$

where the dominant error arises due to the error in the branching ratio. On the other hand, using R_b in Scenario A instead of β gives:

$$R_t = 1.34^{+0.12}_{-0.63}, \qquad |V_{td}| = (12.5^{+1.1}_{-5.9}) \cdot 10^{-3}. \tag{IV.11}$$

The imposition of the R_b constraint eliminates the large values of R_t and $|V_{td}|$ and only values $Br(K^+ \to \pi^+\nu\bar{\nu}) < 1.69 \cdot 10^{-10}$ are allowed.

This significant difference in the impact of β and R_b on the determination of R_t and $|V_{td}|$ found here is mainly due to the fact that a large portion of the range in (IV.10) is inconsistent with the value of R_b. Once the values of $Br(K^+ \to \pi^+\nu\bar{\nu})$ are compatible with the measured value of R_b, as chosen in the scenarios of table IV, the impacts of R_b and β on R_t and $|V_{td}|$ are very similar to each other.

We consider then scenarios I and II of table IV but do not take yet the values for $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ into account. As an additional variable we take β or R_b in the scenario B of table III. In table VIII we give the values of R_t and $|V_{td}|$ resulting from this exercise. Within the shown uncertainties the results obtained in scenario C of table III are the same. The precise value of β or R_b does not matter much in the determination of R_t and $|V_{td}|$, which is evident from the inspection of the $(\bar{\varrho}, \bar{\eta})$ plot. This is also the reason why with the assumed errors on β and R_b the two exercises in table VIII give essentially the same results.

More importantly, while the errors on R_t and $|V_{td}|$ in Scenario I are slightly larger than in the SM column of table V, the corresponding errors in Scenario II are smaller. One should emphasize the clean character of this determination and that R_t and $|V_{td}|$ have been basically found here only from β or R_b and $Br(K^+ \to \pi^+\nu\bar{\nu})$, whereas the results in table V were obtained imposing several different constraints.

In order to judge the precision achievable in the future, it is instructive to show the separate contributions of the uncertainties involved. In general, $|V_{td}|$ is subject to various uncertainties of which the dominant ones are given below

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 0.39 \frac{\sigma(P_c)}{P_c} \pm 0.70 \frac{\sigma(Br(K^+))}{Br(K^+)} \pm \frac{\sigma(|V_{cb}|)}{|V_{cb}|} \ . \tag{IV.12}$$

We find then

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 3.0\%_{P_c} \pm 7.0\%_{Br(K^+)} \pm 1.4\%_{|V_{cb}|}, \qquad \text{(Scenario I)} \tag{IV.13}$$

and

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 2.0\%_{P_c} \pm 3.5\%_{Br(K^+)} \pm 1.0\%_{|V_{cb}|}. \qquad \text{(Scenario II)} \tag{IV.14}$$

Adding the errors in quadrature, we find that $|V_{td}|$ can be determined with an accuracy of $\pm 7.7\%$ and $\pm 4.1\%$, respectively. These numbers are increased to $\pm 8.2\%$ and $\pm 4.2\%$ once the uncertainties due to $m_{\rm t}$, α_s and β (or $|V_{ub}/V_{cb}|$) are taken into account. As a measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ with a precision of 5% is very challenging, the determination of $|V_{td}|$ with an accuracy better than $\pm 5\%$ from $Br(K^+ \to \pi^+\nu\bar{\nu})$ seems very difficult from the present perspective.

We note also that present uncertainty in the determination of $|V_{td}|$ due to P_c amounts to $\pm 7\%$, making an NNLO calculation of P_c very desirable.

It is of interest to compare the precision on $|V_{td}|$ from $K^+ \to \pi^+\nu\bar{\nu}$ just discussed with the one that will be obtained one day directly from the ratio $\Delta M_d/\Delta M_s$. First R_t can be directly obtained by means of (II.44). As ΔM_s should be measured very precisely already in the Run II at Tevatron, the experimental error in (II.44) will be certainly negligible one day, in particular once data from LHC will be available. The fate of the error on R_t will then be solely dependent on the accuracy with which ξ can be calculated. The present uncertainty in ξ is $\pm 6.5\%$. In obtaining $|V_{td}|$ an additional error comes from $|V_{cb}|$. It is roughly $\pm 2\%$ at present and should be decreased in the future. Thus, in the long run, the precision with which $|V_{td}|$ can be determined from $\Delta M_d/\Delta M_s$ will be in the hands of lattice gauge theorists. We expect that a direct measurement of $|V_{td}|$ in this manner, with an accuracy at the level of $4-5\%$, should be possible in the second half of this decade.

In the case of $K^+ \to \pi^+\nu\bar{\nu}$ the future of the accuracy on $|V_{td}|$ will on the other hand be dominated by experimental accuracy on $Br(K^+ \to \pi^+\nu\bar{\nu})$ as, after the NNLO calculation in (Buras *et al.*, 2004b) has been completed, the error on $|V_{td}|$ coming from P_c should be decreased below $\pm 3\%$, as seen in (IV.13) and (IV.14).

3. Impact on UT

The impact of $K^+ \to \pi^+\nu\bar{\nu}$ on the UT is illustrated in Fig. 5, where in the upper part we show the band corresponding to the present central experimental value of $Br(K^+ \to \pi^+\nu\bar{\nu})$ in (I.5) and the present uncertainty due to $P_c(X)$. As expected from the previous discussion, the central value is far from the standard UT fit. In the lower part of Fig. 5 we show the lines corresponding to several selected values of $Br(K^+ \to \pi^+\nu\bar{\nu})$. The construction of the UT from both decays shown there is described below.

E. Impact of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ on the UT

1. $\bar{\eta}$ and ${\rm Im}\lambda_t$

We consider next the impact of a future measurement of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ on the UT. As already discussed in the previous section, this measurement will offer a theoretically clean determinations of $\bar{\eta}$ and in particular of ${\rm Im}\lambda_t$. The relevant formulae are given in (III.17) and (III.18), respectively. Using scenarios I and II of table IV we find

$$\bar{\eta} = 0.351 \pm 0.022, \qquad {\rm Im}\lambda_t = (1.39 \pm 0.08)\cdot 10^{-4} \qquad ({\rm Scenario\ \ I}). \tag{IV.15}$$

$$\bar{\eta} = 0.351 \pm 0.011, \qquad {\rm Im}\lambda_t = (1.39 \pm 0.04)\cdot 10^{-4}. \qquad ({\rm Scenario\ \ II}) \tag{IV.16}$$

The obtained precision in the case of Scenario II is truely impressive. We stress the very clean character of these determinations.

2. Completing the Determination of the UT

In order to construct the UT we need still another input. It could be β, γ, R_b or R_t. It turns out that the most effective in this determination is γ, as in the classification of (Buras *et al.*, 2003c) the $(\bar{\eta},\gamma)$ strategy belongs to the top class together with the (β,γ) pair. The angle γ should be known with high precision in five years. Still it is of interest to see what one finds when β instead of γ is used. R_b is not useful here as it generally gives two solutions for the UT.

In analogy to table VIII we show in table IX the values of $\bar{\varrho}$ and $|V_{td}|$ resulting from scenarios I and II without using $Br(K^+ \to \pi^+\nu\bar{\nu})$. As an additional variable we use β or γ. We observe that, with the assumed errors on β and γ, the use of γ is more effective than the use of β. Moreover, while going from scenario I to scenario II for $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ has a significant impact when β is used, the impact is rather small when γ is used instead. Both features are consistent with the observations made in (Buras *et al.*, 2003c) in the context of $(\beta,\bar{\eta})$ and $(\gamma,\bar{\eta})$ strategies. In particular, the last feature is directly related to the fact that γ is by more than a factor of two larger than β.

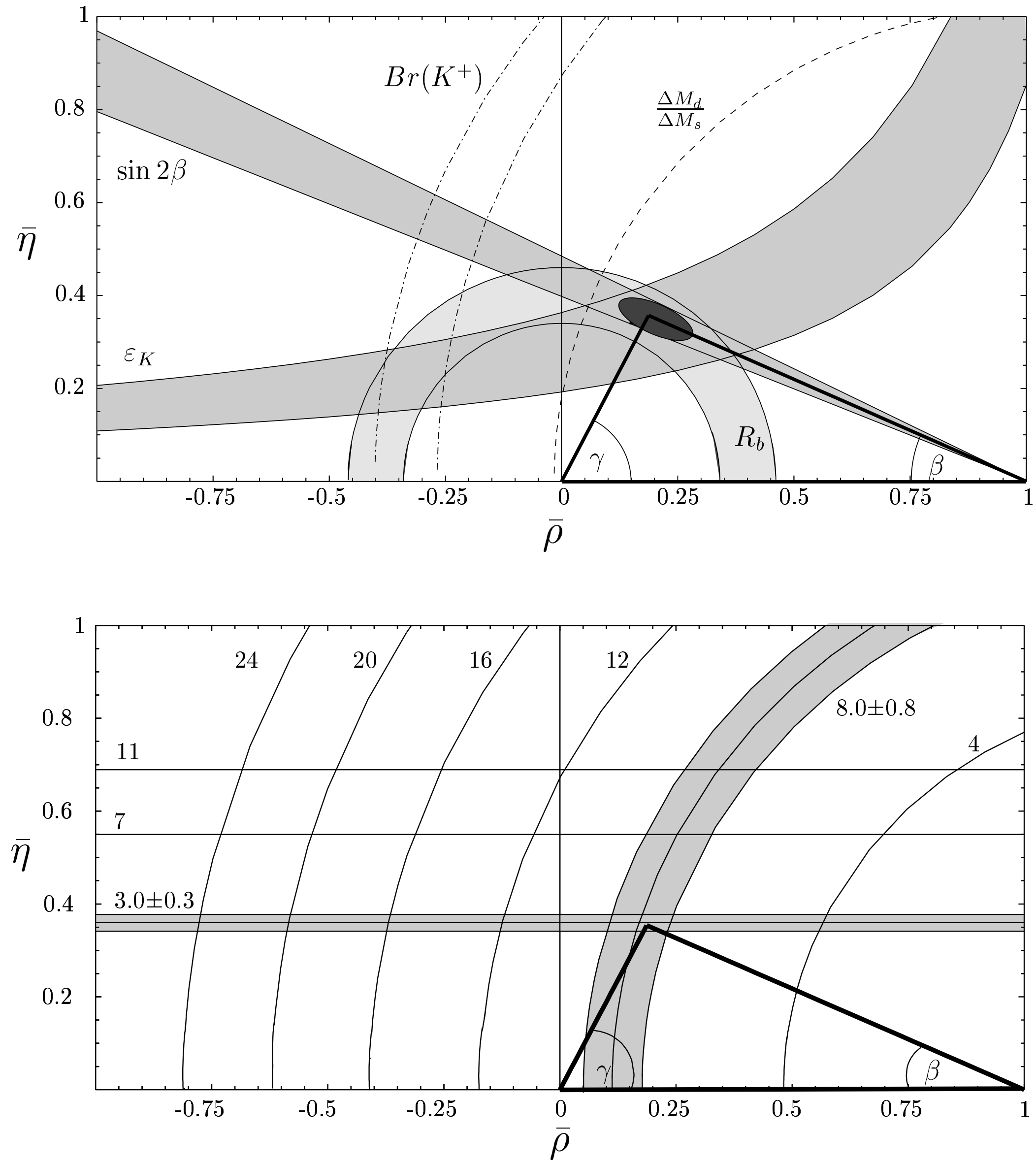

FIG. 5 Upper plot: the result of the standard UT fit as discussed in the text compared to the band resulting from the central experimental value of $Br(K^+ \to \pi^+\nu\bar{\nu})$ including the present uncertainty due to $P_c(X)$. Lower plot: the UT from $K \to \pi\nu\bar{\nu}$ in Scenario I of table IV. Also lines corresponding to several values of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ (in units of 10^{-11}) are shown.

The main message from table IX is that, using a rather precise value of γ, a very precise determination of $|V_{td}|$ becomes possible, where the branching fraction of $K_{\rm L} \to \pi^0\nu\bar{\nu}$ needs to be known only to about 10% accuracy.

3. A Clean and Accurate Determination of $|V_{cb}|$ and $|V_{td}|$

Next, combining β and γ with the values of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and m_t, a clean determination of $|V_{cb}|$ by means of (III.22) is possible. In turn also $|V_{td}|$ can be determined. In table X we show the values of $|V_{cb}|$ and $|V_{td}|$ obtained

TABLE IX The values for $\bar{\varrho}$ and $|V_{td}|/10^{-3}$ (in parentheses) from $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ for various cases considered in the text.

	Scenario I	Scenario II
Scenario B (β)	0.193 ± 0.063 (8.18 ± 0.57)	0.193 ± 0.048 (8.18 ± 0.41)
Scenario C (β)	0.193 ± 0.053 (8.18 ± 0.49)	0.193 ± 0.033 (8.18 ± 0.28)
Scenario B (γ)	0.179 ± 0.042 (8.30 ± 0.37)	0.179 ± 0.041 (8.30 ± 0.41)
Scenario C (γ)	0.179 ± 0.019 (8.30 ± 0.18)	0.179 ± 0.017 (8.30 ± 0.16)

using scenarios I and II for $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ in table IV with β and γ in scenarios B and C of table III.

We observe that the errors on $|V_{cb}|$, except for the pair (C,II), are larger than presently obtained from semi-leptonic B decays. But one should emphasize that this determination is essentially without any theoretical uncertainties. The high precision on $|V_{td}|$ is a result of a very precise measurement of R_t by means of the (β, γ) strategy and a rather accurate value of $|V_{cb}|$ obtained with the help of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$. Again also in this case the determination is theoretically very clean.

TABLE X The values for $|V_{cb}|$ and $|V_{td}|$ (in parentheses) in units of 10^{-3} from $K_{\rm L} \to \pi^0 \nu\bar{\nu}$, β and γ for various cases considered in the text.

	Scenario I	Scenario II
Scenario B	41.2 ± 1.6 (8.24 ± 0.32)	41.2 ± 1.3 (8.24 ± 0.26)
Scenario C	41.2 ± 1.2 (8.24 ± 0.25)	41.2 ± 0.7 (8.24 ± 0.15)

F. Impact of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ on UT

Let us next update the analysis of (Buchalla and Buras, 1996), where the determination of the UT from both decays has been discussed in explicit terms. The relevant formulae have been given in Section III. We consider again two scenarios for which the input parameters are collected in table IV. This time no other parameters beside those given in this table are required for the construction of the UT. The results for the CKM parameters in these two scenarios are given in table XI.

We observe that respectable determinations of all considered quantities except for $\bar{\varrho}$, γ and ${\rm Re}\lambda_t$ in Scenario I can be obtained. Of particular interest are the accurate determinations of $\sin 2\beta$ and of ${\rm Im}\lambda_t$. In the lower part of Fig. 5 we show the resulting UT in the scenario I in question and compare it with its present determination, shown in the upper part of this figure, as discussed at the beginning of this section. Due to the absence of the uncertainty in P_c the measurement of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ with the accuracy of 10% gives a much better determination of $\bar{\eta}$ than the corresponding determination of $\bar{\varrho}$ from $Br(K^+ \to \pi^+ \nu\bar{\nu})$, where P_c matters.

The parameters in the scenario I have been chosen in such a manner that the UT obtained from $K \to \pi\nu\bar{\nu}$ shown in the lower part of Fig. 5 agrees perfectly with the UT determined from the standard analysis of the UT shown in the upper part of this figure. However, as discussed in Section VI.B, this certainly does not have to be the case and each crossing points of various lines coming from $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ indicates the position of the apex of a "fake" unitarity triangle in the case of the departures from the SM expectations. We will return to this topic in Section VI.E.

G. The Angle β from $K \to \pi\nu\bar{\nu}$

Let us next investigate the separate uncertainties in the determination of $\sin 2\beta$ coming from P_c, $Br(K^+ \to \pi^+ \nu\bar{\nu}) \equiv Br(K^+)$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu}) \equiv Br(K_L)$. We find first

$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = \pm 0.31 \frac{\sigma(P_c)}{P_c} \pm 0.55 \frac{\sigma(Br(K^+))}{Br(K^+)} \pm 0.39 \frac{\sigma(Br(K_L))}{Br(K_L)} \; . \tag{IV.17}$$

This leads to

$$\sigma(\sin 2\beta) = 0.017_{P_c} + 0.039_{Br(K^+)} + 0.028_{Br(K_L)} = 0.050 \qquad \text{(Scenario I)} \tag{IV.18}$$

and

$$\sigma(\sin 2\beta) = 0.011_{P_c} + 0.020_{Br(K^+)} + 0.014_{Br(K_L)} = 0.027, \qquad \text{(Scenario II)} \tag{IV.19}$$

where the errors have been added in quadrature. The uncertainties due to $|V_{cb}|$ and m_t are fully negligible.

We observe that

- The present uncertainty in $\sin 2\beta$ due to P_c alone amounts to 0.04, implying that a NNLO calculation of P_c is very desirable.
- The accuracy of the determination of $\sin 2\beta$, after the NNLO result will be available, will depend dominantly on the accuracy with which both branching ratios will be measured. In order to decrease $\sigma(\sin 2\beta)$ down to 0.02 they have to be measured with an accuracy better than 5%.

TABLE XI The determination of CKM parameters from $K \to \pi\nu\bar{\nu}$ for two scenarios of table IV.

	Scenario I	Scenario II
$\bar{\eta}$	0.351 ± 0.022	0.351 ± 0.011
$\bar{\varrho}$	0.167 ± 0.079	0.167 ± 0.042
$\sin 2\beta$	0.716 ± 0.050	0.716 ± 0.027
β	$(22.8 \pm 2.2)^\circ$	$(22.8 \pm 1.1)^\circ$
γ	$(64.2 \pm 10.9)^\circ$	$(64.2 \pm 5.9)^\circ$
R_b	0.389 ± 0.040	0.389 ± 0.020
R_t	0.902 ± 0.072	0.902 ± 0.039
$\lvert V_{td}\rvert/10^{-3}$	8.38 ± 0.65	8.38 ± 0.34
$\mathrm{Im}\lambda_t/10^{-4}$	1.39 ± 0.08	1.39 ± 0.04
$\mathrm{Re}\lambda_t/10^{-4}$	-3.13 ± 0.29	-3.13 ± 0.15

H. The Angle γ from $K \to \pi\nu\bar{\nu}$

Let us next investigate, in analogy to (IV.17), the separate uncertainties in the determination of γ coming from P_c, $Br(K^+ \to \pi^+\nu\bar{\nu})$, $Br(K_\mathrm{L} \to \pi^0\nu\bar{\nu})$ and $|V_{cb}|$. The relevant expression for γ in terms of these quantities is given in (III.29). We find then

$$\frac{\sigma(\gamma)}{\gamma} = \pm 0.75\frac{\sigma(P_c)}{P_c} \pm 1.32\frac{\sigma(Br(K^+))}{Br(K^+)} \pm 0.07\frac{\sigma(Br(K_L))}{Br(K_L)} \pm 4.11\frac{\sigma(|V_{cb}|)}{|V_{cb}|} \pm 2.34\frac{\sigma(m_t)}{m_t}. \tag{IV.20}$$

This gives

$$\sigma(\gamma) = 3.7^\circ_{P_c} + 8.5^\circ_{Br(K^+)} + 0.4^\circ_{Br(K_L)} + 3.8^\circ_{|V_{cb}|} + 2.6^\circ_{m_t} = 10.4^\circ \tag{IV.21}$$

and

$$\sigma(\gamma) = 2.5^\circ_{P_c} + 4.2^\circ_{Br(K^+)} + 0.2^\circ_{Br(K_L)} + 2.5^\circ_{|V_{cb}|} + 0.9^\circ_{m_t} = 5.7^\circ \tag{IV.22}$$

for Scenario I and II, respectively, where the errors have been added in quadrature.

We observe that

- The present uncertainty in γ due to P_c alone amounts to 8.6°, implying that a NNLO calculation of P_c is very desirable.
- The dominant uncertainty in the determination of γ in Scenarios I and II resides in $Br(K^+ \to \pi^+\nu\bar{\nu})$. In order to lower $\sigma(\gamma)$ below 5°, a measurement of this branching ratio with an accuracy of better than 5% is required. The measurement of $Br(K_\mathrm{L} \to \pi^0\nu\bar{\nu})$ has only a small impact on this determination.

TABLE XII The uncertainties in various quantities due to the error in P_c.

$\sigma(P_c)$	± 0.07	± 0.03	± 0.02
$Br(K^+ \to \pi^+ \nu\bar{\nu})/10^{-11}$	± 0.82	± 0.34	± 0.23
$\bar{\eta}$	$-$	$-$	$-$
$\bar{\varrho}$	± 0.067	± 0.029	± 0.019
$\sin 2\beta$	± 0.042	± 0.018	± 0.012
β	$\pm 1.8^\circ$	$\pm 0.8^\circ$	$\pm 0.5^\circ$
γ	$\pm 9.4^\circ$	$\pm 3.8^\circ$	$\pm 2.5^\circ$
R_b	± 0.033	± 0.019	± 0.009
R_t	± 0.061	± 0.026	± 0.017
$\|V_{td}\|/10^{-3}$	± 0.57	± 0.24	± 0.16
$\mathrm{Im}\lambda_t/10^{-4}$	$-$	$-$	$-$
$\mathrm{Re}\lambda_t/10^{-4}$	± 0.25	± 0.11	± 0.07

I. Summary

In this section we have presented a very detailed numerical analysis of the formulae of Section III. First working in three scenarios, A, B and C, for the input parameters that should be measured precisely through B physics observables in this decade, we have shown how the accuracy on the predictions of the branching ratios will improve with time.

In the case of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ there are essentially no theoretical uncertainties and the future of the accuracy of the prediction on this branching ratio within the SM depends fully on the accuracy with which $\mathrm{Im}\lambda_t$ and m_t can be determined from other processes. We learn from table VI that the present error of roughly 20% will be decreased to 9% and 5% when the scenarios B and C will be realized, respectively. As seen in table VII, the progress on the error on $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ will depend importantly on the progress on $|V_{cb}|$.

The case of $K^+ \to \pi^+ \nu\bar{\nu}$ is a bit different as now also the uncertainty in P_c enters. As discussed in Section II, this uncertainty comes on the one hand from the scale uncertainty that can be reduced through a NNLO calculation and on the other hand from the error in m_c. The scale uncertainty dominates at present but after a NNLO result will be available, we expect the error on m_c to be mainly responsible for the error in P_c. Formula (II.17) quantifies this explicitely. The anatomy of parametric uncertainties in $Br(K^+ \to \pi^+ \nu\bar{\nu})$ is presented in table VII. As in the case of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ also here the reduction of the error in $|V_{cb}|$ will be important.

As seen in table VI the present error in $Br(K^+ \to \pi^+ \nu\bar{\nu})$ due to P_c amounts roughly to $\pm 10\%$ an is almost as large as the parametric uncertainty from the CKM elements and m_t. It is also clearly seen in this table that in order to benefit from the improved values of the CKM parameters and of m_t, also the uncertainty in P_c has to be reduced through a NNLO calculation and the improvement of m_c. It appears to us that the present error of 10% due to P_c could be decreased to 4.5% and even 3% one day with the present total error of 15% reduced to 7% and even 4.5%, respectively.

In the main part of this section we have investigated the impact of the future measurements of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ on the determination of the CKM matrix. This analysis was culminated in the table XI but a useful anatomy of various contributions can also be found in the remaining tables of this section and various formulae. These results are self-explanatory and demonstrate very clearly that the $K \to \pi\nu\bar{\nu}$ decays offer powerful means in the determination of the UT and of the CKM matrix.

Clearly, the future determination of various observables by means of $K \to \pi\nu\bar{\nu}$ will crucially depend on the accuracy with which $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ can be measured. Our discussion shows that it is certainly desirable to measure both branching ratios with an accuracy of at least 5%.

On the other hand the uncertainties due to P_c, $|V_{cb}|$ and to a lesser extent m_t are also important ingredients of these investigations. In tables XII and XIII we summarize the uncertainties in various quantities of interest due to errors in P_c and $|V_{cb}|$, respectively.

TABLE XIII The uncertainties in various quantities due to the error in $|V_{cb}|$.

$\sigma(\lvert V_{cb}\rvert)/10^{-3}$	±0.8	±0.6	±0.4
$Br(K^+ \to \pi^+\nu\bar{\nu})/10^{-11}$	±0.44	±0.31	±0.22
$\bar{\eta}$	±0.013	±0.010	±0.007
$\bar{\varrho}$	±0.033	±0.025	±0.016
$\sin 2\beta$	–	–	–
β	–	–	–
γ	$\pm5.3^\circ$	$\pm3.9^\circ$	$\pm2.6^\circ$
R_b	±0.003	±0.002	±0.001
R_t	±0.036	±0.027	±0.018
$\lvert V_{td}\rvert/10^{-3}$	±0.17	±0.12	±0.08
$\mathrm{Im}\lambda_t/10^{-4}$	–	–	–
$\mathrm{Re}\lambda_t/10^{-4}$	–	–	–

V. $K \to \pi\nu\bar{\nu}$ AND MFV

A. Preliminaries

A general discussion of the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ in the framework of minimal flavour violation has been presented in (Buras and Fleischer, 2001). Earlier papers in specific MFV scenarios like two Higgs doublet can be found in (Belanger *et al.*, 1992; Cho, 1998), where additional references are given. Basically, all formulae of Section II and III remain valid except that

- the functions $X(x_t)$ and $S_0(x_t)$ are replaced by the real valued master functions (Buras, 2003a,b, 2004) $X(v)$ and $S(v)$ with v denoting collectively the parameters of a given MFV model,
- if the function $X(v)$ is allowed to take also negative values, the following replacements should effectively be made in all formulae of Sections II and III (Buras and Fleischer, 2001)

$$X \to |X|, \qquad P_c(X) \to \mathrm{sgn}(X) P_c(X), \tag{V.1}$$

- if the function $S(v)$ is also allowed to take negative values, $\mathrm{sgn}(S(v))$ enters some of the expressions given above. We refer to (Buras and Fleischer, 2001) for details.

Here we will assume $S(v) > 0$, as in the SM, because as found in (Buras and Fleischer, 2001; D'Ambrosio *et al.*, 2002) the negative values of $S(v)$ are disfavoured although not yet fully excluded. On the other hand, we will allow for negative values of the function $X(v)$. The values of $S(v)$ can be calculated in any MFV model. On the other hand $S(v)$ can be constrained from the usual UT fit with the result (Buras *et al.*, 2003c)

$$1.3 \leq S(v) \leq 3.8 \text{ (95\% probability region)}, \tag{V.2}$$

to be compared with $S(v) = 2.42 \pm 0.09$ in the SM.

Concerning the UT in the MFV models, we recall that a universal unitarity triangle (UUT) can be be constructed by using only quantities that do not depend on particular parameters of a given MFV model (Buras *et al.*, 2001b). Using then $|V_{cb}|$, $|V_{ub}/V_{cb}|$, (II.50) for $\sin 2\beta$ and the lower bound on ΔM_s together with (II.44) we find the apex of the UUT described by the values of $(\bar{\varrho}, \bar{\eta})$ in the column UUT in table V. The results for various quantities of interest related to this UUT are collected also there. A similar analysis has been performed in (Buras *et al.*, 2003c; D'Ambrosio *et al.*, 2002).

It should be stressed that any MFV model that is inconsistent with the values given in the UUT column in table V is ruled out. We observe that there is little room for MFV models that in their predictions for UT differ significantly from the SM. It is also clear that, to distinguish the SM from the MFV models on the basis of the analysis of the UT presented above, will require considerable reduction of theoretical uncertainties.

TABLE XIV Values of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ in the MFV models in units of 10^{-11} for specific values of $a_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $\mathrm{sgn}(X) = +1$ (-1). We set $P_c(X) = 0.39$.

$Br(K^+ \to \pi^+ \nu\bar{\nu})$ $[10^{-11}]$	$a_{\psi K_S} = 0.69$	$a_{\psi K_S} = 0.74$	$a_{\psi K_S} = 0.79$
5.0	1.4 (5.9)	1.6 (6.9)	2.0 (8.0)
10.0	3.7 (10.1)	4.4 (11.9)	5.3 (13.8)
15.0	6.3 (14.1)	7.5 (16.6)	8.9 (19.4)
20.0	9.0 (18.0)	10.7 (21.2)	12.6 (24.8)
25.0	11.7 (21.9)	13.9 (25.7)	16.5 (30.1)
38.0	19.1 (31.6)	22.7 (37.2)	26.9 (43.6)

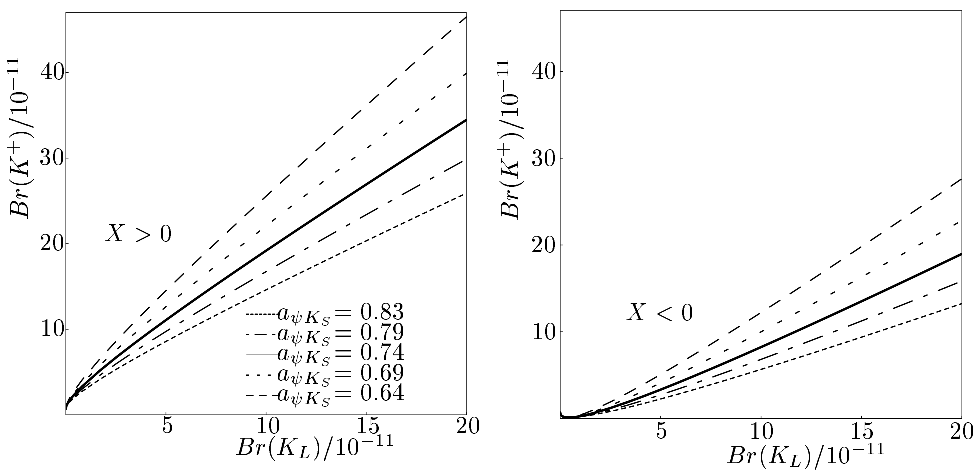

FIG. 6 $Br(K^+ \to \pi^+ \nu\bar{\nu})$ as a function of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ for several values of $a_{\psi K_S}$ in the case of $\mathrm{sgn}(X) = \pm 1$.

B. $K^+ \to \pi^+ \nu\bar{\nu}$ versus $K_{\rm L} \to \pi^0 \nu\bar{\nu}$

An important consequence of (III.26) and (I.1) is the following MFV relation (Buras and Fleischer, 2001)

$$B_1 = B_2 + \left[\frac{\cot\beta\sqrt{B_2} + \mathrm{sgn}(X)\sqrt{\sigma}P_c(X)}{\sigma} \right]^2, \tag{V.3}$$

that, for a given $\sin 2\beta$ extracted from $a_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$, allows to predict $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$. We observe that in the full class of MFV models, independent of any new parameters present in these models, only two values for $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$, corresponding to two signs of X, are possible. Consequently, measuring $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ will either select one of these two possible values or rule out all MFV models.

In (Buras and Fleischer, 2001) a detailed numerical analysis of the relation (V.3) has been presented. In view of the improved data on $\sin 2\beta$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$ we update and extend this analysis. In table XIV, we show values of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ in the MFV models for several values of $a_{\psi K_S}$ and $Br(K^+ \to \pi^+ \nu\bar{\nu})$ consistent with the present data and the two signs of X.

A more detailed presentation is given in Fig. 6, where we show $Br(K^+ \to \pi^+ \nu\bar{\nu})$ as a function of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ for several values of $a_{\psi K_S}$ and two signs of X. These plots are universal for all MFV models. As emphasized in (Buras and Fleischer, 2001), the measurements of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$, $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $a_{\psi K_S}$ will easily allow the distinction between the two signs of X. This is clearly seen in table XIV and Fig. 6. The reduction of the uncertainty due to $P_c(X)$, that is non-negligible, would help in this distinction. In particular, while for $X > 0$, $Br(K^+ \to \pi^+ \nu\bar{\nu})$ is always larger than $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$, this is not always the case for $X < 0$, where the destructive interference between the top and charm contributions to $K^+ \to \pi^+ \nu\bar{\nu}$ can suppress its branching ratio below $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$. We will return to this issue in Section VI in the context of scenarios with new complex phases.

We also observe, as in (Buras and Fleischer, 2001), that the upper bound on $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ following from the data on $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $\sin 2\beta \le 0.785$ is substantially stronger than the model independent bound following from isospin symmetry (Grossman and Nir, 1997)

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) < 4.4 \cdot Br(K^+ \to \pi^+\nu\bar{\nu}). \tag{V.4}$$

With the data in (I.5), that imply

$$Br(K^+ \to \pi^+\nu\bar{\nu}) < 3.8 \cdot 10^{-10}\ (90\%\ \text{C.L.}), \tag{V.5}$$

one finds from (V.4)

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) < 1.7 \cdot 10^{-9}\ (90\%\ \text{C.L.}), \tag{V.6}$$

that is still two orders of magnitude lower than the upper bound from the KTeV experiment at Fermilab (Alavi-Harati *et al.*, 2000b), yielding $Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) < 5.9 \cdot 10^{-7}$.

On the other hand, taking the experimental bound $Br(K^+ \to \pi^+\nu\bar{\nu})$ in (I.5) and $a_{\psi K_S} \le 0.785$, we find from (V.3)

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu})_{\rm MFV} \le \begin{cases} 2.7 \cdot 10^{-10} & \text{sgn}(X) = +1 \\ 4.4 \cdot 10^{-10} & \text{sgn}(X) = -1. \end{cases} \tag{V.7}$$

While the E391a experiment at KEK could give in principle results slightly below the absolute bound in (V.6), it will have certainly hard time to reach the MFV bound in (V.7). Therefore, the observation of any events by this experiment will likely signal effects of new flavour violating interactions.

As $a_{\psi K_S}$ in MFV models determines the true value of β and the true value of γ can be determined in tree level strategies in B decays one day, the true value of $\bar{\eta}$ can also be determined in a clean manner. Consequently, using (III.21) offers probably the cleanest measurement of $|X|$ in the field of weak decays. We will return to this issue in Section VI where this statement can be generalized to most extensions of the SM.

VI. SCENARIOS WITH NEW COMPLEX PHASES IN ENHANCED Z^0-PENGUINS AND $B_d^0 - \bar{B}_d^0$ MIXING

A. Preliminaries

In this section we will consider three simple scenarios beyond the framework of MFV, in which X becomes a complex quantity as given in (I.7), and the universal box function $S(v)$ entering ε_K and $\Delta M_{d,s}$ not only becomes complex but generally becomes non-universal with

$$S_K(v) = |S_K(v)|e^{i2\theta_K}, \quad S_d(v) = |S_d(v)|e^{i2\theta_d}, \quad S_s(v) = |S_s(v)|e^{i2\theta_s}, \tag{VI.1}$$

for $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing, respectively. If these three functions are different from each other, some universal properties found in the SM and MFV models, that have been recently reviewed in (Buras, 2003a,b, 2004), are lost. In addition, the mixing induced CP asymmetries in B decays do not measure the angles of the UT but only sums of these angles and of θ_i.

In order to simplify the presentation we will assume that $S_s = S_0(x_t)$ as in the SM but we will take $S_d(v)$ to be complex with $S_d(v) \neq S_0(x_t)$. This will allow to change the relation between R_t and $\Delta M_d/\Delta M_s$ in (II.44). We will leave open whether $S_K(v)$ receives new physics contributions.

The scenario in which new physics enters dominantly through enhanced Z^0 penguins involving a new CP-violating weak phase was first considered in (Buras *et al.*, 2000, 1998; Buras and Silvestrini, 1999; Colangelo and Isidori, 1998) in the context of rare K decays and the ratio ε'/ε measuring direct CP violation in the neutral kaon system, and was generalized to rare B decays in (Atwood and Hiller, 2003; Buchalla *et al.*, 2001). Recently this particular extension of the SM has been revived in (Buras *et al.*, 2004c,d), where it has been pointed out that the anomalous behaviour in $B \to \pi K$ decays observed by CLEO, BABAR and Belle (Aubert *et al.*, 2002b, 2003b, 2004b; Bornheim *et al.*, 2003; Chao *et al.*, 2004) could be due to the presence of enhanced Z^0 penguins carrying a large new CP-violating phase around -90°.

The possibility of important electroweak penguin contributions behind the anomalous behaviour of the $B \to \pi K$ data has been pointed out already in (Buras and Fleischer, 2000), but only recently has this behaviour been independently observed by the three collaborations in question. Recent discussions related to electroweak penguins can be also found in (Beneke and Neubert, 2003; Yoshikawa, 2003). Other conjectures in connection with these data can be found in (Chiang *et al.*, 2004; Gronau and Rosner, 2003a,b).

The implications of the large CP-violating phase in electroweak penguins for rare K and B decays and $B \to X_s l^+ l^-$ have been analyzed in detail in (Buras *et al.*, 2004c,d) and subsequently the analyses of $B \to X_s l^+ l^-$ and $K_L \to \pi^0 l^+ l^-$ have been extended in (Rai Choudhury *et al.*, 2004) and (Isidori *et al.*, 2004), respectively. It turns out that in this scenario several predictions differ significantly from the SM expectations with most spectacular effects found precisely in the $K \to \pi\nu\bar{\nu}$ system. These effects should easily be identified once the data improve.

On the other hand the scenarios with complex phases in $B_d^0 - \bar{B}_d^0$ mixing have been considered in many papers with the subset of references given in (Bergmann and Perez, 2000, 2001; Bertolini *et al.*, 1987; D'Ambrosio and Isidori, 2002; Laplace, 2002; Laplace *et al.*, 2002; Nir and Silverman, 1990a,b). Most recently this scenario has been discussed in (Fleischer *et al.*, 2003).

In what follows, we will first briefly review the results for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ obtained in (Buras *et al.*, 2004c,d), that were motivated by the $B \to \pi K$ data. Subsequently, we will discuss the implications of this scenario for the $K \to \pi\nu\bar{\nu}$ complex independently of the $B \to \pi K$ system.

Next we will consider scenarios with new physics present only in $B_d^0 - \bar{B}_d^0$ mixing and the function X as in the SM. Here the impact on $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ comes only through modified values of the CKM parameters but, as we will see below, this impact is rather interesting.

Finally we will consider a hybrid scenario with new physics entering both $K \to \pi\nu\bar{\nu}$ decays and $B_d^0 - \bar{B}_d^0$ mixing. In this discussion the (R_b, γ) strategy for the determination of the UT will play a very important role.

B. A Large New CP-Violating Phase θ_X

In this scenario the function X becomes a complex quantity (Buras *et al.*, 1998), as given in (I.7), with θ_X being a new complex phase that originates primarily from new physics contributions to the Z^0 penguin diagrams. An explicit realization of such extension of the SM will be discussed in Section VII. In what follows it will be useful to define the following combination of weak phases,

$$\beta_X \equiv \beta - \beta_s - \theta_X, \tag{VI.2}$$

that generalizes $\beta_{\rm eff}$ to the scenario considered.

Imposing the upper bound on the size of Z^0 penguins from the BaBar and Belle data on $B \to X_s\mu^+\mu^-$ (Aubert *et al.*, 2003a; Kaneko *et al.*, 2003), and taking into account the data on $B \to \pi\pi$ and $B \to \pi K$ decays, one finds (Buras *et al.*, 2004c,d)

$$|X| = 2.17 \pm 0.12, \qquad \theta_X = -(86 \pm 12)^\circ, \qquad \beta_X = (111 \pm 12)^\circ, \tag{VI.3}$$

to be compared with $|X| = 1.53 \pm 0.04$ and $\beta_{\rm eff} = (24.5 \pm 2.0)^\circ$ in the SM. While $|X|$ is only enhanced by a factor of 1.5, the presence of the large new CP violating phase has spectacular implications on the pattern of $K \to \pi\nu\bar{\nu}$ decays. Clearly, in view of significant experimental uncertainties in $B \to \pi\pi$, $B \to \pi K$ and $B \to X_s\mu^+\mu^-$, that led to (VI.3), it is difficult to attach any high confidence level to these results but it is legitimate and certainly interesting to take them seriously and to analyze them.

Following (Buras *et al.*, 2004c), the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ are now given as follows:

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+ \left[\tilde{r}^2 A^4 R_t^2 |X|^2 + 2\tilde{r}\bar{P}_c(X) A^2 R_t |X| \cos\beta_X + \bar{P}_c(X)^2\right] \tag{VI.4}$$

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) = \kappa_L \tilde{r}^2 A^4 R_t^2 |X|^2 \sin^2\beta_X, \tag{VI.5}$$

with κ_+ given in (II.3), κ_L given in (II.19), $\bar{P}_c(X)$ defined in (III.2), β_X in (VI.2) and $\tilde{r}$ in (II.26).

Once $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ have been measured, the parameters $|X|$ and β_X can be determined, subject to ambiguities that can be resolved by considering other processes, such as the non-leptonic B decays and the rare decays discussed in (Buras *et al.*, 2004c). Combining (VI.4) and (VI.5), the generalization of (III.27) to the scenario considered can be found (Buras *et al.*, 2004c, 1998)

$$\sin 2\beta_X = \frac{2\bar{r}_s}{1+\bar{r}_s^2}, \qquad \bar{r}_s = \frac{\varepsilon_1\sqrt{B_1 - B_2} - \bar{P}_c(X)}{\varepsilon_2\sqrt{B_2}} = \cot\beta_X, \tag{VI.6}$$

where $\varepsilon_i = \pm 1$. Moreover,

$$|X| = \frac{\varepsilon_2\sqrt{B_2}}{\tilde{r}A^2 R_t \sin\beta_X}, \quad \varepsilon_2 \sin\beta_X > 0. \tag{VI.7}$$

The "reduced" branching ratios B_i are given in (III.24).

These formulae are valid for arbitrary $\beta_X \neq 0^\circ$. For $\theta_X = 0^\circ$ and $\varepsilon_1 = \varepsilon_2 = 1$, one obtains from (III.27) the SM result in (III.27). In the scenario considered here, we have $99^\circ \leq \beta_X \leq 125^\circ$ and, consequently, $\varepsilon_1 = -1$ and $\varepsilon_2 = 1$. Other ranges of β_X will be considered below.

As in this scenario it is assumed that there are no significant contributions to $B^0_{s,d}$–$\bar{B}^0_{s,d}$ mixings and ε_K, in particular no complex phases, the determination of the CKM parameters through the standard analysis of the unitarity triangle proceeds as in the SM with the input parameters given in Section II.E. Consequently, β and β_s are already known from the usual analysis of the UT and the measurement of $\bar{r}_s$ in $K \to \pi\nu\bar{\nu}$ decays will provide a theoretically clean determination of θ_X and β_X. Similarly, a clean determination of $|X|$, with R_t determined in table V, is possible by means of (VI.7), so that using formulae of (Buras *et al.*, 2003b, 2004c) the electroweak parameters (q, ϕ) in $B \to \pi K$ decays will be determined. Assuming that the measurements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ with $5-10\%$ accuracy will be performed one day, the decays in question will most probably provide the cleanest measurements of (q, ϕ).

Using the results in (VI.3) and the parameters in (II.4) and (II.47)–(II.49), one finds (Buras *et al.*, 2004c)

$$Br(K^+ \to \pi^+\nu\bar{\nu}) = (7.5 \pm 2.1) \cdot 10^{-11}, \quad Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) = (3.1 \pm 1.0) \cdot 10^{-10}. \tag{VI.8}$$

This should be compared with the SM predictions given in (IV.5) and (IV.6).

We observe that, in spite of the enhanced value of $|X|$, $Br(K^+ \to \pi^+\nu\bar{\nu})$ does not significantly differ from the SM estimate because the enhancement of the first term in (VI.4) is to a large extent compensated by the suppression of the second term ($\cos\beta_X \ll \cos(\beta - \beta_s)$) and its reversed sign. Consequently, $Br(K^+ \to \pi^+\nu\bar{\nu})$ is here very strongly dominated by the "top" contribution given by the function X and charm-top interference is either small or even destructive.

On the other hand, we observe a spectacular enhancement of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ by one order of magnitude. Consequently, while $Br(K_{\rm L} \to \pi^0\nu\bar{\nu}) \approx (1/3)Br(K^+ \to \pi^+\nu\bar{\nu})$ in the SM, it is substantially larger than $Br(K^+ \to \pi^+\nu\bar{\nu})$ in this scenario. The huge enhancement of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ seen here is mainly due to the large weak phase $\beta_X \approx 111^\circ$, as

$$\frac{Br(K_{\rm L} \to \pi^0\nu\bar{\nu})}{Br(K_{\rm L} \to \pi^0\nu\bar{\nu})_{\rm SM}} = \left|\frac{X}{X_{\rm SM}}\right|^2 \left[\frac{\sin\beta_X}{\sin(\beta - \beta_s)}\right]^2 \tag{VI.9}$$

and to a lesser extent due to the enhanced value of $|X|$.

Inspecting (VI.4) and (VI.5), one observes (Buras *et al.*, 2004c) that the very strong dominance of the "top" contribution in these expressions implies a simple approximate expression:

$$\frac{Br(K_{\rm L} \to \pi^0\nu\bar{\nu})}{Br(K^+ \to \pi^+\nu\bar{\nu})} \approx 4.4 \times (\sin\beta_X)^2 \approx 4.2 \pm 0.2. \tag{VI.10}$$

We note that $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ is then rather close to its model-independent upper bound (Grossman and Nir, 1997) given in (V.4). It is evident from (VI.6) that this bound is reached when the reduced branching ratios B_1 and B_2 in (III.24) are equal to each other.

A spectacular implication of these findings is a strong violation of the MFV relation (Buchalla and Buras, 1994b) in (I.1). Indeed, one finds (Buras *et al.*, 2004c,d)

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = \sin 2\beta_X = -(0.69^{+0.23}_{-0.41}), \tag{VI.11}$$

in striking disagreement with $(\sin 2\beta)_{\psi K_{\rm S}} = 0.736 \pm 0.049$.

In Fig. 7, we show – in the spirit of the plots in Fig. 6 – $Br(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ for fixed values of β_X that has been presented in (Buras *et al.*, 2004c). As this plot is independent of $|X|$, it offers a direct measurement of the phase β_X. The first line on the left represents the MFV models with $\beta_X = \beta_{\rm eff} = \beta - \beta_s$, already discussed in Section V, whereas the first line on the right corresponds to the model-independent Grossman–Nir bound (Grossman and Nir, 1997) given in (V.4). The central value $\beta_X = 111^\circ$ in (VI.3) is very close to this bound. Note that the value of β_X corresponding to this bound depends on the actual value of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ as at this bound ($B_1 = B_2$) we have (Buras *et al.*, 2004c)

$$(\cot\beta_X)_{\rm Bound} = -\frac{\bar{P}_c(X)}{\varepsilon_2\sqrt{B_2}}. \tag{VI.12}$$

For the central values of $\bar{P}_c(X)$ and B_2 found here the bound corresponds to $\beta_X = 107.3^\circ$. As only $\cot\beta_X$ and not β_X is directly determined by the values of the branching ratios in question, the angle β_X is determined only up to

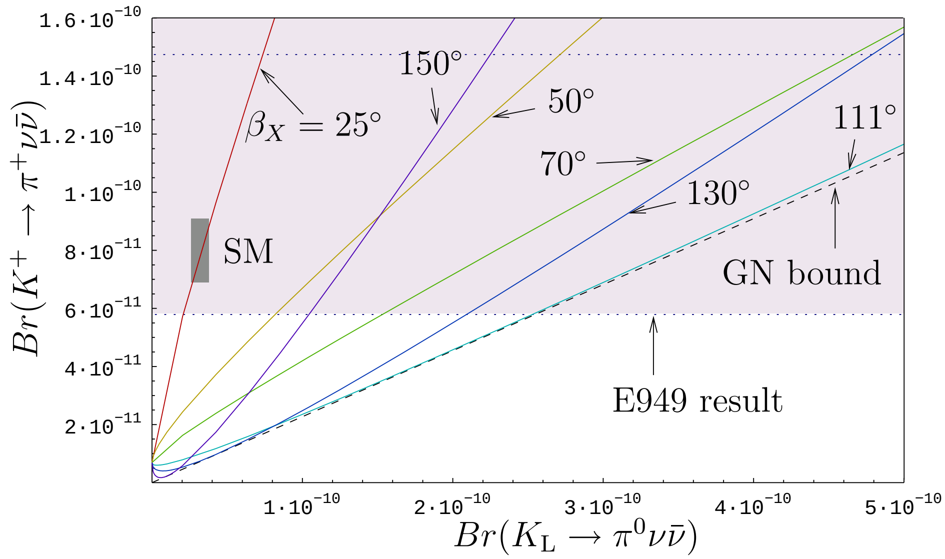

FIG. 7 $Br(K^+ \to \pi^+\nu\bar{\nu})$ as a function of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ for various values of β_X (Buras *et al.*, 2004c). The dotted horizontal lines indicate the lower part of the experimental range (I.4) and the grey area the SM prediction. We also show the bound in (V.4).

discrete ambiguities, seen already in Fig. 7. These ambiguities can be resolved by considering simultaneously other quantities discussed in (Buras *et al.*, 2004c).

Finally, we would like to emphasize one important feature of the correlation between $B \to \pi K$ decays and $K \to \pi\nu\bar{\nu}$ decays pointed out in (Buras *et al.*, 2004c,d). The huge enhancement of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and only a small impact on $Br(K^+ \to \pi^+\nu\bar{\nu})$ is related to $\theta_X < 0$ that is required by the present $B \to \pi K$ data. This is directly a consequence of the negative sign of the phase ϕ in the EW sector of the $B \to \pi K$ system. For $\phi > 0$ and $\theta_X > 0$ one would find $\beta_X < \beta$ and a suppression of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ with $Br(K^+ \to \pi^+\nu\bar{\nu})$ substantially enhanced.

C. General Discussion of θ_X and $|X|$

Clearly the data on $B \to \pi\pi$ and $B \to \pi K$ could change in the future implying different set of the values than given in (VI.3). In view of the data on $B \to X_s\mu^+\mu^-$, it is rather unlikely that $|X|$ could be larger than given in (VI.3). In what follows we will then assume that

$$1.25 \le |X| \le 2.25, \qquad -90° \le \theta_X \le 90°. \tag{VI.13}$$

The inspection of the formulae (VI.4) and (VI.5) reveals then the following simple facts:

- $Br(K^+ \to \pi^+\nu\bar{\nu})$ can be very close to the SM prediction, still allowing for a substantial departure of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ from the SM expectations and strong violation of the relation (I.1).
- If $Br(K^+ \to \pi^+\nu\bar{\nu})$ is found experimentally to be significantly larger than the SM prediction, the bound on $|X|$ in (VI.13) implies that $\cos\beta_X$ must be positive in order that the enhancement of $|X|$ is not compensated by the destructive interference of charm and top contributions. This in turn will also imply that $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ will be less enhanced than found in (Buras *et al.*, 2004c).

In Fig. 8, we show the ratio of the two branching ratios in question as a function of β_X for three values of $|X| = 1.25,\ 1.5,\ 2.0$. We observe that for β_X in the ballpark of $110°$ this ratio is very close to the bound in (V.4). However, even for $\beta_X = 50°$ the ratio is close to unity and by a factor of 3 higher than in the SM.

Finally, in table XV, we give the values of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ for the range in (VI.13) keeping $\beta = 23.5°$ and $|V_{cb}| = 41.5 \cdot 10^{-3}$. In this context we would like to refer to scaling laws for FCNC processes pointed out in (Buras and Harlander, 1992), from which it follows that the dependence of $K \to \pi\nu\bar{\nu}$ branching ratios on $|V_{cb}|$ and $|X|$ is encoded in a single variable

$$Z = A^2|X|. \tag{VI.14}$$

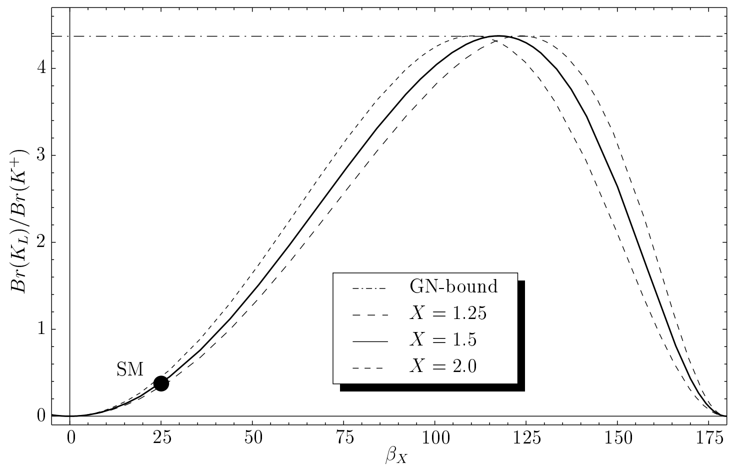

FIG. 8 The ratio of the $K \to \pi\nu\bar{\nu}$ branching ratios as a function of β_X for $|X| = 1.25,\ 1.5,\ 2.0$. The horizontal line is the bound in (V.4).

This observation allows to make the following replacement in table XV

$$|X| \to |X|_{\rm eff} = \left[\frac{|V_{cb}|}{41.5 \cdot 10^{-3}}\right]^2 |X|, \tag{VI.15}$$

so that for $|V_{cb}| \neq 41.5 \cdot 10^{-3}$ the results in this table correspond to different values of $|X|$ obtained by rescaling the values for $|X|$ there by means of (VI.15).

As beyond the SM the uncertainties in the value of $|X|$ are substantially larger than the ones in $|V_{cb}|$, the error in $|V_{cb}|$ can be absorbed into the one of $|X|_{\rm eff}$.

TABLE XV Values of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ (in parentheses) in units of 10^{-11} for different values of θ_X and $|X|$ with $\beta = 23.5^\circ$ and $|V_{cb}| = 41.5 \cdot 10^{-3}$.

$\theta_X/\lvert X\rvert$	1.25	1.50	1.75	2.00	2.25
-90°	2.3	3.3	4.5	6.0	7.6
	(10.1)	(14.5)	(19.8)	(25.8)	(32.7)
-60°	3.8	5.0	6.5	8.3	10.2
	(12.1)	(17.4)	(23.6)	(30.9)	(39.1)
-30°	5.1	6.7	8.4	10.4	12.6
	(8.1)	(11.6)	(15.8)	(20.7)	(26.1)
0°	6.0	7.8	9.7	11.9	14.3
	(2.1)	(3.0)	(4.1)	(5.4)	(6.8)
30°	6.3	8.0	10.0	12.3	14.7
	(0.11)	(0.16)	(0.22)	(0.29)	(0.36)
60°	5.8	7.4	9.3	11.5	13.8
	(4.1)	(5.9)	(8.0)	(10.5)	(13.3)
90°	4.6	6.1	7.8	9.7	11.8
	(10.1)	(14.5)	(19.8)	(25.8)	(32.7)

D. New Complex Phases in the $B_d^0 - \bar{B}_d^0$ Mixing

We next move to the scenario in which $X = X_{\rm SM}$ but there are new contributions to $B_d^0 - \bar{B}_d^0$ mixing. This scenario has been considered in detail in many papers (Bergmann and Perez, 2000, 2001; Bertolini *et al.*, 1987; D'Ambrosio and Isidori, 2002; Laplace, 2002; Laplace *et al.*, 2002; Nir and Silverman, 1990a,b), and lately in (Fleischer *et al.*, 2003). As summarized in the latter paper, this scenario can be realized in supersymmetric models with a) a heavy scale for the soft-breaking terms, b) new sources of flavour symmetry breaking only in the soft-breaking terms which do not involve the Higgs fields and c) Yukawa interactions very similar to the SM case. However, as emphasized in (Fleischer *et al.*, 2003) and discussed briefly in Section VII, this scenario is not representative for all supersymmetric scenarios, in particular those with important mass insertions of the left-right type and Higgs mediated FCNC amplitudes with large $\tan\beta$.

Let us recall that, in the presence of a complex function S_d, the off-diagonal term M_{12}^d in the neutral B_d^0 meson mass matrix has the phase structure

$$M_{12}^d = \frac{\langle B_d^0 | H_{eff}^{\Delta B=2} | \bar{B}_d^0 \rangle}{2 m_{B_d}} \propto e^{i2\beta} e^{i2\theta_d} |S_d| \tag{VI.16}$$

with $|S_d|$ generally differing from $S_0(x_t)$. If S_s remains unchanged, then

- The asymmetry $a_{\psi K_S}$ does not measure β but $\beta + \theta_d$
- The expression for R_t in (II.44) becomes

$$r_d R_t = 0.920\, \tilde{r} \left[\frac{\xi}{1.24}\right] \left[\frac{0.224}{\lambda}\right] \sqrt{\frac{18.4/ps}{\Delta M_s}} \sqrt{\frac{\Delta M_d}{0.50/ps}}, \qquad r_d^2 \equiv \left| \frac{S_d}{S_0(x_t)} \right|. \tag{VI.17}$$

As a consequence of these changes, the true angle β differs from the one extracted from $a_{\psi K_S}$ and also R_t and $|V_{td}|$ will be modified if $r_d \neq 1$.

The parameters (r_d, θ_d) can then be determined for instance as in (Fleischer *et al.*, 2003) by constructing the true unitarity triangle with the help of R_b determined from $|V_{ub}/V_{cb}|$ and γ determined by using the CP asymmetry in $B_d \to \pi^+\pi^-$, the asymmetry $a_{\psi K_S}$ and some input from $B \to \pi K$ decays. As this determination of γ could in principle suffer from new physics contributions, it should be replaced in the future by clean tree level strategies in B decays, that will be available at LHC and BTeV and will be briefly discussed in Section VIII.

As X is not modified with respect to the SM, the impact on $K \to \pi\nu\bar{\nu}$ amounts exclusively to the change of the true $\beta_{\rm eff}$ and R_t in the formulae (III.1) and (III.15). A particular pattern of a possible impact on $K \to \pi\nu\bar{\nu}$ in the scenario in question has been presented in (Fleischer *et al.*, 2003). We summarize the results of this paper in the following.

First let us recall that the measurement of $a_{\psi K_S}$ in (II.50) implies two solutions for $\beta + \theta_d$:

$$\beta + \theta_d \approx 23^\circ, \qquad \beta + \theta_d \approx 67^\circ\ . \tag{VI.18}$$

The authors of (Fleischer *et al.*, 2003) find then that

- In the case of the first solution, γ and R_t are found in the ballpark of the SM expectations and as the function X is not modified, $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ are only insignificantly affected by new physics contributions. One finds then in accordance with our expectations at the end of Section IV.B that $\theta_d > 0$ but it is small and the true value of β is close to 20°.
- In the case of the second solution, γ is found in the ballpark of 125°, R_t is substantially larger than in the SM but the true value of β with roughly 15° is significantly smaller than in the SM implying a large complex phase $\theta_d \approx 50^\circ$. As a result of this pattern, $Br(K^+ \to \pi^+\nu\bar{\nu})$, being sensitive to R_t, is enhanced in this scenario up to $2 \cdot 10^{-10}$. On the other hand $\bar{\eta} = R_t \sin\beta$ is only insignificantly modified so that $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ turns out to be close to the SM expectations although a slight suppression of this branching ratio could be expected. This solution corresponds roughly to the overlap of the $Br(K^+)$ band in the upper part of Fig. 5 with the ε_K and R_b constraints.

While this pattern is clearly possible, it does not represent the most general situation within the scenario considered. The point is that, in the absence of a direct measurement of γ that is not polluted by new physics, the only true values of the CKM parameters that we have to our disposal at present in a model independent manner, are the values of λ, $|V_{cb}|$ and R_b. With the function X given in (II.11), the unitarity of the CKM matrix implies then the following ranges

$$4.2 \cdot 10^{-11} \leq Br(K^+ \to \pi^+ \nu\bar{\nu}) \leq 16.9 \cdot 10^{-11}, \tag{VI.19}$$

$$0 \leq Br(K_{\rm L} \to \pi^0 \nu\bar{\nu}) \leq 5.2 \cdot 10^{-11}. \tag{VI.20}$$

If also $S_K = S_0(x_t)$, the ε_K constraint of (II.41) can also be taken into account implying more stringent ranges

$$5.8 \cdot 10^{-11} \leq Br(K^+ \to \pi^+ \nu\bar{\nu}) \leq 16.5 \cdot 10^{-11}, \tag{VI.21}$$

$$0.6 \cdot 10^{-11} \leq Br(K_{\rm L} \to \pi^0 \nu\bar{\nu}) \leq 3.6 \cdot 10^{-11}. \tag{VI.22}$$

The most recent determination of (r_d, θ_d), that uses the existing constraints on the UT, can be found in (Laplace, 2002; Laplace *et al.*, 2002). Large ranges for these two parameters are still possible. An interesting discussion with a different parametrization is also given in (D'Ambrosio and Isidori, 2002). On the other hand, once both $K \to \pi\nu\bar{\nu}$ branching ratios have been measured, the true values of β and R_t can be determined in this scenario from these decays as in the SM. Comparing subsequently the obtained value of β with the value of $a_{\psi K_S}$ one could determine θ_d. The parameter r_d can then be extracted from (VI.17).

In summary, we do not expect significant enhancement of $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ in this scenario but a substantial suppression of this branching ratio is still possible. On the other hand a significant enhancement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ cannot yet be excluded at present.

E. A Hybrid Scenario

The situation is more involved if new physics effects enter both X and S. Similarly to previous two scenarios, the golden relation in (I.1) is violated, but now the structure of a possible violation is more involved

$$[\sin 2(\beta - \theta_X)]_{\pi\nu\bar{\nu}} \neq [\sin 2(\beta + \theta_d)]_{\psi K_S}. \tag{VI.23}$$

Since θ_X originates in new contributions to the decay amplitude $K \to \pi\nu\bar{\nu}$ and θ_d in new contributions to the $B_d^0 - \bar{B}_d^0$ mixing, it is very likely that $\theta_X \neq \theta_d$.

The most straightforward strategy to disentangle new physics contributions in $K \to \pi\nu\bar{\nu}$ and the $B_d^0 - \bar{B}_d^0$ mixing in this scenario is to use the reference unitarity triangle that results from the (R_b, γ) strategy. Having the true CKM parameters at hand, one can determine θ_X and $|X|$ from $K \to \pi\nu\bar{\nu}$ and θ_d and $|S_d|$ from the $B_d^0 - \bar{B}_d^0$ mixing and $a_{\psi K_S}$.

In order to illustrate these ideas in explicit terms let us investigate, in the rest of this section, how the presence of new contributions in $K \to \pi\nu\bar{\nu}$ and the $B_d^0 - \bar{B}_d^0$ mixing could be signaled in the $(\bar{\varrho}, \bar{\eta})$ plane.

Beginning with $K \to \pi\nu\bar{\nu}$, let us write

$$X = r_X X_{\rm SM} e^{i\theta_X}. \tag{VI.24}$$

Then formulae (VI.4) and (VI.5) apply with

$$|X| \to X_{\rm SM}, \qquad R_t \to r_X R_t. \tag{VI.25}$$

We proceed then as follows:

- From the measured $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ we determine the "fake" angle β in the unitarity triangle with the help of (VI.6). We denote this angle by β_X, that we defined in (VI.2). In what follows we neglect β_s but it can be taken straightforwardly into account if necessary.

- The height of the fake UT from $K \to \pi\nu\bar{\nu}$ is then given by

$$\bar{\eta}_{\pi\nu\bar{\nu}} = r_X R_t \sin\beta_X = \frac{\sqrt{B_2}}{\tilde{r} A^2 X_{\rm SM}}, \tag{VI.26}$$

  where we set $\varepsilon_2 = +1$ in order to be concrete. As seen this height can be found from $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ and $X_{\rm SM}$.

Now let us go to the $B_d^0-\bar{B}_d^0$ mixing where we introduced the parameter r_d defined in (VI.17). We proceed then as follows:

- The asymmetry $a_{\psi K_S}$ determines the fake angle β, that we denote by $\beta_d=\beta+\theta_d$.
- The fake side R_t, to be denoted by $(R_t)_d$, is now given as follows

$$(R_t)_d = r_d R_t. \tag{VI.27}$$

It can be calculated from (VI.17) subject to uncertainties in ξ.

Clearly, generally the fake UT's resulting from $K\to\pi\nu\bar{\nu}$ and the $(\Delta M_d/\Delta M_d,\beta)$ strategy, discussed above, will differ from each other, from the true reference triangle and also from the UT obtained from the (γ,β) and $(\bar{\eta},\gamma)$ strategies, if the determinations of $\bar{\eta}$ and β are polluted by new physics.

We show these five different triangles in Fig. 9. Comparing the fake triangles with the reference triangle, all new physics parameters in $K\to\pi\nu\bar{\nu}$ and $B_d^0-\bar{B}_d^0$ mixing can be easily extracted.

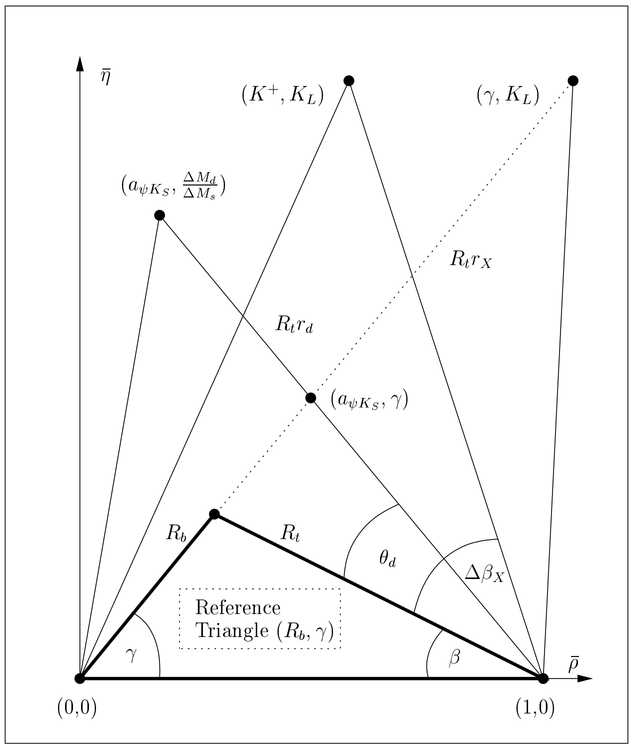

FIG. 9 Fake unitarity triangles as discussed in the text compared to the reference triangle. $\Delta\beta_X=-\theta_X$.

F. Correlation between $Br(K_{\rm L}\to\pi^0\nu\bar{\nu})$ and $Br(B\to X_{s,d}\nu\bar{\nu})$

The branching ratios for the inclusive rare decays $B\to X_{s,d}\nu\bar{\nu}$ can be written in the models with a new complex phase in X as follows (Buras *et al.*, 2004c) $(q=d,s)$

$$Br(B\to X_q\nu\bar{\nu}) = 1.58\cdot 10^{-5}\left[\frac{Br(B\to X_c e\bar{\nu})}{0.104}\right]\left|\frac{V_{tq}}{V_{cb}}\right|^2\left[\frac{0.54}{f(z)}\right]\,|X|^2, \tag{VI.28}$$

where $f(z)=0.54\pm0.04$ is the phase-space factor for $B\to X_c e\bar{\nu}$ with $z=m_c^2/m_b^2$, and $Br(B\to X_c e\bar{\nu})=0.104\pm0.004$.

Formulae (VI.5) and (VI.28) imply interesting relations between the decays $K_{\rm L}\to\pi^0\nu\bar{\nu}$ and $B\to X_{s,d}\nu\bar{\nu}$ that are generalizations of a similar relations within the MFV models (Bergmann and Perez, 2000, 2001; Buras and Fleischer,

2001) to the scenario considered here

$$\frac{Br(K_{\rm L} \to \pi^0 \nu\bar\nu)}{Br(B \to X_s \nu\bar\nu)} = \frac{\kappa_L}{1.58\cdot 10^{-5}} \left[\frac{0.104}{Br(B \to X_c e\bar\nu)}\right] \left[\frac{f(z)}{0.54}\right] A^4 R_t^2 \sin^2\beta_X, \tag{VI.29}$$

$$\frac{Br(K_{\rm L} \to \pi^0 \nu\bar\nu)}{Br(B \to X_d \nu\bar\nu)} = \frac{\kappa_L}{1.58\cdot 10^{-5}} \left[\frac{0.104}{Br(B \to X_c e\bar\nu)}\right] \left[\frac{f(z)}{0.54}\right] \frac{A^4 \tilde r^2}{\lambda^2} \sin^2\beta_X. \tag{VI.30}$$

The experimental upper bound on $Br(B \to X_s \nu\bar\nu)$ reads (Barate *et al.*, 2001)

$$Br(B \to X_s \nu\bar\nu) < 6.4 \cdot 10^{-4} \quad (90\% \text{ C.L.}). \tag{VI.31}$$

Using this bound and setting $R_t = 0.95$, $f(z) = 0.58$ and $Br(B \to X_c e\bar\nu) = 0.10$, we find from (VI.29) the upper bound

$$Br(K_{\rm L} \to \pi^0 \nu\bar\nu) \le 4.4 \cdot 10^{-9} (\sin\beta_X)^2 = \begin{cases} 7.5 \cdot 10^{-10} & \beta_X = 24.5^\circ \\ 3.9 \cdot 10^{-9} & \beta_X = 111^\circ \end{cases} \tag{VI.32}$$

at 90% C.L. for the MFV models and the scenario of Section VI.B, respectively. In the case of the MFV models this bound is weaker than the bound in (V.7) but, as the bound in (VI.31) should be improved in the B-factory era, the situation could change in the next years. Concerning the scenario with a complex phase θ_X of Section VI.B, no useful bound on $Br(K_{\rm L} \to \pi^0 \nu\bar\nu)$ from (VI.31) results at present as the bound in (VI.32) is weaker than the model independent bound in (V.6).

VII. $K \to \pi\nu\bar\nu$ IN SELECTED NEW PHYSICS SCENARIOS

A. Preliminaries

In this section we will briefly review the results for decays $K^+ \to \pi^+\nu\bar\nu$ and $K_{\rm L} \to \pi^0\nu\bar\nu$ in selected new physics scenarios. Our goal is mainly to indicate the size of new physics contributions in the branching ratios in question. Due to several free parameters present in some of these extensions the actual predictions for the branching ratios are not very precise and often depend sensitively on some of the parameters involved. The latter could then be determined or bounded efficiently once precise data on $K \to \pi\nu\bar\nu$ and other rare decays will be available. While we will only present the results for $Br(K^+ \to \pi^+\nu\bar\nu)$ and $Br(K_{\rm L} \to \pi^0\nu\bar\nu)$, most of the analyses discussed below used all available constraints from other observables known at the time of a given analysis. A detailed analysis of these constraints is clearly beyond the scope of this review. A general discussion of $K \to \pi\nu\bar\nu$ beyond the SM can be found in (Grossman and Nir, 1997). In writing this section we also benefited from (D'Ambrosio and Isidori, 2002; Isidori, 2003).

B. MSSM with MFV

There are many new contributions in MSSM such as charged Higgs, chargino, neutralino and gluino contributions. However, in the case of minimal flavour and CP violation it is a good approximation to keep only charged Higgs and chargino contributions.

To our knowledge the first analyses of $K \to \pi\nu\bar\nu$ in this scenario can be found in (Bertolini and Masiero, 1986; Bigi and Gabbiani, 1991; Giudice, 1987; Mukhopadhyaya and Raychaudhuri, 1987), subsequently in (Couture and Konig, 1995; Goto *et al.*, 1998) and the last in (Buras *et al.*, 2001a). In the latter analysis constraints on the supersymmetric parameters from ε_K, $\Delta M_{d,s}$, $B \to X_s\gamma$, $\Delta\varrho$ in the electroweak precision studies and from the lower bound on the neutral Higgs mass have been imposed. Supersymmetric contributions affect both the loop functions like $X(v)$ present in the SM and the values of the extracted CKM parameters like $|V_{td}|$ and ${\rm Im}\lambda_t$. As the supersymmetric contributions to the function $S(v)$ relevant for the analysis of the UT are always positive, the extracted values of $|V_{td}|$ and ${\rm Im}\lambda_t$ are always smaller than in the SM. Consequently, $Br(K^+ \to \pi^+\nu\bar\nu)$ and $Br(K_{\rm L} \to \pi^0\nu\bar\nu)$, that are sensitive to $|V_{td}|$ and ${\rm Im}\lambda_t$, respectively, are generally suppressed relative to the SM expectations. The supersymmetric contributions to the loop function $X(v)$ can compensate the suppression of $|V_{td}|$ and ${\rm Im}\lambda_t$ only for special values of supersymmetric parameters, so that in these cases the results are very close to the SM expectations.

Setting λ, $|V_{ub}|$ and $|V_{cb}|$, all unaffected by SUSY contributions, at their central values one finds (Buras *et al.*, 2001a)

$$0.65 \leq \frac{Br(K^+ \to \pi^+ \nu\bar{\nu})}{Br(K^+ \to \pi^+ \nu\bar{\nu})_{\rm SM}} \leq 1.02, \qquad 0.41 \leq \frac{Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})}{Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})_{\rm SM}} \leq 1.03. \tag{VII.33}$$

We observe that significant suppressions of the branching ratios relative to the SM expectations are still possible. More importantly, finding experimentally at least one of these branching ratios above the SM value would exclude this scenario, indicating new flavour violating sources beyond the CKM matrix. Similarly, in the MSSM based on supergravity a reduction of both $K \to \pi\nu\bar{\nu}$ rates up to 10% is possible (Goto *et al.*, 1998).

Reference (Buras *et al.*, 2001a) provides a compendium of phenomenologically relevant formulae in the MSSM, that should turn out to be useful once the relevant branching ratios have been accurately measured and the supersymmetric particles have been discovered at Tevatron, LHC and the e^+e^- linear collider. The study of the unitarity triangle can be found in (Ali and London, 1999a,b,c, 2001). The inclusion of NLO QCD corrections to the processes discussed in (Buras *et al.*, 2001a) has been performed in (Bobeth *et al.*, 2002). These corrections reduce mainly the renormalization scale uncertainties present in the analysis of (Buras *et al.*, 2001a), without modifying the results in (VII.33) significantly.

C. General Supersymmetric Models

In general supersymmetric models the effects of supersymmetric contributions to rare branching ratios can be larger than discussed above. In these models new CP-violating phases and new operators are present. Moreover the structure of flavour violating interactions is much richer than in the MFV models. Interestingly, due to the $V - A$ structure of the $\bar{\nu}\nu$ current, the only additional new operator is $(\bar{s}d)_{V+A}(\bar{\nu}\nu)_{V-A}$ if the neutrino masses are neglected. As the hadronic matrix elements of $(\bar{s}d)_{V+A}$ and $(\bar{s}d)_{V-A}$ are the same,[4] also the effects of this new operator can be included in the function X. But in contrast to the MFV models, the function X can now be a complex quantity as in the scenarios of Section VI. Moreover as there are new flavour violating interactions, new physics contributions are not governed by the CKM matrix and even if they contain some CKM dependence it is not simply given by an overall factor λ_t. This means that, writing the final expressions in terms of $\lambda_t X$ only, necessarily puts some CKM dependence into X.

The new flavour violating interactions are present because generally the sfermion mass matrices can be non-diagonal in the basis in which all neutral quark-squark-gaugino vertices and quark and lepton mass matrices are flavour diagonal. Instead of diagonalizing sfermion mass matrices it is convenient to consider their off-diagonal terms as new flavour violating interactions. This so–called mass–insertion approximation (Hall *et al.*, 1986) has been reviewed in the classic papers (Gabbiani *et al.*, 1996; Misiak *et al.*, 1998), where further references can be found.

In the context of the $K \to \pi\nu\bar{\nu}$ decays the most extensive analyses using the mass insertion method can be found in (Buras *et al.*, 2000, 1998; Colangelo and Isidori, 1998; Nir and Worah, 1998). It turns out that sizeable enhancements of $K \to \pi\nu\bar{\nu}$ rates can only be generated by chargino-mediated diagrams with a large $\tilde{u}^i_L - \tilde{u}^j_R$ mixing. The most recent of these papers (Buras *et al.*, 2000), finds the upper bounds

$$Br(K^+ \to \pi^+ \nu\bar{\nu}) \leq 1.7 \cdot 10^{-10}, \qquad Br(K_{\rm L} \to \pi^0 \nu\bar{\nu}) \leq 1.2 \cdot 10^{-10}. \tag{VII.34}$$

Larger values are possible, in principle, but rather unlikely. Moreover, as discussed in detail in (Buras *et al.*, 1998), in these models the MFV relation in (I.1) can be violated due to the presence of a new phase θ_X. A rough estimate shows that this phase could be as large as $\pm 25°$. This is not as large as found in (Buras *et al.*, 2004c,d) and in Section VI but still with $\beta_X \approx 50°$, as seen in Fig. 7 and 8, sizable departures from the SM are found.

Very recently the rare decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ have been reanalyzed in a general MSSM with conserved R-parity (Buras *et al.*, 2004a). Working in the mass eigenstate basis and performing the so-called adaptive scanning of a large space of supersymmetric parameters, 16 parameters in the reduced scan and 66 in the extended scan, the authors of (Buras *et al.*, 2004a) find that large departures from the Standard Model expectations are still possible while satisfying all existing constraints. Both branching ratios can be as large as few times 10^{-10} with $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ often larger than $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and close to its model independent upper bound. In particular the results of (Buras *et al.*, 2004c,d) can be obtained. The supersymmetric effects thus turn out to be larger than found in (Buras *et al.*, 2000). This is partly due to the fact that mass insertion approximation in contrast to (Buras *et al.*, 2000) has not been used and consequently no assumptions on the dominance of a single mass insertion as done usually in the

[4] QCD is insensitive to the sign of γ_5.

literature had to be made. The second reason is the exploration of much larger space of parameters that was possible in a reasonable time only by using specially designed Monte Carlo techniques.

As emphasized in (Buras *et al.*, 2000; Buras and Silvestrini, 1999), there exist correlations between $K \to \pi\nu\bar{\nu}$ decays, $K_L \to \mu^+\mu^-$ and ε'/ε, that could bound the size of the enhancement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$. Unfortunately, the hadronic uncertainties in $K_L \to \mu^+\mu^-$ and in particular in ε'/ε lower the usefulness of these correlations at present. More promising, in the context of supersymmetric models and also generally, appear the correlations between $K \to \pi\nu\bar{\nu}$ and rare FCNC semileptonic decays like $B \to X_{s,d}l^+l^-$, $B_{s,d} \to l^+l^-$ and in particular $B \to X_{s,d}\nu\bar{\nu}$, because also in these decays the main deviations from the SM can be encoded in an effective $Z\bar{b}q$ $(q = s, d)$ vertex (Atwood and Hiller, 2003; Buchalla *et al.*, 2001). We have discussed the correlation with $B \to X_{s,d}\nu\bar{\nu}$ in the previous section.

A systematic study of $K \to \pi\nu\bar{\nu}$ decays in flavour supersymmetric models has been performed in (Nir and Raz, 2002; Nir and Worah, 1998). These particular models are designed to solve naturally the CP and flavour problems characteristic for supersymmetric theories.[5] They are more constrained than the general supersymmetric models just discussed, in which parameters are tuned to satisfy the experimental constraints.

Models with exact universality of squark masses at a high energy scale with the A terms proportional to the corresponding Yukawa couplings, models with approximate CP, quark and squark alignment, approximate universality and heavy squarks have been analyzed in (Nir and Raz, 2002; Nir and Worah, 1998) in general terms. It has been concluded that in most of these models the impact of new physics on $K \to \pi\nu\bar{\nu}$ is sufficiently small so that in these scenarios one can get information on the CKM matrix from these decays even in the presence of supersymmetry. On the other hand, supersymmetric contributions to $B_d^0 - \bar{B}_d^0$ mixing in models with alignment, with approximate universality and heavy squarks can significantly affect the asymmetry $a_{\psi K_S}$, so that in these models the golden relation (I.1) can be violated.

Finally, in supersymmetric models with non-universal A terms, enhancements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ up to $1.5 \cdot 10^{-10}$ and $2.5 \cdot 10^{-10}$ are possible, respectively (Chen, 2002). Significant departures from the SM expectations have also been found in supersymmetric models with R-parity breaking (Deandrea *et al.*, 2004).

D. Models with Universal Extra Dimensions

The decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ have been studied in the SM model with one extra universal dimension in (Buras *et al.*, 2003d). In this model (ACD) (Appelquist *et al.*, 2001) all the SM fields are allowed to propagate in all available dimensions and the relevant penguin and box diagrams receive additional contributions from Kaluza-Klein (KK) modes. This model belongs to the class of MFV models and the only additional free parameter relative to the SM is the compactification scale $1/R$. Extensive analyses of the precision electroweak data, the analyses of the anomalous magnetic moment of the muon and of the $Z \to b\bar{b}$ vertex have shown the consistency of the ACD model with the data for $1/R \geq 300\,\text{GeV}$. We refer to (Buras *et al.*, 2004e, 2003d) for the list of relevant papers.

For $1/R = 300\,\text{GeV}$ and $1/R = 400\,\text{GeV}$ the function X is found with $m_t = 167\,\text{GeV}$ to be $X = 1.67$ and $X = 1.61$, respectively. This should be compared with $X = 1.53$ in the SM. In contrast to the analysis in the MSSM discussed in (Buras *et al.*, 2001a) and above, this $5-10\%$ enhancement of the function X is only insignificantly compensated by the change in the values of the CKM parameters. Consequently, the clear prediction of the model are the enhanced branching ratios $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$, albeit by at most 15% relative to the SM expectation. These enhancements allow to distinguish this scenario from the MSSM with MFV.

The enhancement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ in the ACD model is interesting in view of the experimental results in (I.5) with the central value by a factor of 1.8 higher than the central value in the SM. Even if the errors are substantial and this result is compatible with the SM, the ACD model with a low compactification scale is closer to the data. In table XVI we show the upper bound on $Br(K^+ \to \pi^+\nu\bar{\nu})$ in the ACD model obtained in (Buras *et al.*, 2003d) by means of the formula (III.10), with X replaced by its enhanced value in the model in question. To this end $|V_{cb}| \leq 0.0422$, $P_c(X) < 0.47$, $m_{\rm t}(m_t) < 172$ GeV and $\sin 2\beta = 0.734$ have been used. Table XVI illustrates the dependence of the bound on the nonperturbative parameter ξ, $1/R$ and ΔM_s. We observe that for $1/R = 300$ GeV and $\xi = 1.30$ the maximal value for $Br(K^+ \to \pi^+\nu\bar{\nu})$ in the ACD model is rather close to the central value in (I.5).

Clearly, in order to distinguish these results and the ACD model from the SM, other quantities, that are more sensitive to $1/R$, should be simultaneously considered. In this respect, the sizable downward shift of the zero ($\hat{s}_0$) in the forward-backward asymmetry $A_{\rm FB}$ in $B \to X_s\mu^+\mu^-$ and the suppression of $Br(B \to X_s\gamma)$ by roughly 20% at $1/R = 300\,\text{GeV}$ appear to be most interesting (Buras *et al.*, 2004e).

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[5] See the review in (Grossman *et al.*, 1998b).

TABLE XVI Upper bound on $Br(K^+ \to \pi^+ \nu\bar{\nu})$ in units of 10^{-11} for different values of ξ, $1/R$ and $\Delta M_s = 18/\mathrm{ps}$ (21/ps) from (Buras *et al.*, 2003d).

ξ	$1/R = 300$ GeV	$1/R = 400$ GeV	SM
1.30	12.0 (10.7)	11.3 (10.1)	10.8 (9.3)
1.25	11.4 (10.2)	10.7 (9.6)	10.3 (8.8)
1.20	10.7 (9.6)	10.1 (9.1)	9.7 (8.4)
1.15	10.1 (9.0)	9.5 (8.5)	9.1 (7.9)

E. Models with Lepton-Flavour Mixing

In the presence of flavour mixing in the leptonic sector, the transition $K_L \to \pi^0 \nu_i \bar{\nu}_j$, with $i \neq j$ could receive significant CP-conserving contributions (Grossman and Nir, 1997). Subsequently this issue has been analyzed in (Perez, 1999, 2000) and recently in (Grossman *et al.*, 2004). Here we summarize briefly the main findings of these papers.

In (Perez, 1999, 2000) the effect of light sterile right-handed neutrinos leading to scalar and tensor dimension-six operators has been analyzed. As shown there, the effect of these operators is negligible, if the right-handed neutrinos interact with the SM fields only through their Dirac mass terms.

Larger effects are expected from the operators

$$O_{sd}^{ij} = (\bar{s}\gamma_\mu d)(\bar{\nu}_L^i \gamma_\mu \nu_L^j), \tag{VII.35}$$

that for $(i \neq j)$ create a neutrino pair which is not a CP eigenstate. As shown in (Grossman *et al.*, 2004) the condition for a non-vanishing $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ rate in this case is rather strong. One needs either CP violation in the quark sector or a new effective interaction that violates both quark and lepton universality. One finds then the following pattern of effects:

- If the source of universality breaking is confined to mass matrices, the effects of lepton-flavour mixing get washed out in the $K \to \pi\nu\bar{\nu}$ rates after the sum over the neutrino flavours has been done. There are in principle detectable effects of lepton mixing only in cases where there are two different lepton-flavour mixing matrices, although they cannot be large.

- In models in which simultaneous violation of quark and lepton universality proceeds entirely through Yukawa couplings, the CP conserving effects in $K \to \pi\nu\bar{\nu}$ are suppressed by Yukawa couplings. As explicitly shown in (Grossman *et al.*, 2004) even in the MSSM with flavour violation and large $\tan\beta$ these types of effects are negligible.

- In exotic scenarios, such as R-parity violating supersymmetric models, lepton flavour mixing could generate sizable CP-conserving contributions to $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ and generally in $K \to \pi\nu\bar{\nu}$ rates.

F. Other Models

There exist other numerous analyses of $K \to \pi\nu\bar{\nu}$ decays within various extensions of the SM. For completeness we briefly describe them here.

In (Carlson *et al.*, 1996) the rate for $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ has been calculated in several extensions of the SM Higgs sector, including the Liu-Wolfenstein two-doublet model of spontaneous CP- violation and the Weinberg three doublet model. It has been concluded that although in the usual two Higgs doublet model, with CP-violation governed by the CKM matrix, some measurable effects could be seen, in models in which CP-violation arises either entirely or predominantly from the Higgs sector the decay rate is much smaller than in the SM.

Similarly, in supersymmetric models with large $\tan\beta$ and no new CP-violating phases, the new physics contributions to $K \to \pi\nu\bar{\nu}$ decays are negligible (Buras *et al.*, 2002, 2003a). This is because the usual charged Higgs and chargino contributions to these decays are suppressed for large $\tan\beta$ and the flavour changing Higgs penguins, that for $\tan\beta \approx 40$ can enhance $B_{d,s} \to \mu^+\mu^-$ rates by 2-3 orders of magnitude, give negligible contributions to $K \to \pi\nu\bar{\nu}$ because of the tiny neutrino masses.

The study of $K \to \pi\nu\bar{\nu}$ in models with four generations, extra vector-like quarks and isosinglet down quarks can be found in (Aguilar-Saavedra, 2003; Hattori *et al.*, 1998; Hawkins and Silverman, 2002; Huang *et al.*, 2001; Hung and

Soddu, 2002; Yanir, 2002). In particular in four generation models (Hattori *et al.*, 1998; Huang *et al.*, 2001; Yanir, 2002) due to three additional mixing angles and two additional complex phases, $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ can be enhanced by 1-2 orders of magnitude with respect to the SM expectations and also $Br(K^+ \to \pi^+ \nu\bar{\nu})$ can be significantly enhanced. Unfortunately, due to many free parameters, the four generation models are not very predictive. Very recently a new analysis of $K \to \pi\nu\bar{\nu}$ in a model with an extra isosinglet down quark appeared in (Deshpande *et al.*, 2004). Putting all the available constraints on the parameters of this model, the authors conclude that $Br(K^+ \to \pi^+ \nu\bar{\nu})$ can still be enhanced up to the present experimental central value, while $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ can reach $1 \cdot 10^{-10}$.

The decays $K \to \pi\nu\bar{\nu}$ have also been investigated in a seesaw model for quark masses (Kiyo *et al.*, 1999). In this model there are scalar operators $(\bar{s}d)(\bar{\nu}_\tau \nu_\tau)$, resulting from LR box diagrams, that make the rate for $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ non-vanishing even in the CP conserving limit and in the absence of lepton-flavour mixing. But the enhancement of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ due to these operators is at most of order 30% even for $M_{W_R} = 500\,{\rm GeV}$ with a smaller effect in $Br(K^+ \to \pi^+ \nu\bar{\nu})$.

The effects of the electroweak symmetry breaking on rare K and B decays, including $K \to \pi\nu\bar{\nu}$, in the presence of new strong dynamics, have been worked out in (Buchalla *et al.*, 1996b; Burdman, 1997). Deviations from the SM in $K \to \pi\nu\bar{\nu}$ have been shown to be correlated with the ones in B decays (Burdman, 1997).

The implications of a modified effective $Zb\bar{b}$ vertex on $K \to \pi\nu\bar{\nu}$, in connection with the small disagreement between the SM and the measured asymmetry A^b_{FB} at LEP, have been discussed in (Chanowitz, 1999, 2001). While the predictions are rather uncertain, an enhancement of $Br(K^+ \to \pi^+ \nu\bar{\nu})$ by a factor of two, towards the central experimental value, is possible.

Recently (He and Valencia, 2004) the decays $K \to \pi\nu\bar{\nu}$ have been analyzed in models that are variations of left-right symmetric models in which right-handed interactions, involving in particular a heavy Z' boson, single out the third generation (He and Valencia, 2002, 2003). The contributions of these new non-universal FCNC interactions appear both at the tree and one-loop level. The tree level contributions involving Z' of the type $(\bar{s}d)_{V+A}(\bar{\nu}_\tau \nu_\tau)_{V+A}$ can be severely constrained by other rare decays, ε_K and in particular $B^0_s - \bar{B}^0_s$ mixing, when it will be measured. Still, they can enhance $Br(K^+ \to \pi^+ \nu\bar{\nu})$ to the central experimental value in (I.5) and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ could be as high as $1.4 \cdot 10^{-10}$. These enhancements are accompanied by an enhancement of ΔM_s and finding ΔM_s in the ballpark of the SM expectations would significantly weaken these enhancements. On the other hand new one loop contributions involving Z' boson are not constrained by $B^0_s - \bar{B}^0_s$ mixing and can give significant enhancements of both branching ratios even if $\Delta M_s \approx (\Delta M_s)_{SM}$. Unfortunately the presence of many free parameters in these new one loop contributions does not allow to make definite predictions.

Enhancement of both $K \to \pi\nu\bar{\nu}$ branching ratios up to 50% has been found in a five dimensional split fermions scenario (Chang and Ng, 2002) and the decay $K^+ \to \pi^+ \nu\bar{\nu}$ turns out to be the best for providing the constraints on the bulk SM in the Randall-Sundrum scenario (Burdman, 2002).

G. Summary

We have seen in this and the previous section that many scenarios of new physics allow still for significant enhancements of both $Br(K^+ \to \pi^+ \nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$: $Br(K^+ \to \pi^+ \nu\bar{\nu})$ can still be enhanced by factors of 2-3 and $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$ could be by an order of magnitude larger than expected within the SM. While for obvious reasons most of the papers concentrate on possible enhancements of both branching ratios, their suppressions in several scenarios are still possible. This is in particular the case of the MSSM with MFV and in several models in which CP violation arises from the Higgs sector.

Because most models contain several free parameters, definite predictions for $K \to \pi\nu\bar{\nu}$ can only be achieved by considering simultaneously as many processes as possible so that these parameters are sufficiently constrained. Interesting in this respect, as emphasized in Section VI, is the correlation between the sign of the new complex phase in the EW penguin sector signaled by data on $B \to \pi K$ decays and the enhancement of $Br(K_{\rm L} \to \pi^0 \nu\bar{\nu})$.

VIII. COMPARISON WITH OTHER DECAYS

After this exposition of $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_{\rm L} \to \pi^0 \nu\bar{\nu}$ decays in the SM and its most studied extensions we would like to compare the potential of these two clean rare decays in extracting the CKM parameters and in testing the SM and its extensions with other prominent K and B decays for which a rich literature exists. A subset of relevant references will be given below.

A. K Decays

In the K system, the most investigated in the past are the parameters ε_K and the ratio ε'/ε that describe respectively the indirect and direct CP violation in $K_L \to \pi\pi$ decays and the rare decays $K_L \to \mu^+\mu^-$ and $K_L \to \pi^0 e^+ e^-$. None of them can compete in the theoretical cleanness with the decays considered here but some of them are still useful.

First, the decay $K_L \to \mu^+\mu^-$ is fully dominated by the absorptive part that can be very reliably predicted in terms of the $K_L \to \gamma\gamma$ rate but is insensitive to the short distance physics. The short distance contributions to $K_L \to \mu^+\mu^-$, that originate in Z^0 penguins and box diagrams as in the $K \to \pi\nu\bar{\nu}$ decays, are by itself theoretically very clean and are known including NLO QCD corrections (Buchalla and Buras, 1999). Unfortunately, these contributions are hidden in the dispersive contribution to $K_L \to \mu^+\mu^-$ that contains also a long distance component which is very difficult to estimate. In spite of many efforts to estimate the room left for the short distance part in the rate for this decay, the situation is rather unsatisfactory (D'Ambrosio *et al.*, 1998b; Gomez Dumm and Pich, 1999; Greynat and de Rafael, 2003; Isidori and Unterdorfer, 2004; Knecht *et al.*, 1999; Valencia, 1998). In the SM one finds $Br(K_L \to \mu^+\mu^-)_{\rm SD} = (8 \pm 3) \cdot 10^{-10}$. While the chapter on the extraction of this component from the data is certainly not closed, let us quote the estimate of (D'Ambrosio *et al.*, 1998b; Isidori and Unterdorfer, 2004), which reads

$$Br(K_L \to \mu^+\mu^-)_{\rm SD} \le 2.5 \cdot 10^{-9}. \tag{VIII.36}$$

It can be used to bound new physics contributions, in particular those coming from enhanced Z^0 penguins. Here the master function $Y(v)$ (Buras, 2003a,b, 2004), instead of $X(v)$, plays the crucial role. It is slightly more sensitive to new physics contributions than $X(v)$ due to smaller importance of box diagrams but possible upper bounds on $Y(v)$ resulting from (VIII.36) should be considered with care. In any case one should not expect that $K_L \to \mu^+\mu^-$ will play an important role in the determination of the CKM parameters unless some important progress in understanding non-perturbative dynamics will be made. On the other hand it can be used as a rough tool in excluding certain new physics scenarios, as it already played in devising the GIM mechanism thirty years ago.

Much more promising is the decay $K_L \to \pi^0 e^+ e^-$ that has been a subject of interest already for many years (Cohen *et al.*, 1993; D'Ambrosio *et al.*, 1998a; Donoghue and Gabbiani, 1995; Ecker *et al.*, 1987; Gilman and Wise, 1980) It is similarly to $K_{\rm L} \to \pi^0\nu\bar{\nu}$ dominated by CP-violating (CPV) contributions but the direct CPV component is subdominant at least in the SM. The decay $K_{\rm L} \to \pi^0 e^+ e^-$ has recently been reconsidered within the SM (Buchalla *et al.*, 2003) in view of new NA48 data on $K_{\rm S} \to \pi^0 e^+ e^-$ and $K_{\rm L} \to \pi^0\gamma\gamma$ (Lai *et al.*, 2003; Lazzeroni, 2003), which allow a much better evaluation of the CP-conserving (CPC) and indirectly (mixing) CPV contributions. The CPC part is found to be below $3\cdot 10^{-12}$. Moreover, in the SM the indirectly (mixing) CPV contribution and the interference of both CPV contributions dominate the branching ratio in question, while the directly CPV contribution alone is significantly smaller and in the ballpark of $4 \cdot 10^{-12}$. Very recently an independent analysis in (Friot *et al.*, 2004) obtained similar results. In particular the intereference between the indirectly and directly CPV contributions has been found to be constructive in accordance with the findings in (Buchalla *et al.*, 2003).

In the first scenario of Section VI, the SM pattern of CPV contributions is significantly changed (Buras *et al.*, 2004c), with the directly CPV contribution becoming dominant. Indeed, similar to $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$, the directly CPV contribution to $Br(K_{\rm L} \to \pi^0 e^+ e^-)$ is enhanced by more than one order of magnitude.

Explicit expression for the branching ratio in the SM including all contributions can be found in (Buchalla *et al.*, 2003) and its generalization to the first scenario of Section VI has been presented in (Buras *et al.*, 2004c). Again, as in the case of $K_L \to \mu^+\mu^-$, the short distance directly CPV component can be calculated very reliably, although due to the presence of ordinary γ penguins, the NLO result of (Buras *et al.*, 1994a) has slightly larger theoretical uncertainties than the results for $K \to \pi\nu\bar{\nu}$ decays. However, the presence of the indirectly CPV contribution that is dominant in the SM and its most extensions, limits the precision on this decay at least at present. A precise measurement of $Br(K_S \to \pi^0 e^+ e^-)$, hopefully available from KLOE at Frascati one day, that could give a good estimate of the indirectly CPV component, could promote $K_L \to \pi^0 e^+ e^-$ to the leading decays in testing the SM and its extensions. But the fact that the directly CPV component is generally subdominant in this decay does not allow $K_L \to \pi^0 e^+ e^-$ to compete with $K^+ \to \pi^+\nu\bar{\nu}$ and in particular with $K_{\rm L} \to \pi^0\nu\bar{\nu}$.

The present experimental bound from KTeV (Alavi-Harati *et al.*, 2004),

$$Br(K_{\rm L} \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10} \quad (90\% \text{ C.L.}), \tag{VIII.37}$$

should be compared with the SM prediction (Buchalla *et al.*, 2003; Isidori *et al.*, 2004),

$$Br(K_{\rm L} \to \pi^0 e^+ e^-)_{\rm SM} = (3.7^{+1.1}_{-0.9}) \cdot 10^{-11} \tag{VIII.38}$$

and with a very similar result $(3.7 \pm 0.4) \cdot 10^{-11}$ obtained recently in (Friot *et al.*, 2004).

In the first scenario of Section VI one finds (Buras *et al.*, 2004c; Isidori *et al.*, 2004)

$$Br(K_L \to \pi^0 e^+ e^-)^{\rm NP}_{\rm EXP} = (9.0 \pm 1.6) \cdot 10^{-11}, \tag{VIII.39}$$

which is lower than the upper bound in (VIII.37) by only a factor of 3.

Very recently also the decay $K_L \to \pi^0 \mu^+ \mu^-$ has been reconsidered within the SM (Isidori *et al.*, 2004). Again, here the three components are present with the CPC contribution playing significantly more important role than in $K_L \to \pi^0 e^+ e^-$. Fortunately this component can be estimated rather reliably (Isidori *et al.*, 2004) and one finds for the full branching ratio (Isidori *et al.*, 2004)

$$Br(K_{\rm L} \to \pi^0 \mu^+ \mu^-)_{\rm SM} = (1.5 \pm 0.3) \cdot 10^{-11}, \tag{VIII.40}$$

to be compared with the experimental bound (Alavi-Harati *et al.*, 2000a)

$$Br(K_{\rm L} \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10} \quad (90\% \text{ C.L.}). \tag{VIII.41}$$

The indirectly CPV and the CPC contributions are roughly of the same size and contribute together 2/3 of the full branching ratio. The rest comes from the interference of the CPV contributions and the direct CPV contribution alone. Of particular interest is the finding that the pure direct CPV component in $K_{\rm L} \to \pi^0 \mu^+ \mu^-$ contributes roughly 14% to the branching ratio to be compared with 7% in the case of $K_{\rm L} \to \pi^0 e^+ e^-$. Consequently $K_{\rm L} \to \pi^0 \mu^+ \mu^-$ is more sensitive to new physics contributions. Indeed, in the new physics scenario of (Buras *et al.*, 2004c,d), that we discussed in Section VI.B, the authors of (Isidori *et al.*, 2004) find

$$Br(K_{\rm L} \to \pi^0 \mu^+ \mu^-)^{\rm NP}_{\rm EWP} = (4.3 \pm 0.7) \cdot 10^{-11}, \tag{VIII.42}$$

that is an enhancement of the SM branching ratio by a factor of 3 to be compared to 2.4 in the case of the $\pi^0 e^+ e^-$ channel. Similarly to the latter case the direct CPV component dominates $Br(K_{\rm L} \to \pi^0 \mu^+ \mu^-)$ in this scenario.

While not as clean as $K \to \pi \nu \bar\nu$, the decays $K_{\rm L} \to \pi^0 \mu^+ \mu^-$ and $K_{\rm L} \to \pi^0 e^+ e^-$ will certainly play an important role in future investigations. In particular, as emphasized in (Isidori *et al.*, 2004), the simultaneous consideration of these two decays can similarly to the $K \to \pi \nu \bar\nu$ complex give deeper insight into short distance dynamics. A plot of $Br(K_{\rm L} \to \pi^0 \mu^+ \mu^-)$ versus $Br(K_{\rm L} \to \pi^0 e^+ e^-)$ in the spirit of Fig. 7 can be found in (Isidori *et al.*, 2004).

Much worse is the situation with ε'/ε. In principle, ε'/ε could be a very good quantity to bound the size of the electroweak penguins as it is very sensitive to this component, but a very poor knowledge of the hadronic matrix elements of QCD-penguin operators, that dominate this ratio, precludes its usefulness in testing the SM and its extensions efficiently at present. This is unfortunate because experimentalists, after many efforts, succeeded in measuring ε'/ε with an accuracy of $\pm 10\%$ (Alavi-Harati *et al.*, 1999, 2003; Batley *et al.*, 2002; Lai *et al.*, 2001). A recent review with relevant references can be found in (Buras and Jamin, 2004).

Finally a comment on ε_K should be made. Our view here is probably more optimistic than some other views exposed in the literature. The dominant uncertainty here is the parameter $\hat B_K$ that is known still only to $\pm 15\%$. The remaining uncertainties connected with $|V_{cb}|$ and m_t should be decreased in the coming years to an acceptable level. The fate of the usefulness of ε_K lies then in the hands of lattice gauge theorists. However, the fact that the non-perturbative uncertainties here can be collected in one single factor, makes ε_K superior to many non-leptonic B decays for which hadronic matrix elements are very difficult to calculate in QCD.

In summary, in the K system, $K \to \pi \nu \bar\nu$ decays have no competition from other decays, but the $K_L \to \pi^0 l^+ l^-$ complex and the parameter ε_K may in due time contribute to the precision tests of flavour dynamics.

B. B Decays

The situation with B decays is very different. First of all there are many more channels than in K decays, which allows to eliminate or reduce many hadronic uncertainties by simultaneously considering several decays and using flavour symmetries. Also the fact that now the b quark mass is involved in the effective theory allows to calculate hadronic amplitudes in an expansion in the inverse power of the b quark mass and invoke related heavy quark effective theory, heavy quark expansions, QCD factorization for non-leptonic decays, perturbative QCD approach and others. During the last years considerable advances in this field have been made (Battaglia *et al.*, 2003). While in semi-leptonic tree level decays this progress allowed to decrease the errors on the elements $|V_{ub}|$ and $|V_{cb}|$ (Battaglia *et al.*, 2003), in the case of prominent radiative decays like $B \to X_s \gamma$ and $B \to X_s l^+ l^-$, these methods allowed for a better estimate of hadronic uncertainties. In addition during last decade and in this decade theoretical uncertainties in these decays have been considerably reduced through the computations of NLO and in certain cases NNLO QCD corrections (Ali, 2003; Buchalla, 2003; Buchalla *et al.*, 1996a; Buras, 1998; Fleischer, 2002, 2004; Hurth, 2003; Nir, 2001).

In the case of non-leptonic decays, various strategies for the determination of the angles of the unitarity triangle have been proposed. Excellent reviews of these strategies have been given by Fleischer in (Fleischer, 2002, 2004). See also (Buras, 2003a,b, 2004) and (Ali, 2003; Buchalla, 2003; Hurth, 2003; Nir, 2001). These strategies generally use simultaneously several decays and are based on plausible dynamical assumptions that can be furthermore tested by invoking still other decays.

There is no doubt that these methods will give us considerable insight into flavour and QCD dynamics but it is fair to say that most of them cannot match the $K \to \pi\nu\bar{\nu}$ decays with respect to the theoretical cleanness. On the other hand there exist a number of strategies for the determination of the angles and also sides of the unitarity triangle that certainly can compete with the $K \to \pi\nu\bar{\nu}$ complex and in certain cases are even slightly superior to it, provided corresponding measurements can be made precisely. These are the strategies which we will briefly discuss in what follows.

Let us first recall that, among the theoretically cleanest strategies for the determination of the angles of the unitarity triangle, are in principle the decays of B_d^0 and B_s^0 into CP eigenstates. With the help of time dependent CP asymmetries in these decays a theoretically clean measurement of the CKM phases can be made, provided the decay amplitude is dominated by a single weak phase. The exposition of these methods can be found in (Ali, 2003; Anikeev *et al.*, 2001; Ball *et al.*, 2000; Buchalla, 2003; Buras, 2003a,b, 2004; Fleischer, 2002, 2004; Harrison and Quinn, 1998; Hurth, 2003; Nir, 2001) and the original papers quoted below.

The classic example here is the mixing induced CP asymmetry in the decays $B_d^0(\bar{B}_d^0) \to \psi K_S$ that allows within the SM a direct measurement of the angle β in the UT without any theoretical uncertainties (Bigi and Sanda, 1981; Carter and Sanda, 1980, 1981). As discussed recently (Boos *et al.*, 2004), even at the level of experimental precision of $\sigma(\sin 2\beta) = 0.005$, theoretical uncertainties in the determination of β through $a_{\psi K_S}$ can be neglected. As the decay amplitude is dominated by the tree diagrams, the new physics effects in the decay amplitude are likely to be unimportant but of course new physics could enter through $B_d^0 - \bar{B}_d^0$ mixing implying that in such a case the asymmetry in question measures not β but $\beta + \theta_d$ with θ_d being a new complex phase in the $B_d^0 - \bar{B}_d^0$ mixing. We have considered such a scenario in Section VI. The important point is that $\beta + \theta_d$ can be measured in a theoretically clean manner, but the fact that the measurement is in principle subject to "new physics" pollution does not guarantee the extraction of the true angle β from this asymmetry.

The direct determination of the angle α by means of the time dependent CP asymmetry in the decays $B_d^0(\bar{B}_d^0) \to \pi^+\pi^-$ is a different story. As indicated by the data from Belle and BaBar (Abe *et al.*, 2003a, 2004; Jawahery, 2003), in this decay, in addition to tree diagram contributions also QCD penguin diagrams play a significant role.

A clean measurement of α in the presence of this "QCD penguin pollution" is therefore impossible from this decay alone. The well known strategy to deal with this "penguin problem" is the isospin analysis of Gronau and London (Gronau and London, 1990). It requires however the measurement of $Br(B^0 \to \pi^0\pi^0)$, which for years was not available. With the recent measurements of this branching ratio at BaBar and Belle first steps in this direction could be made but the necessary input is incomplete yet and a precise determination of α using this method appears from the present perspective rather difficult. For this reason several, rather involved, strategies have been proposed. They are reviewed in (Ali *et al.*, 2004; Ball *et al.*, 2000; Buras and Fleischer, 1998; Harrison and Quinn, 1998). A subset of these references can be found in (Charles, 1999; Fleischer and Mannel, 1997; Gronau, 1993; Gronau *et al.*, 2001; Grossman and Quinn, 1998). It is to be seen which of these methods will eventually allow us to measure α with a respectable precision.

More promising appear the strategies in which the $B \to \pi\pi$ system is used in conjuction with the angle β from $a_{\psi K_S}$ and with some minimal information on $B \to \pi K$ decays to extract the angle γ from these decays. This strategy has been inspired by the analyses performed in (Fleischer, 1999c, 2000; Fleischer *et al.*, 2003; Fleischer and Matias, 2002) and refined recently in (Buras *et al.*, 2004c,d), where $\gamma = (64.7 \pm 6.6)^\circ$ has been found. Similar results have been subsequently obtained in (Ali *et al.*, 2004). The decay $B_d \to \pi^+\pi^-$ was also discussed in (Botella and Silva, 2003; Buchalla and Safir, 2004) in the context of bounds on γ and the UT. For other analyses see (Luo and Rosner, 2003) and references therein.

Next we should mention here $B_d \to \phi K_S$, that within the SM also measures the angle β with very small hadronic uncertainties (Barbieri and Strumia, 1997; Ciuchini *et al.*, 1997; Fleischer, 1997; Grossman *et al.*, 1998a; Grossman and Worah, 1997; London and Soni, 1997) but the fact that it is a pure penguin decay makes it subject to new physics uncertainties. The departure of $(\sin 2\beta)_{\phi K_S}$ from $(\sin 2\beta)_{\psi K_S}$, as seen by Belle, may indicate indeed the presence of new physics in this decay (Datta, 2002; Fleischer and Mannel, 2001; Grossman *et al.*, 2003; Hiller, 2002; Khalil and Kou, 2003; Raidal, 2002) but, as the BaBar result does not indicate a significant departure of $(\sin 2\beta)_{\phi K_S}$ from $(\sin 2\beta)_{\psi K_S}$, the situation is rather unclear at present (Abe *et al.*, 2003b; Aubert *et al.*, 2004a; Browder, 2004). An analogue of $B_d \to \psi K_S$ in B_s-decays is $B_s \to \psi\phi$. The corresponding CP asymmetry measures here η (Buras, 1995) in the Wolfenstein parametrization or equivalently the phase β_s. It is very small, however, and this fact makes it a good place to look for the physics beyond the SM. In particular the CP violation in $B_s^0 - \bar{B}_s^0$ mixing from new sources beyond the SM could be probed in this decay. Another useful channel for β is $B_d \to D^+D^-$.

There are other decays of B_d^0 and B_s^0 to CP eigenstates but it appears at present that only $B_d^0 \to \psi K_S$ can provide a measurement of a CKM phase that could be put in a golden class together with $K \to \pi\nu\bar{\nu}$ decays.

Next decays of neutral B mesons B_d^0 and B_s^0 to CP non-eigenstates should be considered. The first prominent strategy of this type is based on the full time dependent analysis of $B_d^0 \to D^\pm\pi^\mp$ and $\bar{B}_d^0 \to D^\pm\pi^\mp$ (Diehl and Hiller, 2001; Dunietz, 1998; Dunietz and Sachs, 1988; Sachs, 1985). The fact that B_d^0 and $\bar{B}_d^0$ can decay to the same final state and the presence of only tree diagrams in these decays makes it possible to determine $2\beta + \gamma$ without any hadronic uncertainties. Taking β from $a_{\psi K_S}$, the angle γ can be determined. Unfortunately, the relevant interefences between amplitudes are $\mathcal{O}(\lambda^2)$ and the execution of this strategy is a very difficult experimental task. See (Silva *et al.*, 2003) for an interesting discussion.

The corresponding strategy in the $B_s^0(\bar{B}_s^0)$ sector is the one based on $B_s^0 \to D_s^\pm K^\mp$ and $\bar{B}_s^0 \to D_s^\pm K^\mp$ (Aleksan *et al.*, 1992; Falk and Petrov, 2000; Fleischer and Dunietz, 1996; London *et al.*, 2000). It measures $2\beta_s + \gamma$ without hadronic uncertainties and is experimentally more promising than the previous strategy as the relevant interference effects are larger. It is the leading strategy for the determination of the angle γ at LHC and BTeV. The angle β_s is expected to be very small as already discussed in Section II and this can be tested in the CP asymmetry in $B_s \to \psi\phi$.

We have then two strategies for γ that use the neutral B_d^0 and B_s^0 mesons that are theoretically very clean. Moreover, as the decays proceed solely through tree diagrams, the pollution from new physics in the decay amplitudes is likely to be negligible. But of course new physics can enter through new complex phases in $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings. As this new complex phases are universal and can be extracted from the asymmetries in $B_d \to \psi K_S$ and $B_s \to \psi\phi$, respectively, the angle γ can be extracted from both strategies in a theoretically clean manner without new physics pollution.

Clearly in order to avoid the pollution from new physics and hadronic uncertainties directly, we need strategies involving charged B decays that proceed only through tree diagrams, as in the last two strategies, but in contrast to them are not affected by the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings and consequently are free from any new phases. One would think that in this case it would be impossible to find a clean strategy, as $B^\pm$ decays, in which CP violation is only in the decay amplitudes, are generally subjects to large hadronic uncertainties. But here this problem can be avoided by considering several channels simultaneously, a luxury that is not given to us in K decays.

Indeed, by replacing the spectator s-quark in the strategy involving $B_s^0 \to D_s^\pm K^\mp$ and $\bar{B}_s^0 \to D_s^\pm K^\mp$ (Aleksan *et al.*, 1992; Falk and Petrov, 2000; Fleischer and Dunietz, 1996; London *et al.*, 2000) through the u-quark one arrives at decays of $B^\pm$ that can be used to extract the angle γ (Gronau and Wyler, 1991). One can easily check that this strategy is unaffected by penguin contributions. Moreover, as particle-antiparticle mixing is absent here, γ can be measured directly. Both these features make it plausible that this strategy, not involving to first approximation any loop diagrams, is particularly suited for the determination of γ without any new physics pollution.

By considering six decay rates $B^\pm \to D_{CP}^0 K^\pm$, $B^+ \to D^0 K^+$, $\bar{D}^0 K^+$ and $B^- \to D^0 K^-$, $\bar{D}^0 K^-$ where $D_{CP}^0 = (D^0 + \bar{D}^0)/\sqrt{2}$ is a CP eigenstate, the well known triangle construction due to Gronau and Wyler (Gronau and Wyler, 1991) allows to determine γ. However, the method is not without problems. The detection of D_{CP}^0, that is necessary for this determination, is experimentally challenging. Moreover, the small branching ratios of the colour suppressed channels $B^+ \to D^0 K^+$ and its charge conjugate, and the absence of this suppression in the two remaining channels imply a rather squashed triangle thereby making the extraction of γ very difficult. Still in view of the great potential of this strategy in determining the true angle γ, all efforts should be made to realize it. Variants of this method that could be more promising are discussed in (Atwood *et al.*, 1997; Dunietz, 1991; Gronau and London., 1991).

The three strategies discussed above can be generalized to other decays. In particular (Dunietz, 1991; Fleischer, 2003a,b,c; Gronau and London., 1991)

- $2\beta + \gamma$ and γ can be measured in

$$B_d^0 \to K_S D^0,\ K_S \bar{D}^0, \qquad B_d^0 \to \pi^0 D^0,\ \pi^0 \bar{D}^0 \tag{VIII.43}$$

  and the corresponding CP conjugated channels,

- $2\beta_s + \gamma$ and γ can be measured in

$$B_s^0 \to \phi D^0,\ \phi \bar{D}^0, \qquad B_s^0 \to K_S^0 D^0,\ K_S \bar{D}^0 \tag{VIII.44}$$

  and the corresponding CP conjugated channels,

- γ can be measured by generalizing the Gronau–Wyler construction to $B^\pm \to D^0\pi^\pm, \bar{D}^0\pi^\pm$ and to B_c decays (Fleischer and Wyler, 2000):

$$B_c^\pm \to D^0 D_s^\pm,\ \bar{D}^0 D_s^\pm, \qquad B_c^\pm \to D^0 D^\pm,\ \bar{D}^0 D^\pm\ . \tag{VIII.45}$$

It appears that the methods for γ discussed here may give useful results at later stages of CP-B investigations, in particular at LHC-B and BTeV.

Finally a few comments on the strategies for γ that use the U-spin symmetry should be made here. Although not as clean as the strategies discussed above, they could be the first to offer a respectable direct determination of this angle. They have been first proposed in (Fleischer, 1999a,b,c). The first strategy involves the decays $B^0_{d,s} \to \psi K_S$ and $B^0_{d,s} \to D^+_{d,s} D^-_{d,s}$. The second strategy involves $B^0_s \to K^+K^-$ and $B^0_d \to \pi^+\pi^-$. They are mainly limited by U-spin breaking effects. They are promising for Run II at Tevatron and in particular for LHCB and BTeV.

A method of determining γ, using $B^+ \to K^0\pi^+$ and the U-spin related processes $B^0_d \to K^+\pi^-$ and $B^0_s \to \pi^+K^-$, was presented in (Chiang and Wolfenstein, 2000; Gronau and Rosner, 2000). A general discussion of U-spin symmetry in charmless B decays and more references to this topic can be found in (Fleischer, 2002, 2004; Gronau, 2000).

Useful constraints for the UT within the SM and its extensions will come from the future measurements of ΔM_s and the branching ratios for $B \to X_{s,d}\nu\bar{\nu}$ and $B_{s,d} \to \mu^+\mu^-$. The relevant formulae within the SM and MFV are given in (II.44), (III.13) and (III.14), respectively. The fate of the usefulness of the measurement of ΔM_s depends on the accuracy with which ξ can be calculated. A similar comment applies to $B_{s,d} \to \mu^+\mu^-$. As shown in (Buras, 2003c), by combining $\Delta M_{s,d}$ with $B_{s,d} \to \mu^+\mu^-$ the uncertainties due to F_{B_d} and F_{B_s} in all these quantities can be eliminated making the tests cleaner, but of course some information is lost in these combinations and it is very desirable to have clean individual predictions for the four observables in question.

The case of $B \to X_{s,d}\nu\bar{\nu}$ is a different story. Here the relevant branching ratios are theoretically clean and their ratio offers probably the cleanest measurement of R_t but the experiments are very difficult. Using instead exclusive decays like $B \to K^*\nu\bar{\nu}$ or $B \to \rho\nu\bar{\nu}$, that are easier to measure, brings in theoretical uncertainties in the relevant formfactors. It should also be stated that $B \to X_s\nu\bar{\nu}$ and $B \to X_d\nu\bar{\nu}$ taken separately are not as clean as $K \to \pi\nu\bar{\nu}$ decays due to significant dependence on m_c^2/m_b^2 in the process of the normalization to tree level decays. Still the measurements of the branching ratios in question would be an important advance.

One should also emphasize that in models with non-minimal flavour violation the appearance of significant contributions from new operators can introduce new hadronic uncertainties that cannot be canceled in the ratios of the relevant observables. This is the case of the ratio $\Delta M_d/\Delta M_s$ that in the supersymmetric models with large $\tan\beta$ is related to R_t in a more complicated manner (Buras *et al.*, 2002, 2003a) than given in (II.44) or (VI.17).

Finally, we should comment on the decays $B \to X_{s,d}\gamma$ and $B \to X_{s,d}l^+l^-$ and their exclusive counterparts. These decays played already an important role in bounding the parameters of the SM and its extensions like supersymmetry. They will certainly continue to play this role. On the other hand, from the present perspective, the accuracy of the tests that can be made with the help of these decays are at best at the level of $\pm 10\%$ and consequently in the long run they will not be able to compete with the $K \to \pi\nu\bar{\nu}$ decays that are theoretically much cleaner. This is in particular the case of $K_{\rm L} \to \pi^0\nu\bar{\nu}$ that eventually should be more powerful in searching for new physics than $B \to X_{s,d}l^+l^-$ as stressed in (D'Ambrosio *et al.*, 2002).

IX. LONG DISTANCE CONTRIBUTIONS TO $K \to \pi\nu\bar{\nu}$

In this section we discuss briefly the long distance contributions to the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ that we have neglected so far. More detailed discussions and explicit calculations have been presented in (Buchalla and Isidori, 1998; Ecker *et al.*, 1988; Fajfer, 1997; Geng *et al.*, 1996; Hagelin and Littenberg, 1989; Lu and Wise, 1994; Rein and Sehgal, 1989). These effects can be potentially interesting especially when the NNLO calculation anticipated in Section IV is actually performed and $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ are measured with an accuracy of 5%.

Accordingly, we begin with the discussion of $K^+ \to \pi^+\nu\bar{\nu}$, where there can be, in principle, two additional contributions to the branching ratio:

- Effects through soft u quarks in the penguin loop that induce an on shell $K^+ \to \pi^+Z^0 \to \pi^+\nu\bar{\nu}$ transition. These effects are addressed (Ecker *et al.*, 1988; Fajfer, 1997; Geng *et al.*, 1996; Hagelin and Littenberg, 1989; Lu and Wise, 1994; Rein and Sehgal, 1989) in chiral perturbation theory.

- Higher dimensional operators contributing to the OPE in the charm sector (Falk *et al.*, 2001).

In particular, discussing only the contributions involving the Z^0 neutral current, which receive a $\Delta I = 1/2$ enhancement, the coupling of the Z^0 can be composed into the electromagnetic component and the left handed weak current:

$$\mathcal{L}_{int} = \frac{\sqrt{g_1^2+g_2^2}}{2} Z^{0\mu} \left[J^L_\mu - 2\sin^2\theta_W J^{em}_\mu \right] , \tag{IX.46}$$

where

$$J_\mu^L = \bar{u}_L \gamma_\mu u - \bar{d}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu s \equiv J_{8\mu}^L + J_{1\mu}^L, \tag{IX.47}$$

$$J_\mu^{em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s\,. \tag{IX.48}$$

Here, J_μ^L and J_μ^{em} transform as $(8_L, 1_R)$ and $(1_L, 1_R)$ with respect to the chiral symmetry group $SU(3)_L \times SU(3)_R$.

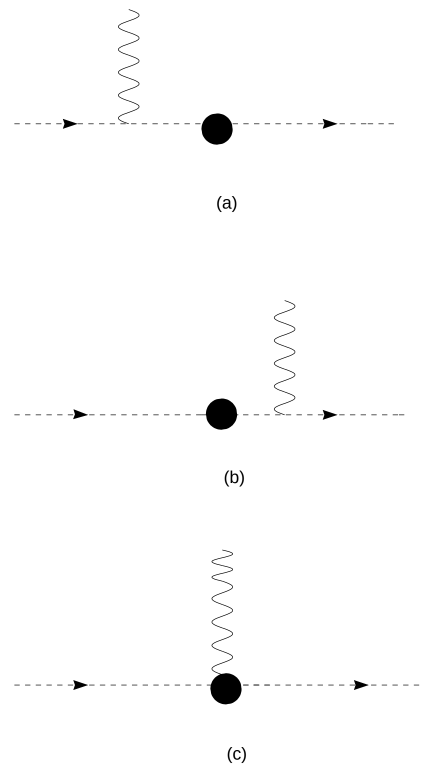

FIG. 10 Leading order chiral perturbation theory diagrams contributing to a $K^+ \to \pi^+ Z^0$ vertex (from (Lu and Wise, 1994)). The dashed lines denote the pion and kaon, while the wavy line denotes the Z^0, and the dot indicates the insertion of a flavor changing effective vertex.

The leading order diagrams for the $K^+ \to \pi^+ Z^0$ vertex are shown in Fig 10. A calculation of these diagrams for the electromagnetic current (Ecker *et al.*, 1988) yields that the sum of all contributions cancels. The octet contribution of the weak current can be calculated and is found to be numerically about 5% of the charm contribution:

$$\frac{Br(K^+ \to \pi^+ \nu\bar{\nu})_{LD}}{\kappa_+ (V_{us}^* V_{ud})^2} = \left(\frac{g^8 \pi^2}{3}\right)^2 \left(\frac{F_\pi}{M_W}\right)^4, \tag{IX.49}$$

where g^8 is known to be $|g^8| \simeq 5.1$ and F_π is the pion decay constant. Unfortunately, the analogous calculation for the singlet piece is not possible, due to the axial $U(1)$ anomaly that spoils chiral symmetry. Lu and Wise (Lu and Wise, 1994) circumvent this problem by considering the large N_c limit, where the $U(1)$ symmetry is restored. Calculating then the diagrams gives a contribution that precisely cancels the octet piece in (IX.49). One would expect that some cancellations take place also in the physical case of three colors.

On the other hand, there are the effects from higher dimensional operators in the OPE (Falk *et al.*, 2001), which would naturally to be expected comparable in size to the effects just discussed. They have to be considered only in the charm contributions, if one assumes a natural scaling of M_K^2/m_q^2 in the Wilson coefficients. The scaling of the Inami-Lim functions then leads to an overall scaling of M_K^2/M_W^4, which is independent of the quark masses. The top contribution is then simply suppressed by CKM factors.

Going to dimension eight, one finds two operators that appear when expanding the penguin and box diagrams:

$$O_1^l = \bar{s}\gamma^\nu(1-\gamma^5)d(i\partial)^2\left(\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l\right)$$
$$O_2^l = \bar{s}\gamma^\nu(1-\gamma^5)(iD)^2 d\;\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l +$$

$$2\bar{s}\gamma^\nu(1-\gamma^5)(iD^\mu)d\;\bar{\nu}_l\gamma_\nu(1-\gamma^5)(\partial_\mu)\nu_l +$$
$$\bar{s}\gamma^\nu(1-\gamma^5)d\;\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial)^2\nu_l\,, \tag{IX.50}$$

where D^μ is the covariant derivative involving the gluon field. While the matrix element of O_1^l can be rather reliably estimated and gives a negligible contribution compared to the leading dimension six terms, the matrix element of O_2^l is harder to estimate, due to the gluon appearing in the covariant derivative. It is found that in a very pessimistic scenario these contributions could have effects that are comparable to the uncertainties from the charm mass. The final word here will be spoken from the lattice community.

Let us now turn to $K_{\rm L} \to \pi^0\nu\bar{\nu}$. Long distance contributions here are mostly equivalent to CP conserving effects and have been comprehensively studied in (Buchalla and Isidori, 1998). As for $K^+ \to \pi^+\nu\bar{\nu}$, there are effects from soft up quarks, that are treated in chiral perturbation theory, and higher dimensional operators in the charm sector, which are actually short distance effects. It is found that they are suppressed by several effects, reinforcing the theoretically clean character of this decay. Let us briefly describe these effects.

The contributions from soft up quarks in the penguin loops have been studied in (Buchalla and Isidori, 1998) and in (Geng *et al.*, 1996). As is the case for $K^+ \to \pi^+\nu\bar{\nu}$, the leading diagrams appear at one loop order. They are calculated explicitely by Buchalla and Isidori (Buchalla and Isidori, 1998), who find, taking into account also phase space suppression, that the CP conserving long distance contributions are suppressed by approximately a factor of 10^{-5} compared to the dominant top contribution.

The next contribution that can be important are then higher dimensional operators in the OPE. As (Buchalla and Isidori, 1998) studies only CP conserving contributions only one operator that is antisymmetric in neutrino momenta, survives from the expansion of the box diagrams (contributions from Z^0-penguins also drop out for the same reason):

$$H_{CPC} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}\lambda_c\,\ln\frac{m_c}{\mu}\,\frac{1}{M_W^2}T_{\alpha\mu}\bar{\nu}(\overleftarrow{\partial^\alpha}-\partial^\alpha)\gamma^\mu(1-\gamma_5)\nu\;, \tag{IX.51}$$

$$T_{\alpha\mu} = \bar{s}\overleftarrow{D}_\alpha\gamma_\mu(1-\gamma_5)d - \bar{d}\gamma_\mu(1-\gamma_5)D_\alpha s\;. \tag{IX.52}$$

There arise now several suppression factors: First, there is the naive suppression of the operator scaling, which is estimated to be $\mathcal{O}(\lambda_c M_K^2/{\rm Im}\lambda_t M_W^2) \approx 10\%$ compared to the leading top contribution. Here, the smallness of M_K/M_W is compensated by the ratio of CKM factors $\lambda_c/{\rm Im}\lambda_t$.

The suppression is more severe when the matrix elements are calculated, since the leading order $K_L - \pi^0$ matrix element in chiral perturbation theory is found to be:

$$\langle\pi^0(p)|T_{\alpha\mu}|K_L(k)\rangle = -\frac{i}{2}[(k-p)_\alpha(k+p)_\mu + \frac{1}{4}m_K^2 g_{\alpha\mu}]\;, \tag{IX.53}$$

which vanishes when multiplied with the leptonic current in the operator due to the equations of motion and the negligible neutrino masses. The chiral suppression of the NLO (p^4) terms leads to an additional reduction of higher dimensional operator contributions by about $m_K^2/(8\pi^2 f_\pi^2) \approx 20\%$. Finally, one has to take into account also phase space effects, which further suppress these terms.

Estimating the $\mathcal{O}(p^4)$ matrix elements and performing the phase space calculations, the authors of (Buchalla and Isidori, 1998) find that short distance CP conserving effects are suppressed by a factor of 10^{-5} compared to the dominant top contribution and conclude that they are "safely negligible, by a comfortably large margin".

It is then fair to say, from the present perspective, that long distance effects are rather well under control especially in $K_{\rm L} \to \pi^0\nu\bar{\nu}$, but also in $K^+ \to \pi^+\nu\bar{\nu}$, where an improved estimate of the relevant hadronic matrix elements could increase our confidence in the negligibility of these effects. In view of the precision that should be achieved by the NNLO calculation it seems, however, mandatory to keep an eye on the contributions of higher dimensional operators in $K^+ \to \pi^+\nu\bar{\nu}$.

X. CONCLUSIONS AND OUTLOOK

In the present review we have summarized the present status of the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$, paying in particular attention to theoretical and parametric uncertainties. Our analysis reinforced the importance of these decays in testing the SM and its extensions. We have pointed out that the clean theoretical character of these decays remains valid in essentially all extensions of the SM, whereas this is often not the case for non-leptonic two-body B decays used to determine the CKM parameters through CP asymmetries and/or other strategies. Here, in extensions of the SM in which new operators and new weak phases are present, the mixing induced asymmetry $a_{\phi K_S}$ and other similar asymmetries suffer from potential hadronic uncertainties that make the determination of the relevant parameters problematic unless the hadronic matrix element can be calculated with sufficient precision. In spite of advances in non-perturbative calculations of non-leptonic amplitudes for B decays (Bauer *et al.*, 2002a,b,c; Beneke *et al.*, 1999, 2002; Beneke and Feldmann, 2003; Beneke and Neubert, 2003; Keum *et al.*, 2001a,b; Stewart, 2003), we are still far away from precise calculations of non-leptonic amplitudes from first principles. On the other hand the branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ can be parametrized in essentially all extensions of the SM by a single complex function X (real in the case of MFV models) that can be calculated in perturbation theory in any given extension of the SM.

There exists, however, a handful of strategies in the B system that similarly to $K \to \pi\nu\bar{\nu}$, are very clean. We have discussed them briefly in the previous section. Moreover, in contrast to $K \to \pi\nu\bar{\nu}$, there exist strategies involving B decays that allow not only a theoretically clean determination of the UT but also one free from new physics pollution.

Our main findings are as follows:

- Our present predictions for the branching ratios read

$$Br(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (7.8 \pm 1.2) \cdot 10^{-11}, \qquad (X.54)$$

$$Br(K_{\rm L} \to \pi^0\nu\bar{\nu})_{\rm SM} = (3.0 \pm 0.6) \cdot 10^{-11}. \qquad (X.55)$$

  This is an accuracy of $\pm 15\%$ and $\pm 20\%$, respectively.

- Our analysis of theoretical uncertainties in $K \to \pi\nu\bar{\nu}$, that come almost exclusively from the charm contribution to $K^+ \to \pi^+\nu\bar{\nu}$, reinforces the desire to perform a NNLO analysis of this contribution (Buras *et al.*, 2004b). Indeed the $\pm 18\%$ uncertainty in $P_c(X)$ coming dominantly from the scale uncertainties and the value of $m_c(m_c)$, translates into an uncertainty of $\pm 7.0\%$ in the determination of $|V_{td}|$, ± 0.04 in the determination of $\sin 2\beta$ and $\pm 10\%$ in the prediction for $Br(K^+ \to \pi^+\nu\bar{\nu})$. We believe that an NNLO analysis of the charm component and further progress on the determination of $m_c(m_c)$ could reduce the error in $P_c(X)$ down to $\pm 5\%$, implying the reduced error in $|V_{td}|$ of $\pm 2\%$, in $\sin 2\beta$ of ± 0.011 and $\pm 3\%$ in $Br(K^+ \to \pi^+\nu\bar{\nu})$.

- Further progress on the determination of the CKM parameters, that in the next few years will dominantly come from BaBar, Belle and Tevatron and later from LHC and BTeV, should allow eventually the predictions for $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ with the uncertainties of roughly $\pm 5\%$ or better. It should be emphasized that this accuracy cannot be matched by any other rare decay branching ratio in the field of meson decays.

- We have analyzed the impact of precise measurements of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ on the unitarity triangle and other observables of interest, within the SM. The results of this study are summarized in table XI. In particular we have analyzed the accuracy with which $\sin 2\beta$ and the angle γ could be extracted from these decays. Provided both branching ratios can be measured with the accuracy of $\pm 5\%$, an error on $\sin 2\beta$ of 0.025 could be achieved. The determination of γ requires an accurate measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and the reduction of the errors in $P_c(X)$ and $|V_{cb}|$. With a $\pm 5\%$ measurement of $Br(K^+ \to \pi^+\nu\bar{\nu})$ and the reduction of the errors in $P_c(X)$ and $|V_{cb}|$ anticipated in the coming years, γ could be measured with an error of $\pm 5^\circ - 6^\circ$.

- We have emphasized that the simultaneous investigation of the $K \to \pi\nu\bar{\nu}$ complex, the mass differences $\Delta M_{d,s}$ and the angles β and γ from clean strategies in two body B decays, should allow to disentangle different new physics contributions to various observables and determine new parameters of the extensions of the SM. The (R_t, β), (R_b, γ), (β, γ) and $(\bar{\eta}, \gamma)$ strategies for UT when combined with $K \to \pi\nu\bar{\nu}$ decays are very useful in this goal. This is in particular the case for the (R_b, γ) strategy that is related to the reference unitarity triangle (Barenboim *et al.*, 1999; Cohen *et al.*, 1997; Goto *et al.*, 1996; Grossman *et al.*, 1997). A graphical representation of these investigations is given in Fig. 9.

- We have presented a new "golden relation" between β, γ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$, given in (III.20), that with improved values of m_t and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ should allow very clean test of the SM one day. Another new relation is the one between β, γ and $Br(K^+ \to \pi^+\nu\bar{\nu})$, that is given in (III.11). Although not as clean as the golden relation in (III.20) because of the presence of P_c, it should play a useful role in future investigations.

- We have presented the results for both decays in models with minimal flavour violation and in three scenarios with new complex phases in enhanced Z^0 penguins and/or $B_d^0 - \bar{B}_d^0$ mixing. The effects of the new complex phase in Z^0 penguins in $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ turn out to be truely spectacular (Buras *et al.*, 2004c), with a smaller effect in $Br(K^+ \to \pi^+\nu\bar{\nu})$.

- We have reviewed the results for $Br(K^+ \to \pi^+\nu\bar{\nu})$ and $Br(K_{\rm L} \to \pi^0\nu\bar{\nu})$ in a number of specific extensions of the SM. In particular we have discussed supersymmetry with MFV, more general supersymmetric models with new complex phases, models with universal extra dimensions and models with lepton-flavour mixing. Each of these models has some characteristic predictions for the branching ratios in question, so that it should be possible to distinguish between various alternatives. Simultaneous investigations of other observables should be very helpful in this respect.
- Finally we have compared the usefulness of $K \to \pi\nu\bar{\nu}$ decays in testing various models with the one of other decays. While in the K system $K \to \pi\nu\bar{\nu}$ decays have no competition, there is a handful of B decays and related strategies that are also theoretically very clean. It is precisely the comparison between the results of these clean strategies in the B system with the ones obtained one day from $K \to \pi\nu\bar{\nu}$ decays that will be most interesting.

We hope we have convinced the reader that the very clean rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_{\rm L} \to \pi^0\nu\bar{\nu}$ deserve a prominent status in the field of flavour and CP violation and that precise measurements of their branching ratios are of utmost importance. Let us hope that our waiting for these measurements will not be too long.

XI. ACKNOWLEDGMENTS

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APPENDIX A: Remarks on $P_c(X)$

The results in tables I and II have been obtained using renormalization group evolution first from M_W down to $\mu_b = \mathcal{O}(m_b)$ in a $f = 5$ theory and subsequent evolution from $\mu_b = \mathcal{O}(m_b)$ down to $\mu_c = \mathcal{O}(m_c)$ in a $f = 4$ theory. The resulting analytic expressions are rather complicated and will not be presented here. However, it has been found in (Buchalla and Buras, 1994a, 1999) and confirmed by the last two authors here, that within the accuracy of 0.1% one can proceed instead as follows.

- For a fixed value of $\alpha_s(M_Z)$ in a $f = 5$ theory one finds the corresponding $\alpha_s(\mu_c)$ in a $f = 4$ theory incorporating the threshold at μ_b.
- One uses then the formulae (6)–(10) for X^l_{NL} of (Buchalla and Buras, 1999) in the $f = 4$ theory in the full range $\mu_c \le \mu \le M_W$ without incorporating the threshold at μ_b. To this end the value of $\alpha_s(M_W)$ has to be evaluated from $\alpha_s(\mu_c)$ obtained in the first step. As now no threshold at μ_b is incorporated, the resulting $\alpha_s(M_W)$ differs from the one one would obtain in the first step of this procedure. It turns out that this difference corrects almost exactly for the neglect of the threshold in explicit formulae for X_{NL} so that numerically the resulting X^l_{NL} and $P_c(X)$ are within 0.1% equal to the ones obtained with the μ_b threshold taken into account.

For the convenience of the reader we recall the formulae for X^l_{NL} of (Buchalla and Buras, 1999). First

$$X^l_{NL} = C_{NL} - 4B^{(1/2)}_{NL}, \tag{A.1}$$

where C_{NL} and $B^{(1/2)}_{NL}$ correspond to the Z^0 penguin and the box-type contribution, respectively. We have ($x_c = m_c^2/M_W^2$)

$$\begin{aligned} C_{NL} = \frac{x_c(m_c)}{32} K_c^{\frac{24}{25}} \Bigg[& \left(\frac{48}{7} K_+ + \frac{24}{11} K_- - \frac{696}{77} K_{33} \right) \left(\frac{4\pi}{\alpha_s(\mu)} + \frac{15212}{1875} (1 - K_c^{-1}) \right) \\ & + \left(1 - \ln \frac{\mu^2}{m_c^2} \right) (16K_+ - 8K_-) - \frac{1176244}{13125} K_+ - \frac{2302}{6875} K_- + \frac{3529184}{48125} K_{33} \\ & + K \left(\frac{56248}{4375} K_+ - \frac{81448}{6875} K_- + \frac{4563698}{144375} K_{33} \right) \Bigg], \end{aligned} \tag{A.2}$$

where

$$K = \frac{\alpha_s(M_W)}{\alpha_s(\mu)}, \qquad K_c = \frac{\alpha_s(\mu)}{\alpha_s(m_c)}, \tag{A.3}$$

$$K_+ = K^{\frac{6}{25}}, \qquad K_- = K^{\frac{-12}{25}}, \qquad K_{33} = K^{\frac{-1}{25}}, \tag{A.4}$$

$$B_{NL}^{(1/2)} = \frac{x(m)}{4} K_c^{\frac{24}{25}} \left[3(1-K_2)\left(\frac{4\pi}{\alpha_s(\mu)} + \frac{15212}{1875}(1-K_c^{-1})\right) - \ln\frac{\mu^2}{m^2} - \frac{r\ln r}{1-r} - \frac{77}{3} + \frac{15212}{625}K_2 + \frac{4364}{1875}KK_2 \right]. \tag{A.5}$$

Here $K_2 = K^{-1/25}$, $r = m_l^2/m_c^2(m_c)$ and m_l is the lepton mass. In (A.2) – (A.5) the scale is $\mu = \mathcal{O}(m_c)$. The two-loop expression for $\alpha_s(\mu)$ is given by

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2}} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\ln\frac{\mu^2}{\Lambda^2}}{\ln\frac{\mu^2}{\Lambda^2}} \right], \tag{A.6}$$

$$\beta_0 = 11 - \frac{2}{3}f, \qquad \beta_1 = 102 - \frac{38}{3}f. \tag{A.7}$$

The effective number of flavours to be used in the expressions above is $f = 4$. The QCD scale in (A.6) is $\Lambda = \Lambda_{\overline{MS}}^{(4)}$ with the values collected in table XVII below. To the considered order, the explicit $\ln(\mu^2/m_c^2)$ terms in (A.2) and (A.5) cancel the μ-dependence of the leading terms.

In table XVII, we give the values of the relevant quantities involved in this simplified analysis that inserted in the formulae (A.1)–(A.7) reproduce to an excellent accuracy the numbers in tables I and II. Additionally, for completeness, we also give in table XVIII the values of $X_{\rm NL}^e$ and $X_{\rm NL}^\tau$ that enter the formula for $P_c(X)$ in (II.12).

TABLE XVII Values of $\alpha_s^{(f)}(\mu)$ and $\Lambda_{\overline{MS}}^{(f)}$ corresponding to given values of $\alpha_s^{(5)}(M_{\rm Z})$ with $m_c = 1.30\,{\rm GeV}$ and $m_b = 4.2\,{\rm GeV}$.

$\alpha_s^{(5)}(M_Z)$	0.115	0.116	0.117	0.118	0.119	0.120	0.121
$\Lambda_{\overline{\rm MS}}^{(5)}$[MeV]	190	202	214	226	239	253	267
$\alpha_s^{(4)}(m_c)$	0.356	0.368	0.381	0.394	0.409	0.425	0.442
$\Lambda_{\overline{\rm MS}}^{(4)}$[MeV]	277	292	308	323	340	357	374
$\alpha_s^{(4)}(M_W)$	0.1119	0.1129	0.1138	0.1147	0.1156	0.1166	0.1175

TABLE XVIII The functions $X_{\rm NL}^e$ and $X_{\rm NL}^\tau$ for various $\alpha_s(M_{\rm Z})$ and $m_{\rm c}$.

	$X_{\rm NL}^e/10^{-4}$			$X_{\rm NL}^\tau/10^{-4}$		
$\alpha_s^{(5)}(M_{\rm Z})$ \ $m_{\rm c}$ [GeV]	1.25	1.30	1.35	1.25	1.30	1.35
0.115	10.46	11.33	12.22	7.09	7.81	8.56
0.116	10.35	11.22	12.11	6.98	7.70	8.44
0.117	10.24	11.10	11.99	6.87	7.59	8.33
0.118	10.12	10.98	11.87	6.75	7.47	8.21
0.119	9.99	10.85	11.74	6.63	7.34	8.08
0.120	9.86	10.72	11.61	6.49	7.20	7.94
0.121	9.72	10.58	11.47	6.35	7.06	7.80

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