A 2 Megawatt Multi-Stage Proton Accumulator

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A 2 Megawatt Multi-Stage Proton Accumulator

1 Introduction

1.1 Motivation

The delivery of high intensity proton beams for neutrino experiments is a core element of the Fermilab physics program for the next decade and beyond. This document outlines a plan which will greatly enhance the intensity capability beyond the year 2010 should budget and approval for the Proton Driver Linac fail to materialize. In order to reduce costs and to minimize disruption to the ongoing program, the plan uses existing infrastructure (tunnel enclosures, service buildings, power, utilities, etc.). The cost scale is estimated to be less than $100M, and the plan could be fully implemented by 2012 without the need for an extended shutdown period.

The use of existing infrastructure allows the plan to be broken into stages. Project staging has the important benefit of providing a fraction of the total performance at a fraction of the total cost. The schedule for each stage is driven by physics need and funding availability.

1.2 Concept

Multi-turn injection into the Booster is the current process for obtaining high intensity proton bunches in the Main Injector for neutrino experiments. Because of the relatively small aperture of the Booster and the large space charge tune shift at Booster injection, proton loss at injection limits the number of protons per bunch. Since space charge effects rapidly decrease with energy, it is more desirable to increase the proton intensity at higher energies. Due to the rapid cycling nature of the Booster, many Booster batches can be quickly combined at the Booster extraction energy. Because the bunch length requirements for neutrino experiments are not strict, the best technique to combine multiple Booster batches is to coalesce them longitudinally.

Slip stacking multiple Booster batches is the central concept of the Proton Plan[1]. In Stage 1, while the collider program is still running, nine Booster batches will be slipped stacked in the Main Injector for the neutrino program. In Stage 2, when the Recycler becomes available after the collider program is concluded, the slip stacking will be done in the Recycler which can handle 33% more batches with a 30% decrease in the cycle time. The number of batches stacked into the Recycler can not be increased further by slip stacking because of the rather severe amount of emittance dilution that is fundamental to the slip stacking process.

Another large increase in proton intensity is possible after the collider program concludes because the present antiproton production complex can be converted into a multi-stage proton accumulator for injection into the Main Injector. This accumulator would have three major components, each re-using or replacing existing machines:

- The Accumulator ring as a RF momentum stacker.
- The Recycler ring as a box-car stacker.
- The Debuncher ring would be replaced with a wide aperture booster.

1.2.1 *RF Momentum Stacking in the Accumulator*

The center piece of this concept is RF momentum stacking in the Accumulator. The key features of RF momentum stacking are a large momentum aperture and injection system located at high dispersion. Because the same features are required for stochastic momentum stacking of antiprotons, RF momentum stacking in the Accumulator would be possible with only minor modifications to the Accumulator.

During momentum stacking, a Booster batch is placed on the injection orbit of the Accumulator and accelerated towards the high energy aperture as shown in Figure 1-1. Another Booster batch is injected onto the injection orbit and accelerated towards the high energy aperture and deposited adjacent to the previous batch. The limit to how many Booster batches can be stacked is not the Accumulator aperture but the momentum aperture of the Main Injector at the transition energy. With present Booster performance, the Main Injector momentum aperture can comfortably handle over four Booster batches. This large number of batches can be combined using momentum stacking because momentum stacking has very little longitudinal emittance dilution.

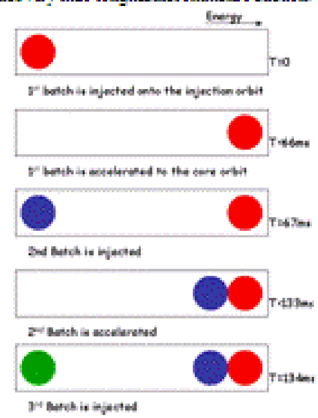


Figure 1-1 Cross section of the transverse aperture in high dispersion of the Accumulator during momentum stacking.

1.2.2 *Box Car Stacking in the Recycler*

Once at least three to four Booster batches have been momentum stacked in the Accumulator, the coalesced proton stack would be transferred to the Recycler. Since the Accumulator circumference is one seventh of the Recycler circumference, five more Accumulator stacks can be placed one after the other in the Recycler (box-car style) while leaving one seventh of the ring as an abort gap. The Recycler fully loaded in this manner would contain twenty four Booster batches which is twice the number of batches

Motivation

- Develop an alternative plan to provide high intensity proton beams for the neutrino program beyond 2010 should budgets and approvals for the planned projects fail to materialize.
- The proposal needs to have the following important features
  - It must be inexpensive (< $100M or so)
  - It must be completed quickly (before 2012)
  - It should not shutoff the collider complex or the neutrino program for an extended period of time.
- These goals can be accomplished only if:
  - It uses the present Fermilab infrastructure (tunnel enclosures, service buildings, power, utilities, etc.)
  - The project is staged

Concept

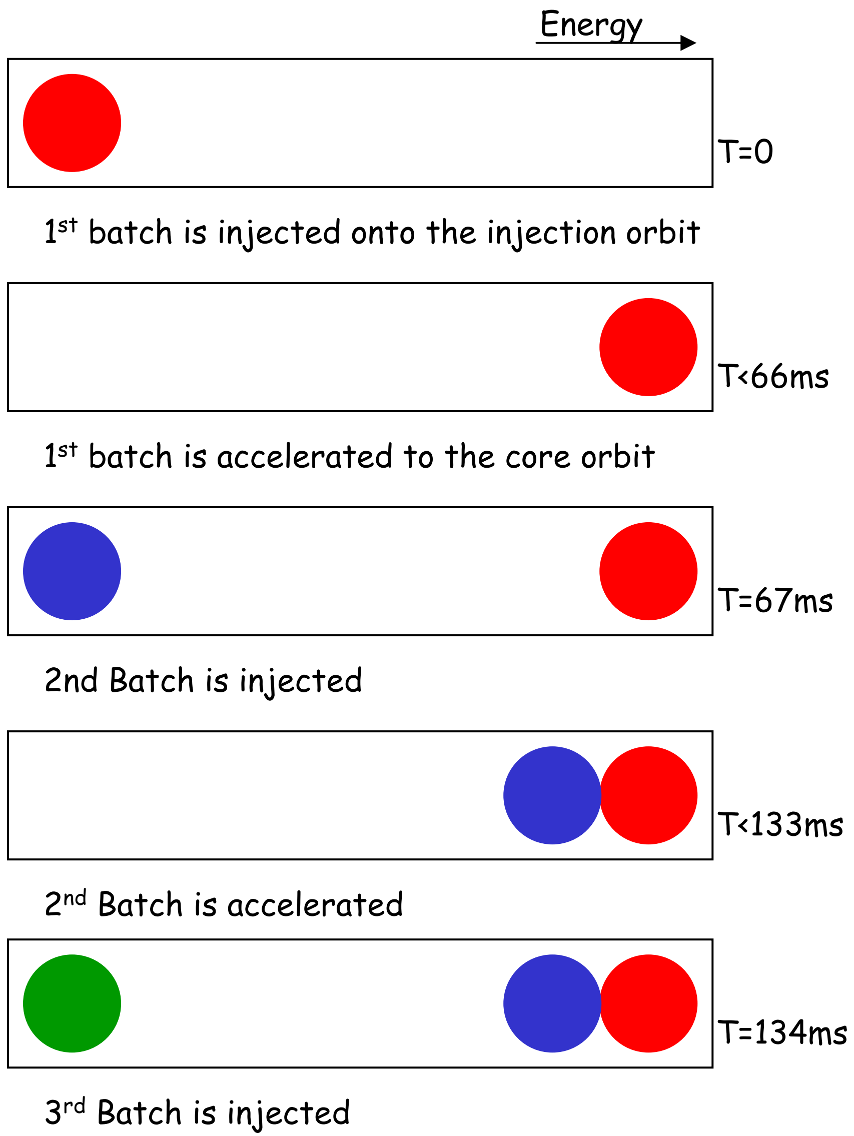
- After the collider program concludes, the present antiproton production complex can be converted into a multi-stage proton accumulator for injection into the Main Injector.
  - Debuncher ->Wide Aperture Booster
  - Accumulator -> Momentum Stacker
  - Recycler -> Box Car Stacker

Momentum Stacking in the Accumulator

- After acceleration in the Booster the beam will be transferred to the Accumulator ring.
- Using the Accumulator as a proton accumulator reduces the peak intensity requirement in the Booster
- Results in a smaller required aperture for the Booster
  - Smaller space charge tune shift
  - Reduced requirements on acceleration efficiency
- The Accumulator was designed for momentum stacking
  - Large momentum aperture ~ 84 x 2.8 eV-Sec
  - Injection kickers are located in 9m of dispersion
  - Injection kickers do not affect core beam

Mechanics of Momentum Stacking

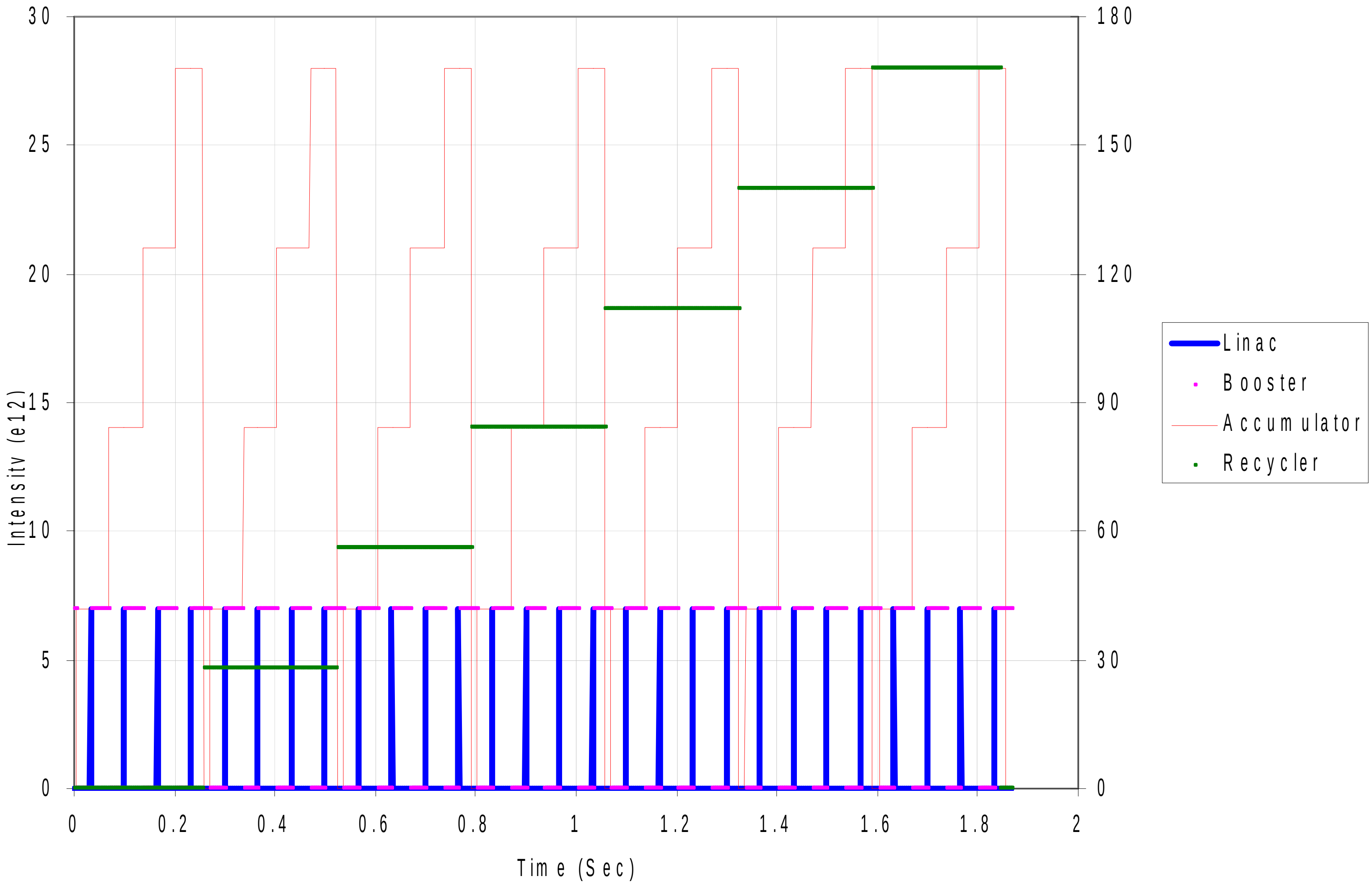
- Inject in a newly accelerated Booster batch every 67 mS onto the low momentum orbit of the Accumulator
- The freshly injected batch is accelerated towards the core orbit where it is merged and debunched into the core orbit
- Momentum stack 3-4 Booster batches



Multi-stage Proton Accumulator Scheme

- Momentum stack in the Accumulator
  - Inject in a newly accelerated Booster batch every 67 mS onto the high momentum orbit of the Accumulator
  - Decelerate new batch towards core orbit and merge with existing beam
  - Momentum stack 3-4 Booster batches
  - Extract a single Accumulator batch
    - Every 200 – 270 mS
    - At an intensity of 3-4x a single Booster batch
- Box Car Stack in the Recycler
  - Load in a new Accumulator batch every 200-270mS
  - Place six Accumulator batches sequentially around the Recycler
- Load the Main Injector in a single turn

Multi-stage Proton Accumulator Production Cycle



Project Staging

- Because the concept uses existing infrastructure the performance can be broken into stages
- Project staging has the important benefit of providing
  - a fraction of the total performance
  - at a fraction of the total cost
- The schedule for each stage is driven by physics need and funding availability
- **Each stage is based on standard accelerator technology and accelerator parameters that are currently achievable.**

Stages of the Present Proton Plan

- Present - Present Booster -> Main Injector
  - 6.4×10^{16} pph
  - 162kW 120 GeV Beam
  - 1.1×10^{16} pph Collider
  - 2.1×10^{16} pph BNB
- Stage 1 - Proton Plan Booster -> Main Injector
  - 13.5×10^{16} pph
  - 370kW 120 GeV Beam
  - 1.4×10^{16} pph Collider
  - 5.1×10^{16} pph BNB
- Stage 2 - Proton Plan Booster -> Recycler -> Main Injector
  - 13.5×10^{16} pph
  - 705kW 120 GeV Beam

New Stages for the Multi-Stage Proton Accumulator

- Stage 3 - Proton Plan Booster -> Accumulator -> Recycler -> Main Injector
  - 21.6×10^{16} pph
  - 1140kW 120 GeV Beam
- Stage 4 - New Booster -> Accumulator -> Recycler -> Main Injector
  - 43.2×10^{16} pph
  - 2300kW 120 GeV Beam

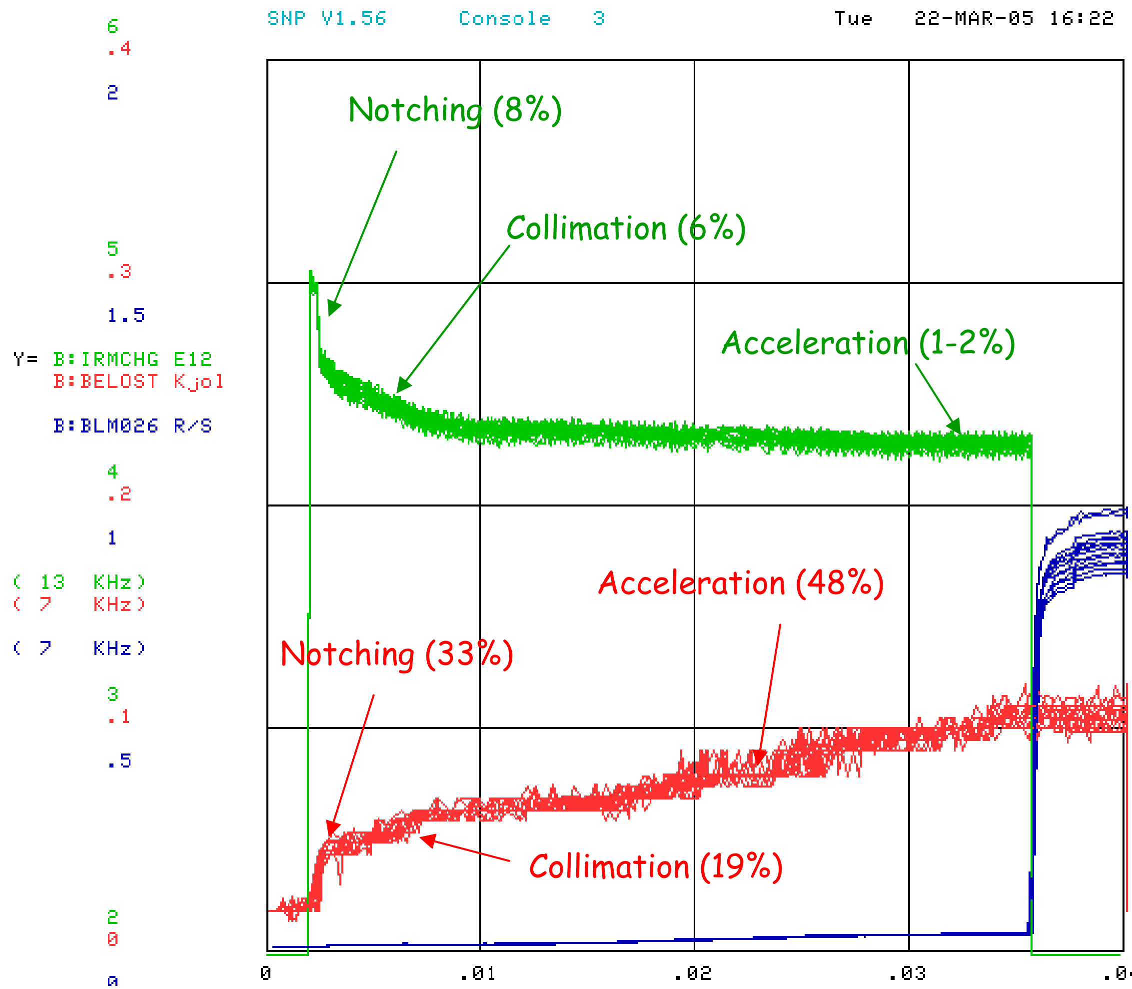
Booster Performance

- Amount of beam power lost per pulse is inversely proportional to the repetition rate

$$P_L = J_L R$$

- For simplicity the beam loss can be divided into two categories,
  - beam loss due to creating the beam gap (notch) for extraction
  - beam lost transversely during acceleration

$$J_L = E_n \Delta N_n + E_A \Delta N_A$$



Booster Performance

- The total efficiency of the Booster is:

$$\frac{N_{ext}}{N_{inj}} = \left(1 - f_n - f_A\right)$$

  - f_n is the ratio of the amount of beam loss during notching to the injection intensity
  - f_A is the ratio of the amount of beam loss during acceleration to the injection
- For a given notching fraction, the fraction of beam loss during acceleration that can be tolerated is:

$$f_A = \frac{P_L - \left(N_{ext} E_n R + P_L\right) f_n}{N_{ext} E_A R + P_L}$$

- Assuming a guassian profile as a simple approximation, the amount of beam in the halo that is outside the aperture is:

$$f_h = e^{-3\frac{A}{\varepsilon_{95}}}$$

Booster Performance

- The amount of beam that is permitted to be in the halo is:

$$f_h = \frac{\Delta N_A}{2\left(N_{ext} + \Delta N_A\right)} = \frac{f_A}{2\left(1 - f_n\right)}$$

- The aperture required is :

$$A = \frac{S_f \varepsilon_{95}}{3} \ln\left(\frac{2\left(1 - f_n\right)}{f_A}\right)$$

- The half-aperture of the magnets is proportional to
  - The transverse acceptance,
  - The momentum acceptance
  - The closed orbit displacement

$$\Delta x = \sqrt{\frac{A_n}{\beta\gamma}\beta_{max}} + \frac{\Delta p}{p} D_{max} + c.o.d.$$

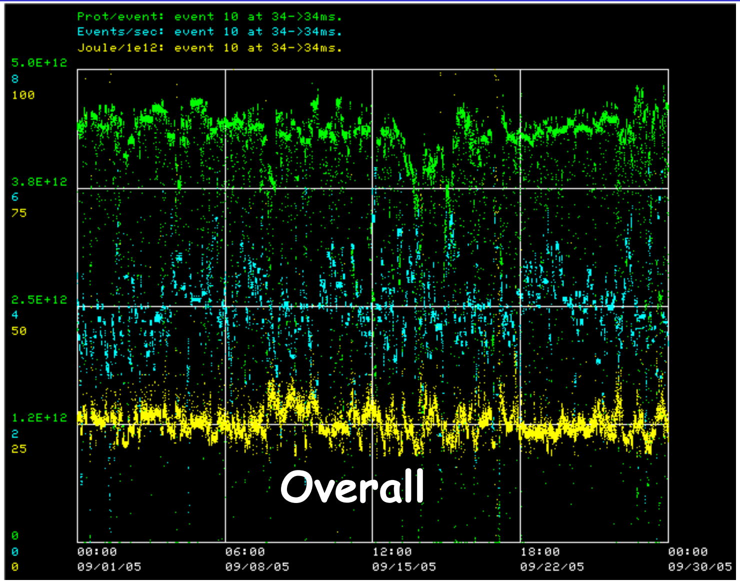
- Compare designs with the same space charge tune shift

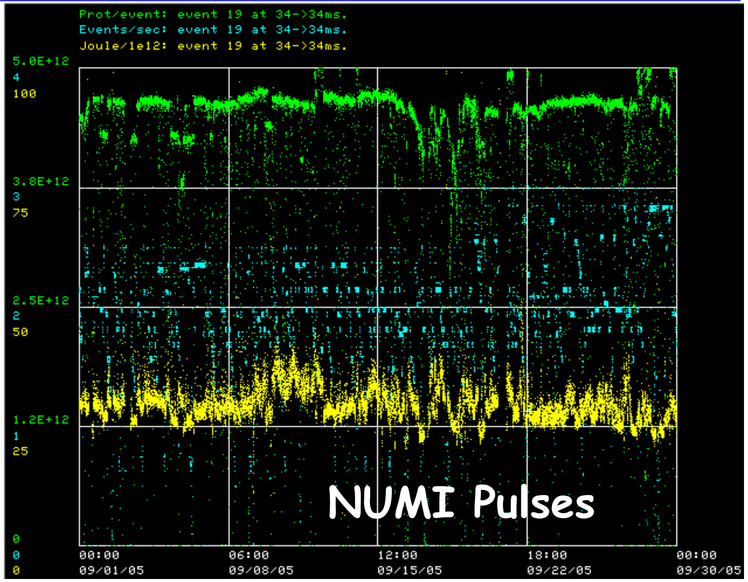
$$\varepsilon_n \propto \frac{N_{inj}}{\beta\gamma^2 \Delta\nu}$$

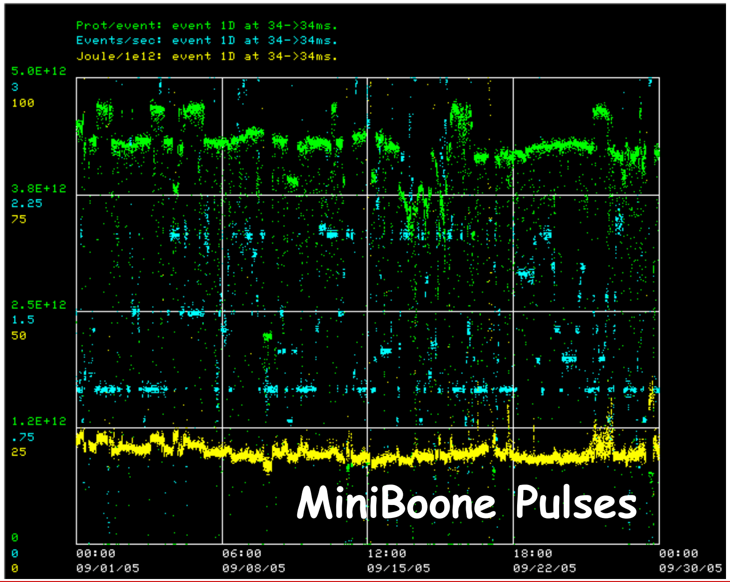
Booster Throughput Scenarios

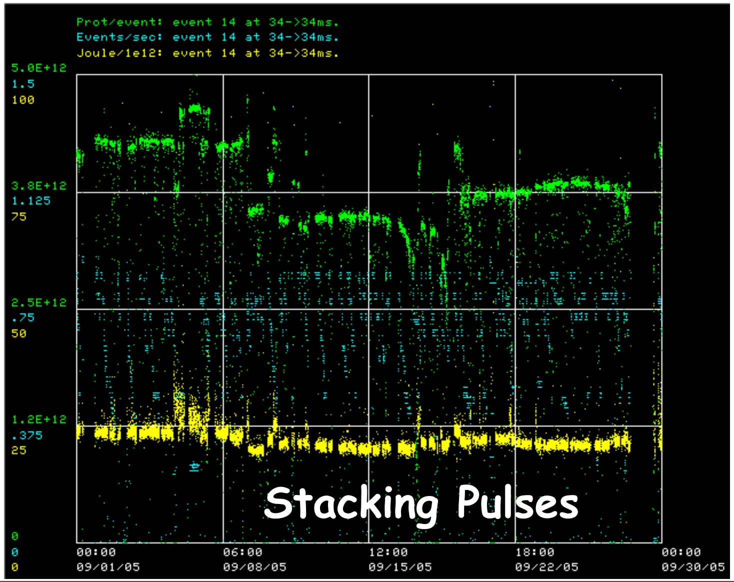
- The vertical aperture in the present Booster is
  - 1.64 inches for the F magnets
  - 2.25 inches for the D magnets
- The horizontal good field aperture is
  - 4.3 inches for the F magnets
  - 3 inches for the D magnets
- The RF cavities in the Booster are located between two D magnets
  - The horizontal beta function is at a minimum
  - The vertical beta function is a maximum.
  - The RF cavity aperture is 2.25 inches.
- To increase Booster throughput, we have two knobs available
  - Increase beam power lost in the Booster tunnel
  - Increase the effective Booster aperture (or decrease the closed orbit distortion tolerance)

Booster Performance Sept. 2005









Booster Throughput Scenarios

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
Booster Flux	6.38	13.53	13.54	21.59	43.18	36.36	$x10^{16}$/Hr
Collider Flux	1.09	1.41	0.00	0.00	0.00	0.00	$x10^{16}$/Hr
NUMI Flux	3.21	7.05	13.54	21.59	43.18	36.36	$x10^{16}$/Hr
NUMI Beam Power	162	367	704	1140	2280	1920	kW
MiniBoone Flux	2.08	5.07	0.00	0.00	0.00	0.00	$x10^{16}$/Hr

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
Slip Stack Final Intensity	6.9	8	0	0	0	0	$x10^{12}$
NUMI Final Intensity	22	42	55	95	190	150	$x10^{12}$
MI Cycle Time	2.6	2.2	1.5	1.6	1.6	1.5	Sec
Slip Stack Batches	2	2	0	0	0	0	
NUMI Batches	5	9	12	24	24	6	
Slip Stack Efficiency	88	93	100	100	100	100	%
NUMI Efficiency	95	97.5	97.5	99	99	99	%

Booster Intensity Requirements

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
Extraction Intensity	4.43	4.70	4.70	4.00	8.00	25.25	$x10^{12}$
Rep. Rate	4	8	8	15	15	4	Hz
Average Beam Power Lost	440	440	440	440	440	440	Watts
Notch Bunches	7	4	4	0	0	2	
Notch Energy	450	450	450	450	450	650	MeV
Acceleration Loss Energy	1050	1050	1050	1050	1050	1050	MeV
Injection Energy	400	400	400	400	400	600	MeV
Allowed Tune Shift	0.47	0.47	0.47	0.47	0.47	0.47	
Bunching Factor	2	2	2	2	2	2	

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
Acceleration loss	8.7	4.3	4.3	4.2	2.1	1.0	%
Efficiency	83.0	90.9	90.9	95.8	97.9	96.6	%
Injection Intensity	5.3	5.2	5.2	4.2	8.2	26.1	$x10^{12}$
Norm. Emittance at Inj	11.4	11.1	11.1	8.9	17.5	38.1	π-mm-mrad
Norm Acceptance at Inj	18.9	22.7	22.7	18.7	43.0	108.4	π-mm-mrad

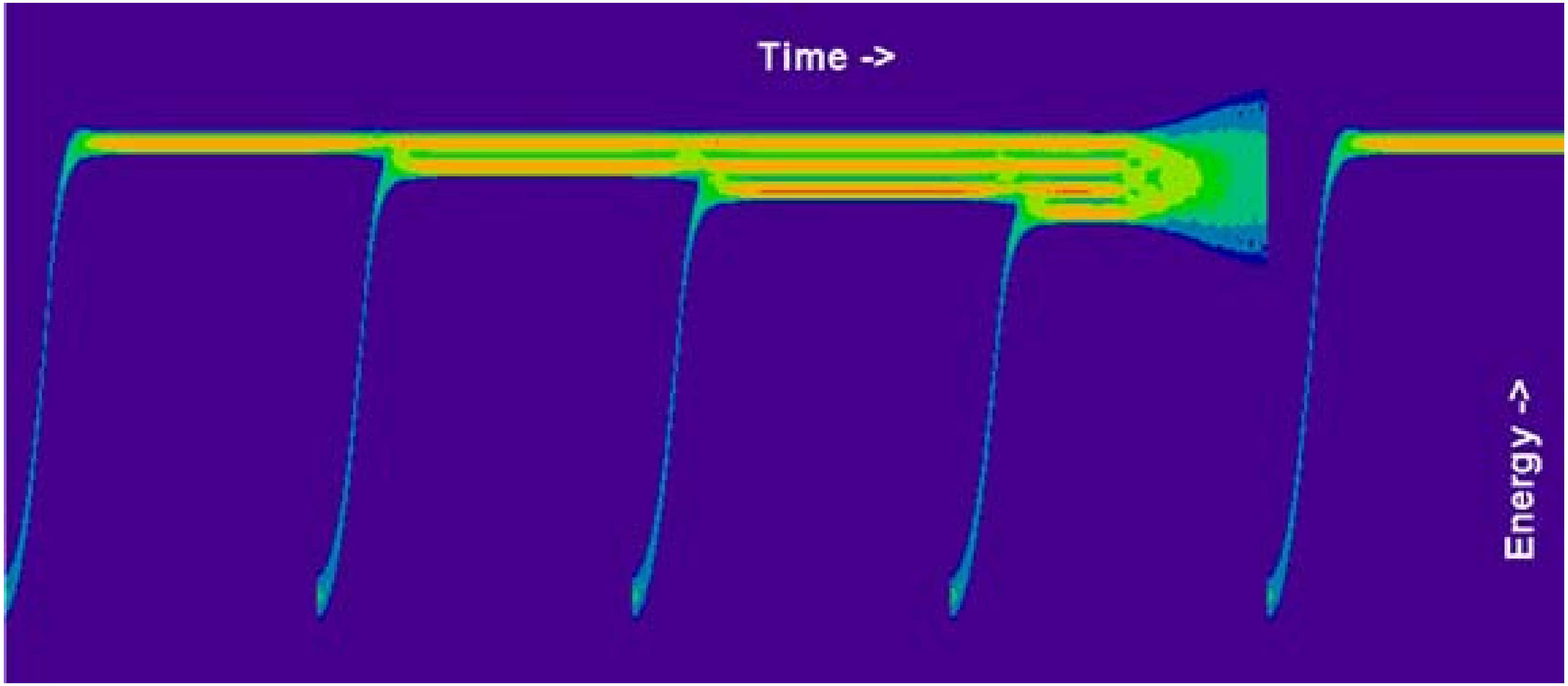
Booster Aperture Requirements

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
F magnet β_x	33	33	33	33	15	15	m
F magnet β_y	14	14	14	14	20	20	m
F magnet D_x	3	3	3	3	2.5	2.5	m
D magnet β_x	14	14	14	14	15	15	m
D magnet β_y	22	22	22	22	20	20	m
D magnet D_x	2.5	2.5	2.5	2.5	2.5	2.5	m
Momentum Acceptance	0.4	0.4	0.4	0.4	1.2	2.4	%
Misalignment & c.o.d.	10	6.5	6.5	9.5	20	20	mm

Parameter	Present	Stage 1	Stage 2	Stage 3	Stage 4	PD2	
F Aperture Width	2.81	2.87	2.87	2.79	3.95	5.93	in
F Aperture Height	1.66	1.65	1.65	1.64	3.08	4.00	in
D Aperture Width	2.06	2.04	2.04	2.03	3.95	5.93	in
D Aperture Height	1.98	2.00	2.00	1.96	3.08	4.00	in

Momentum Stacking

Output longitudinal emittance = 84 * 0.38 eV-sec



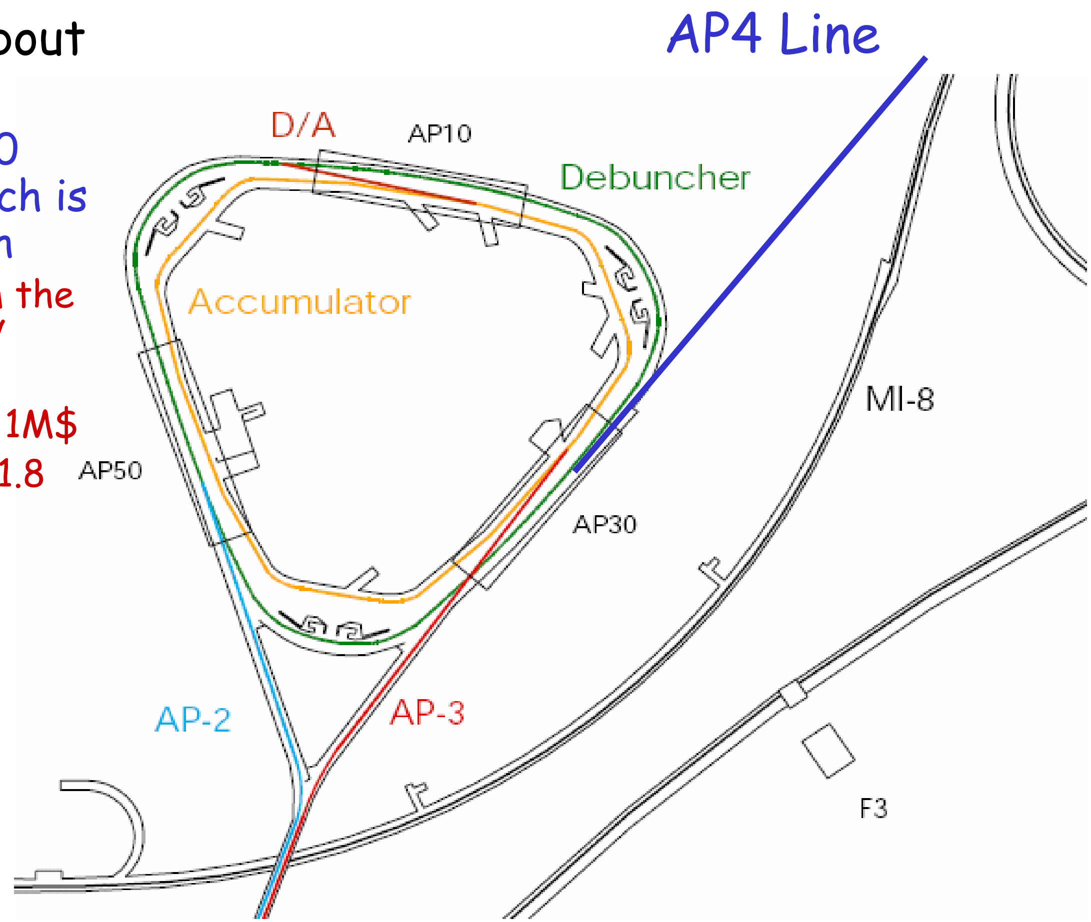
Input longitudinal emittance = 84 * 0.08 eV-sec

Advantages of Momentum Stacking

- Transient Beam Loading
  - Slip stacking or barrier bucket stacking requires manipulating intense beams with low RF voltages in a mostly empty circumference
  - In momentum stacking, the circumference is always uniformly loaded
- Speed of process
  - Injected beam can be decelerated quickly towards the core beam
- Longitudinal emittance dilution
  - The core beam can be debunched during stacking process reducing the amount of "white spaces"
- Cogging in the Booster
  - Prior to injection into the Accumulator, the injection orbit of the Accumulator is empty
  - The Accumulator injection system can be phase-locked to the Booster which eliminates the need for cogging in the Booster
  - The Booster notch can be made in the Linac

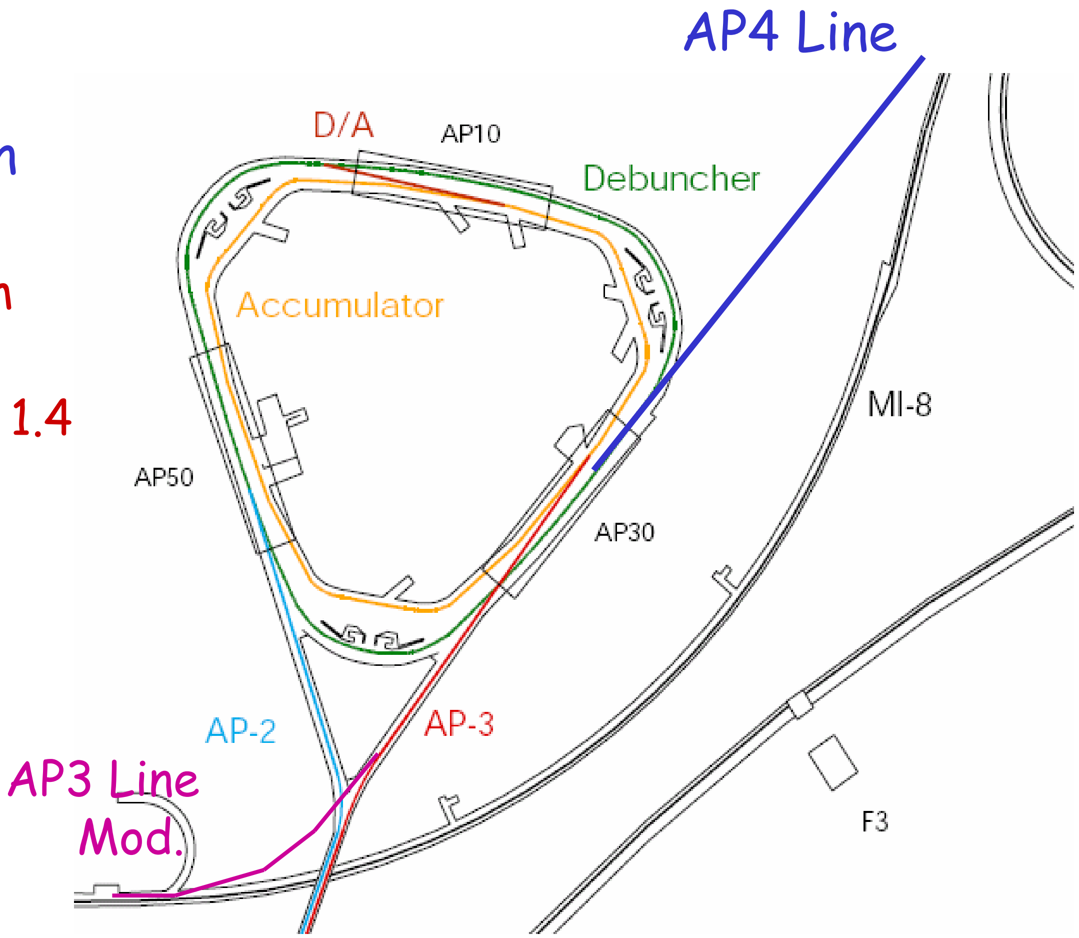
AP-4 Line

- The old Booster is connected to the new Booster via a re-built AP4 Line
- The new AP4 line is about 240 meters in length
  - Compared to the 600 MeV line of PD2 which is 250 meters in length
    - Use Magnets from the AP2 line for 8 GeV operation
    - 600 MeV magnets 1M$
    - Civil Construction 1.8 M$

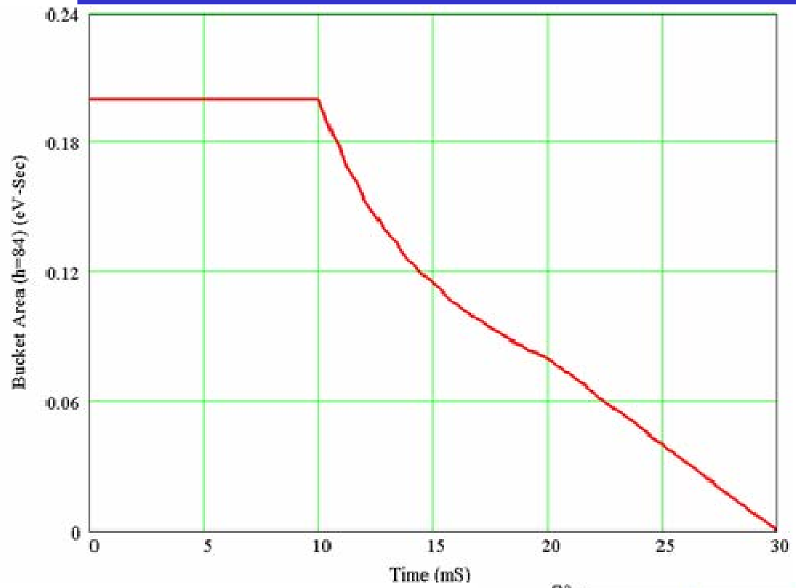


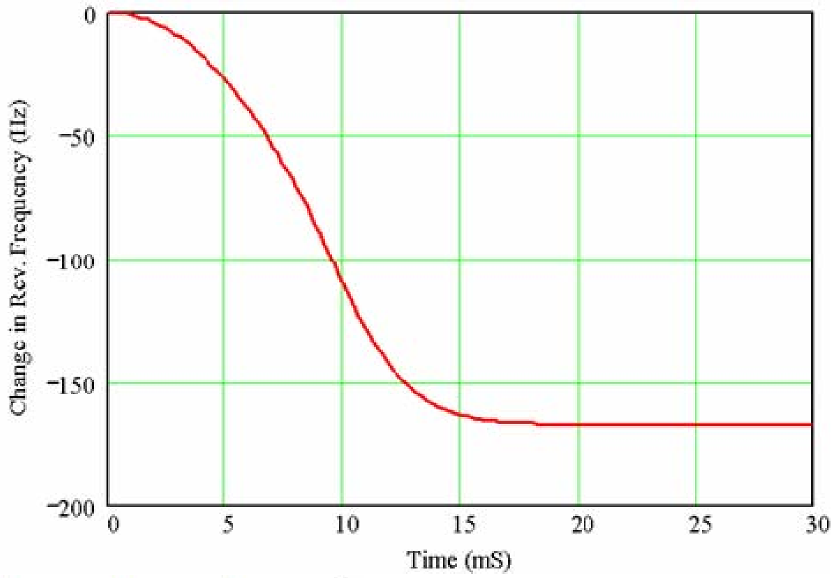
AP-3 Line Modification

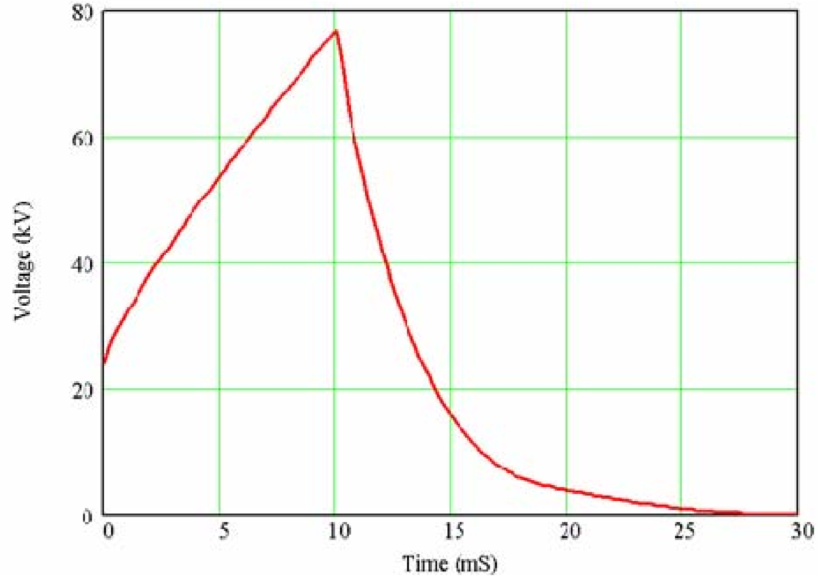
- The AP3 line needs to be connected to the MI-8 line
  - The modification is about 100 meters in length
    - Use magnets from the rest of AP3
    - Civil Construction 1.4 M$



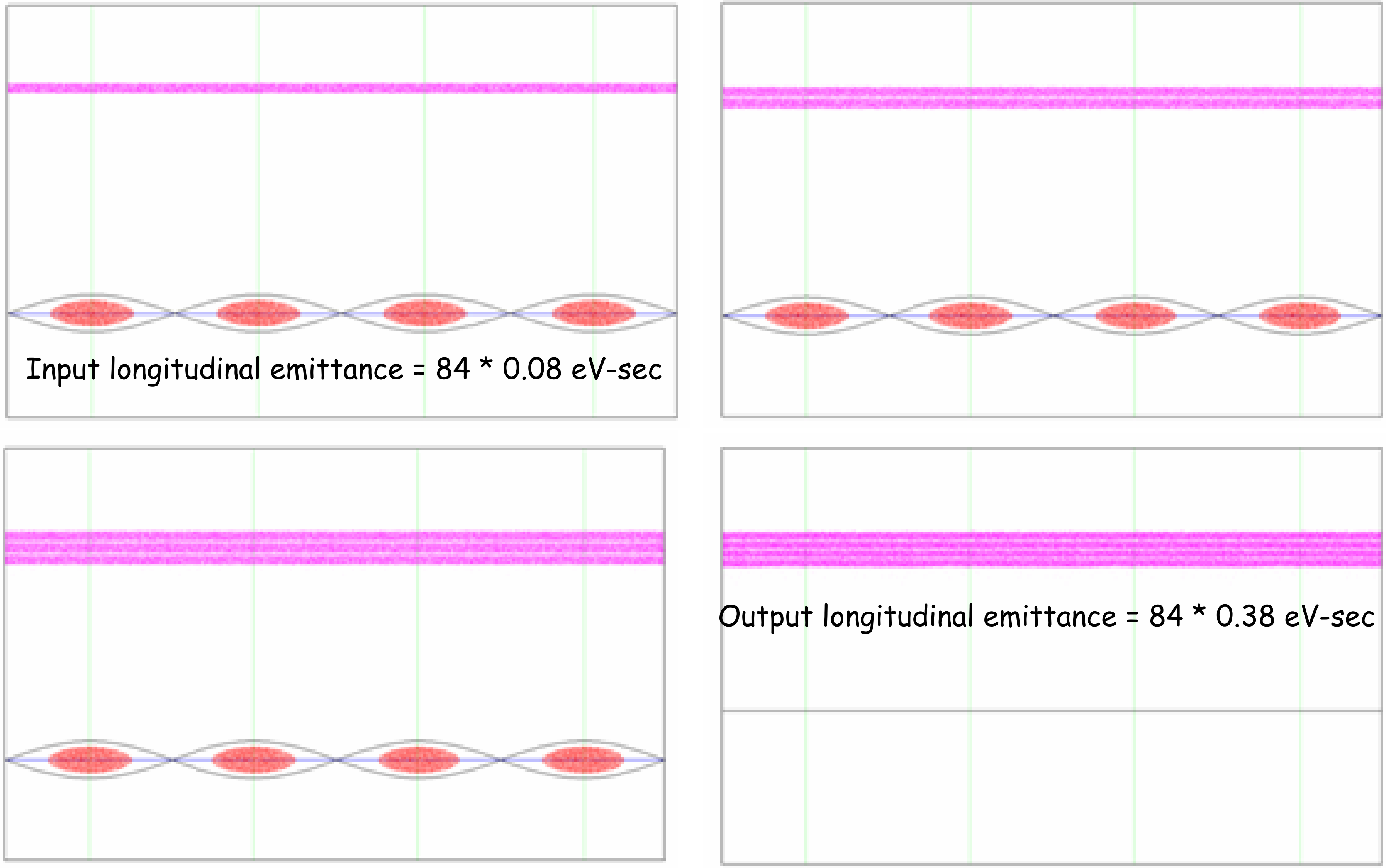
Momentum Stacking RF Curves







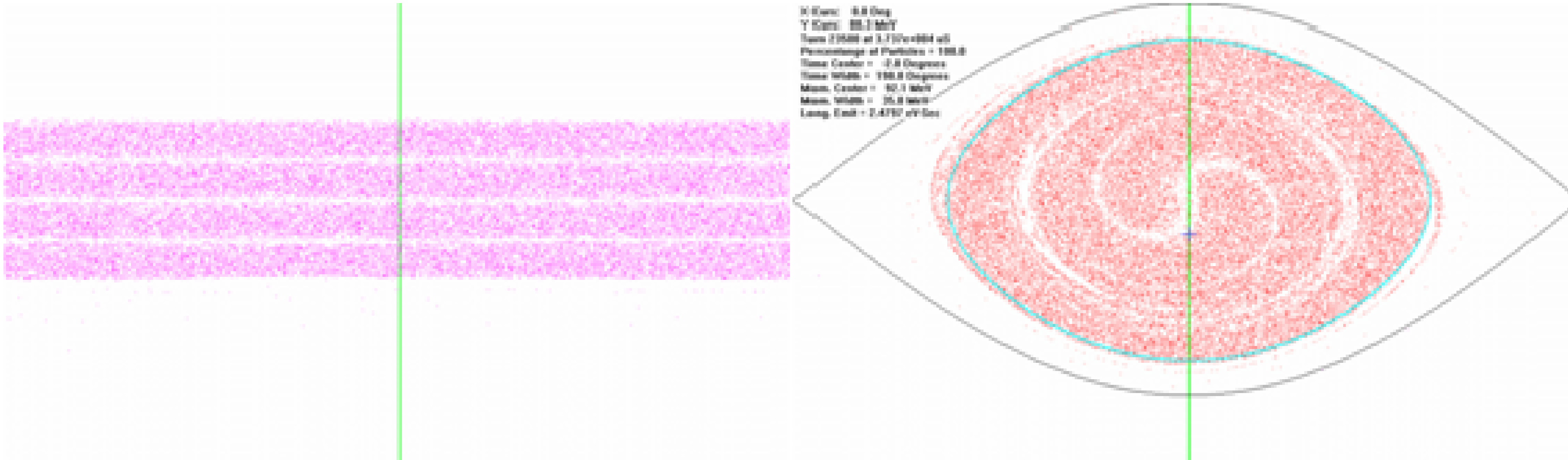
Momentum Stacking Phase Space

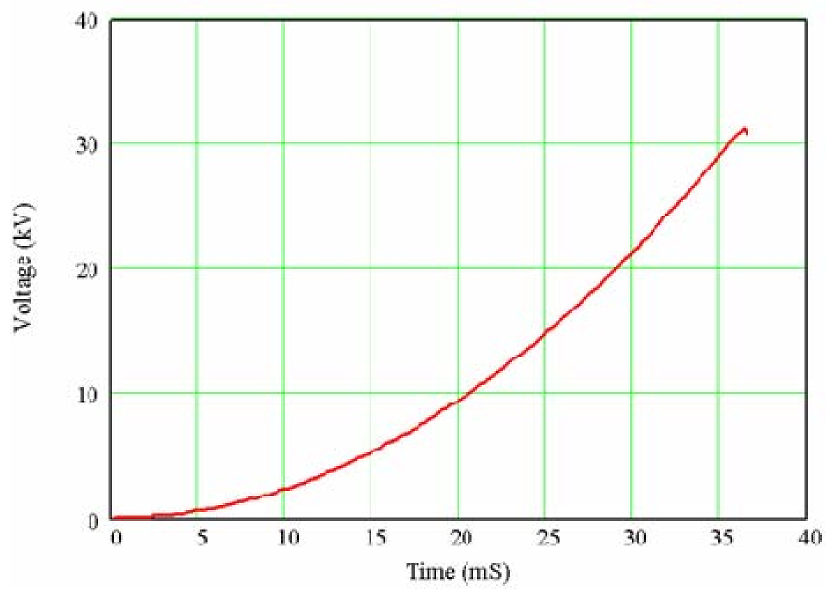


Extraction From the Accumulator

- After 4 batches have been stacked begin preparing to extract all the beam from the Accumulator to the Recycler
- Re-bunch the entire stacked beam at h=12 in the Accumulator (7.5 MHz)
  - Low harmonic for a large enough gap between buckets which could accommodate kicker rise time
  - High harmonic for fast synchrotron period for the speed of the process
  - New system –need 30 kV/Turn in Accumulator

Extraction Phase Space





Box Car Stacking in the Recycler

- After 4 booster batches have been momentum stacked in the Accumulator, the beam would be transferred to the Recycler.
  - 7.5 MHz MHz synchronous transfer
    - New system
    - Need 80kV/Turn for a 4.2 eV-sec bucket
  - Accumulator phase ALIGNED to the Recycler
- The Accumulator is 1/7 of the Recycler's circumference
- Boxcar stack six of the Accumulator batches leaving 1/7 of the Recycler ring for an abort gap.
- After 6 Accumulator batches have been stacked into the Recycler debunch 7.5 MHz beam in >80mS
- Re-capture in 53 MHz buckets for acceleration.
  - Need 500 kV for 0.6 eV-sec
  - Use three Tevatron RF cavities

Stage 3 Cost Estimate in k$

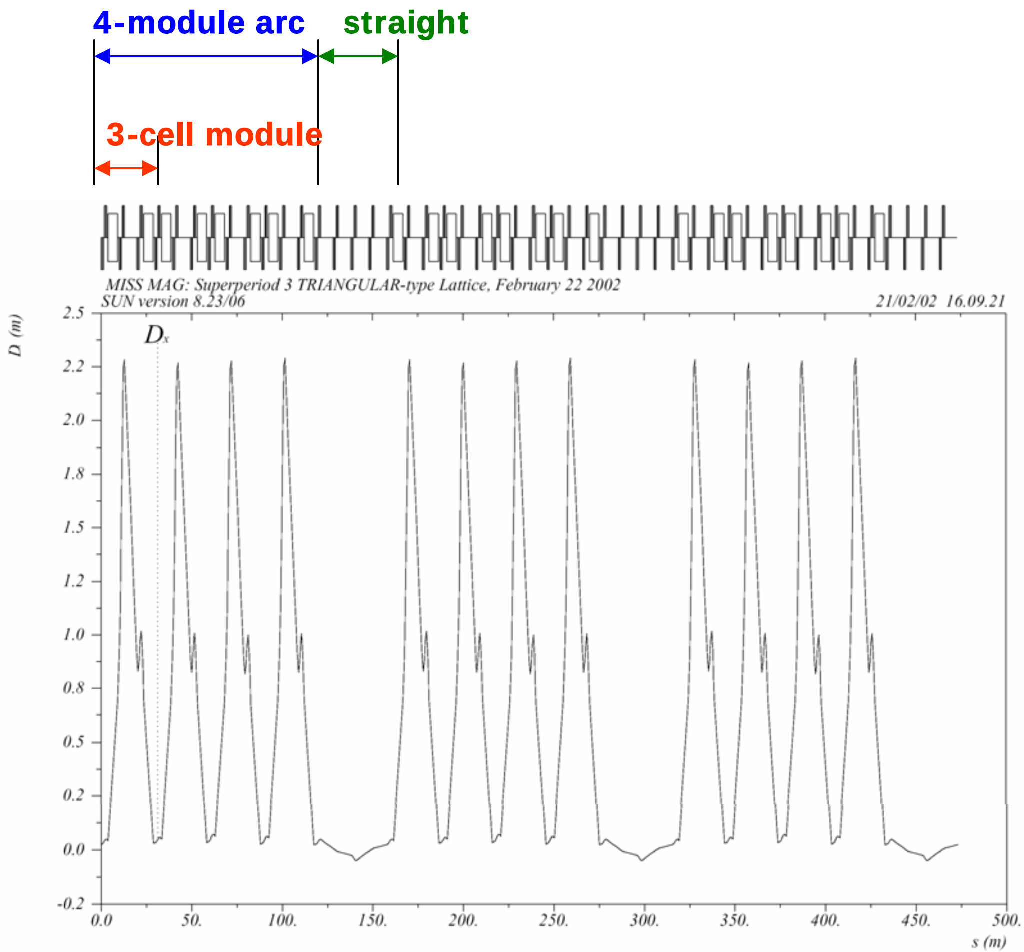
Description	Cost
Linac Notching	100
Booster Extraction Upgrade	1,000
AP4 Line Civil	1,800
AP4 Tie In & Installation	500
AP3 Modification Civil	1,400
AP3 Tie In & Installation	500
Accumulator Shielding	3,000
Accumulator Kickers	1,000
Accumulator 53 MHz RF	400
Accumulator 7.5 MHz RF	400
Accumulator Instrumentation	200
Recycler 7.5 MHz RF	1,000
Recycler 53 MHz Installation	300
Recycler Instrumentation	200
Total	11,800

Stage 4 New Booster

- Triangular shape to fit to the Debuncher footprint
- No transition crossing (γ_t = 18.1)
- Zero dispersion straights
- Simple: 1 type of dipole, 1 type of quad (QF and QD same length)
- Doublet lattice, 90° phase advance per cell
- 3 cells per module, with missing dipole in the mid-cell, 270° phase advance per module
- 4 modules per arc, 6π phase advance per arc
- Plenty of free space: 24 x 7 m
- Low beta-function (15 m) and dispersion (2.3 m)
- Good optical properties (large dynamic aperture, weak dependence of lattice functions on amplitude and dp/p)

Courtesy of Weiren Chou

Stage 4 Triangular Lattice



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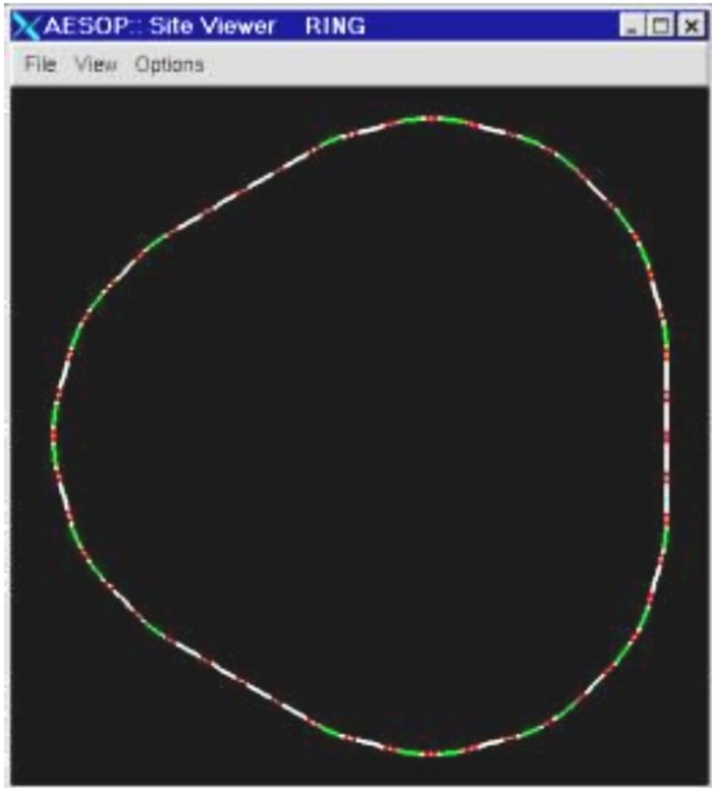

Cost Comparison Between Stage 4 and PD2

- Reuse the pbar tunnel
  - Saving $37M in civil
  - However include $3M for radiation shielding
- Reduce magnet aperture from 4" x 6" to 3" x 5"
  - Saving $20M in magnet and power supply cost.
- Use new type of beam pipe
  - Saving $1M in vacuum
- Reuse utilities
  - Existing in Pbar Tunnel
  - Lower beam power.
  - saving $4.4M
- Reuse controls
  - Saving $2M

Courtesy of Weiren Chou

PD2 to Stage 4 Cost Comparison

WBS	Description	Level 3	Level 2	Level 1
1	Technical Systems			98,986
1.1	8 GeV Synchrotron		78,997	
1.1.1	Magnets	27,329		
1.1.2	Power supplies	25,968		
1.1.3	RF	5,115		
1.1.4	Vacuum	6,061		
1.1.5	Collimators	325		
1.1.6	Injection system	938		
1.1.7	Extraction system	2,189		
1.1.8	Instrumentation	2,393		
1.1.9	Controls	2,468		
1.1.10	Utilities	4,931		
1.1.11	Installation	1,280		
1.2	Linac Improvements and Upgrade		17,500	
1.2.1	Front end and RFQ	3,000		
1.2.2	New drift tube Tank #1	1,000		
1.2.3	Transfer line to new CCL	1,800		
1.2.4	New CCL modules and klystrons	11,100		
1.2.5	Controls and diagnostics	600		
1.3	600 MeV Transport Line		900	
1.3.1	Magnets	720		
1.3.2	Power supplies	180		
1.4	8 GeV Transport Line		1,589	
1.4.1	Magnets	1,271		
1.4.2	Power supplies	318		
2	Civil Construction			37,593
2.1	8 GeV Synchrotron		17,500	
2.1.1	Enclosure	7,000		
2.1.2	Service buildings	7,000		
2.1.3	Utility support building	3,500		
2.2	Linac extension		2,500	
2.3	600 MeV Transport Line		1,800	
2.4	8 GeV Transport Line		2,200	
2.5	Site work		4,800	
2.6	Subcontractors OH&P		5,760	
2.8	Environmental controls and permits		3,033	
	TOTAL ($k)			136,579

WBS	Description	Level 3	Level 2	Level 1
1	Technical Systems			72,317
1.1	8 GeV Synchrotron		71,597	
1.1.1	Magnets	17,329		
1.1.2	Power supplies	15,968		
1.1.3	RF	25,115		
1.1.4	Vacuum	5,061		
1.1.5	Collimators	325		
1.1.6	Injection system	938		
1.1.7	Extraction system	2,189		
1.1.8	Instrumentation	2,393		
1.1.9	Controls	468		
1.1.10	Utilities	531		
1.1.11	Installation	1,280		
1.2	Linac Improvements and Upgrade		0	
1.2.1	Front end and RFQ	0		
1.2.2	New drift tube Tank #1	0		
1.2.3	Transfer line to new CCL	0		
1.2.4	New CCL modules and klystrons	0		
1.2.5	Controls and diagnostics	0		
1.3	600 MeV Transport Line		720	
1.3.1	Magnets	720		
1.3.2	Power supplies	0		
1.4	8 GeV Transport Line		0	
1.4.1	Magnets	0		
1.4.2	Power supplies	0		
2	Civil Construction			3,000
2.1	8 GeV Synchrotron		3,000	
2.1.1	Enclosure	3,000		
2.1.2	Service buildings	0		
2.1.3	Utility support building	0		
2.2	Linac extension		0	
2.3	600 MeV Transport Line		0	
2.4	8 GeV Transport Line		0	
2.5	Site work		0	
2.6	Subcontractors OH&P		0	
2.8	Environmental controls and permits		0	
	TOTAL ($k)			75,317

Stage 4 Booster vs BNL AGS Booster

	Stage 4	AGS Booster
Circumference (m)	505	201
Injection (MeV)	400	200
Extraction (GeV)	8	1.5
Rep rate (Hz)	15	7.5
Total dipoles	24×5.2 m = 124.8 m	36×2.4 m = 86.4 m
Total quads	96×1.24 m = 119 m	48×0.5 m = 24 m
Beam pipe aperture	3 in × 5 in	2.8 in × 5.9 in
Max β function (m)	14.8 / 15.2	13.9 / 13.7
Max dispersion (m)	2.3	2.9
Transition γ	18.1	4.79
Beam intensity	8×10^{12}	2×10^{13}
Year constructed	TBD	1991
Construction cost	$75M (estimated)	$32M (1991)
Civil cost included?	No	Yes

Courtesy of Weiren Chou

- Adjust $32M in 1991 for 14 years of 4% of inflation = $55M
- Scale the cost as the length in magnets = $122M
- Remove civil construction = $122M - $37M = $85M

Booster Neutrino Beamline (BNB) Option

- After Stage 4 is complete, the old Booster is no longer needed
- The present Linac can support pulse lengths in excess of 50 μS
  - A 40mA Linac beam pulse for 50 μS has 12.4×10^{12} particles
- From a single Linac pulse, a chopper placed in the 400 MeV line would be able to send
  - 8.2×10^{12} protons to the new booster
  - 4.2×10^{12} protons to the old booster.
    - Gets BNB to 21.6×10^{16} protons/hour

20 Hz Option

- Because of the lack of a transition crossing, the longitudinal emittance in Stage 4 might be well below 0.06 eV-sec per 53 MHz bunch.
    - With this small of a longitudinal emittance, it may be possible to momentum stack five booster batches in the Accumulator.
    - Momentum stacking five booster batches in the Accumulator at 15 Hz would require a Main Injector cycle time of 2.0 seconds which is the same flux as stacking four batches for a Main Injector cycle time of 1.6 seconds.
- However, the new Booster could be designed to run at a 20 Hz rate. Momentum stacking five booster batches at 20 Hz in the Accumulator would permit a Main Injector cycle time of 1.5 seconds.
    - The proton flux would increase by 33 percent to provide enough protons for a 3.0 megawatt 120 GeV beam.
    - The aperture requirements of the Stage 4 booster would remain about the same because the aperture is based mostly on peak intensity and is only weakly related to repetition rate.

Summary

- The present antiproton production complex can be converted into a multi-stage proton accumulator
  - That supplies enough protons for a 1.1 MW 120 GeV beam for a cost of about $12M
  - That can be upgraded to provide enough protons for a 2.3 MW 120 GeV beam for an additional cost of about $75M
- Because the concept uses existing infrastructure the performance can be broken into stages
  - Project staging has the important benefit of providing
    - a fraction of the total performance
    - at a fraction of the total cost
  - The schedule for each stage is driven by physics need and funding availability
  - **Each stage is based on standard accelerator technology and accelerator parameters that are currently achievable.**
- Integrating the present Booster into this scheme could in addition provide 8 GeV protons in 1.6 μS bursts at a rate of 21×10^{16} protons/hour for a Booster Neutrino Beam (BNB)