Review of sensitivity Computations

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What said today does not
always come true tomorrow.

Outline

- What we want to measure
- What we can measure
- What we really measure
- How to compute that using GLoBES
- How to extract physical parameters
- Some results for the Wide Band Beam

What we want to measure

- Mass splittings
- Mixing angles
- CP phase
- Mass hierarchy
- New physics
- ...

What we can measure

$$
\begin{aligned}
P_{\mu e} \;\simeq\;& \sin^2 2\theta_{13}\,\sin^2\theta_{23}\sin^2\Delta_{31} \\
\mp\;& \alpha\,\sin 2\theta_{12}\,\sin 2\theta_{13}\,\sin\delta\,\sin 2\theta_{23}\,\Delta_{31}\,\sin^2\Delta_{31} \\
-\;& \alpha\,\sin 2\theta_{12}\,\sin 2\theta_{13}\,\cos\delta\,\sin 2\theta_{23}\,\Delta_{31}\,\cos\Delta_{31}\sin\Delta_{31} \\
+\;& \alpha^2\,\sin^2 2\theta_{12}\,\cos^2\theta_{23}\,\Delta_{31}^2,
\end{aligned}
$$

with

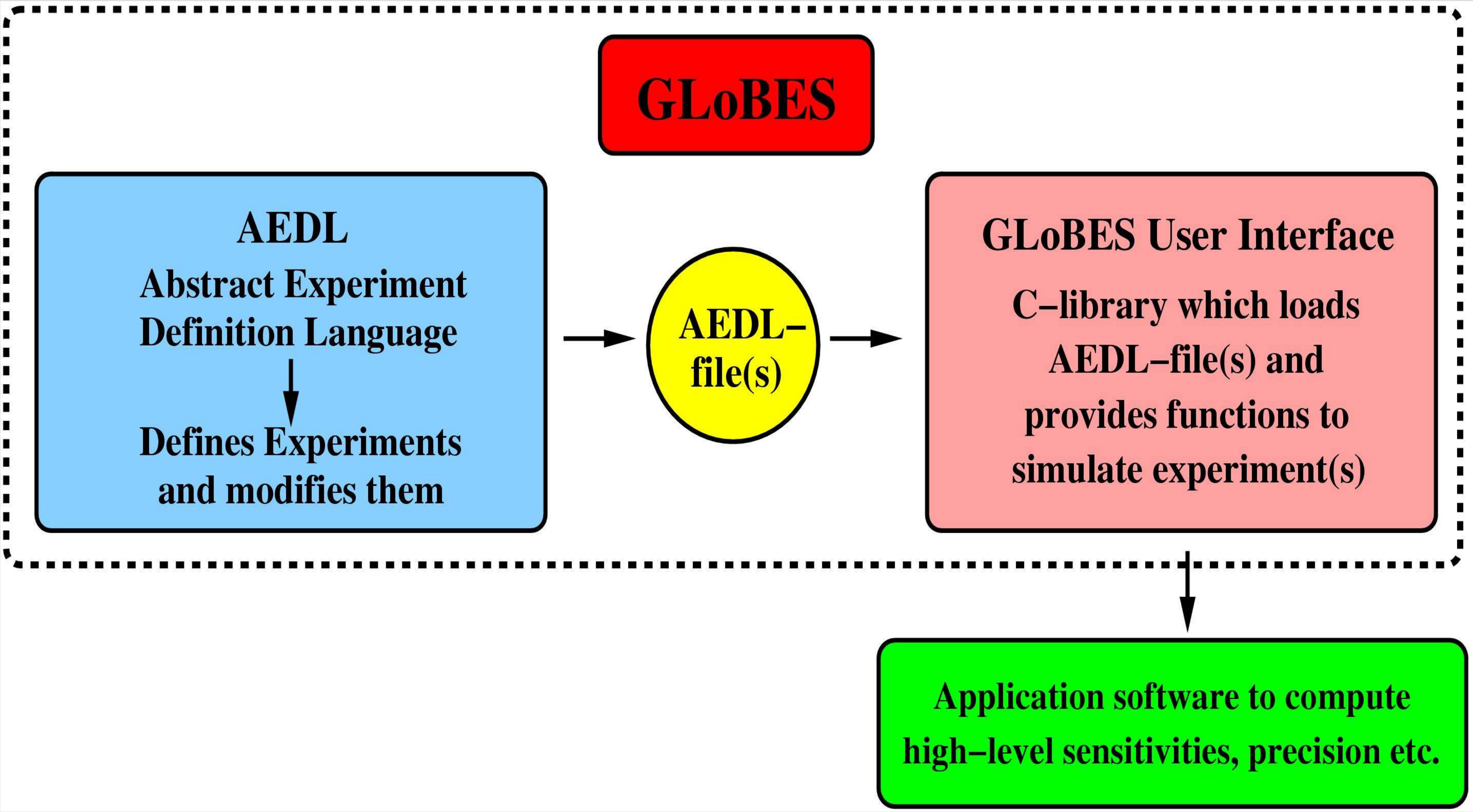
$$
\alpha \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}}\,, \qquad \Delta_{31} \equiv \frac{\Delta m^2_{31} L}{4E_\nu}
$$

What we really measure

- A 'real' experiment does NOT measure probabilities but event rates
- The rates are a convolution of probability, flux, x-section, detector effects ...

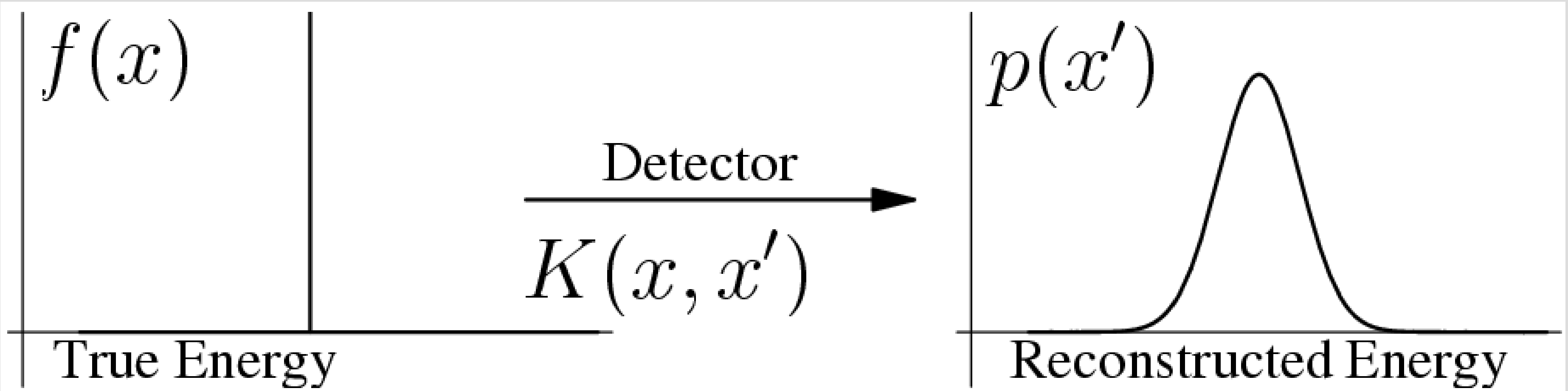
$$
\begin{aligned}
\frac{dn_\beta^{\mathrm{IT}}}{dE'} = \quad & N \int_0^\infty \int_0^\infty dE\, d\hat{E} \quad \underbrace{\Phi_\alpha(E)}_{\text{Production}} \times \\
& \underbrace{\frac{1}{L^2} P_{(\alpha\to\beta)}(E, L, \rho; \theta_{23}, \theta_{12}, \theta_{13}, \Delta m^2_{31}, \Delta m^2_{21}, \delta_{\mathrm{CP}})}_{\text{Propagation}} \times \\
& \underbrace{\sigma_f^{\mathrm{IT}}(E) k_f^{\mathrm{IT}}(E - \hat{E})}_{\text{Interaction}} \times \\
& \underbrace{T_f(\hat{E}) V_f(\hat{E} - E')}_{\text{Detection}},
\end{aligned}
$$

Basic concept behind GLoBES



What does a detector do?

- It maps some incident, true particle flux X into a detected, reconstructed flux Y
- The particles in X and Y may be different
- There may be many different Y's for one X

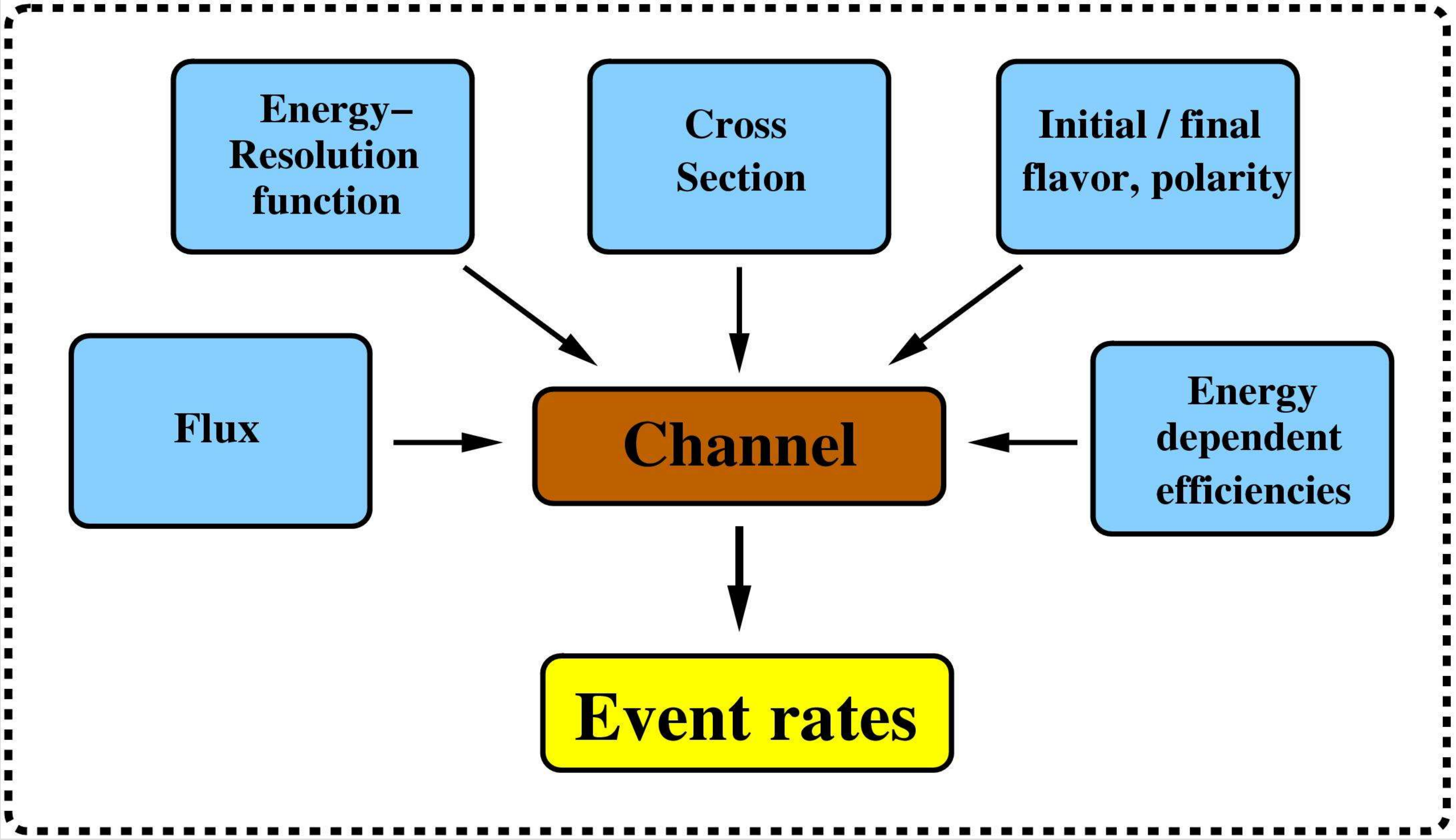


$$p(x') = \int dx\, f(x) K(x, x')$$

Intermediate states?

- This mapping usually involves intermediate states, eg. hadron showers, muons asf.
- The are only important in as far as they determine the properties of the mapping function
- Goal: Fast detector “simulation”

AEDL / Channels



AEDL Channels cont.

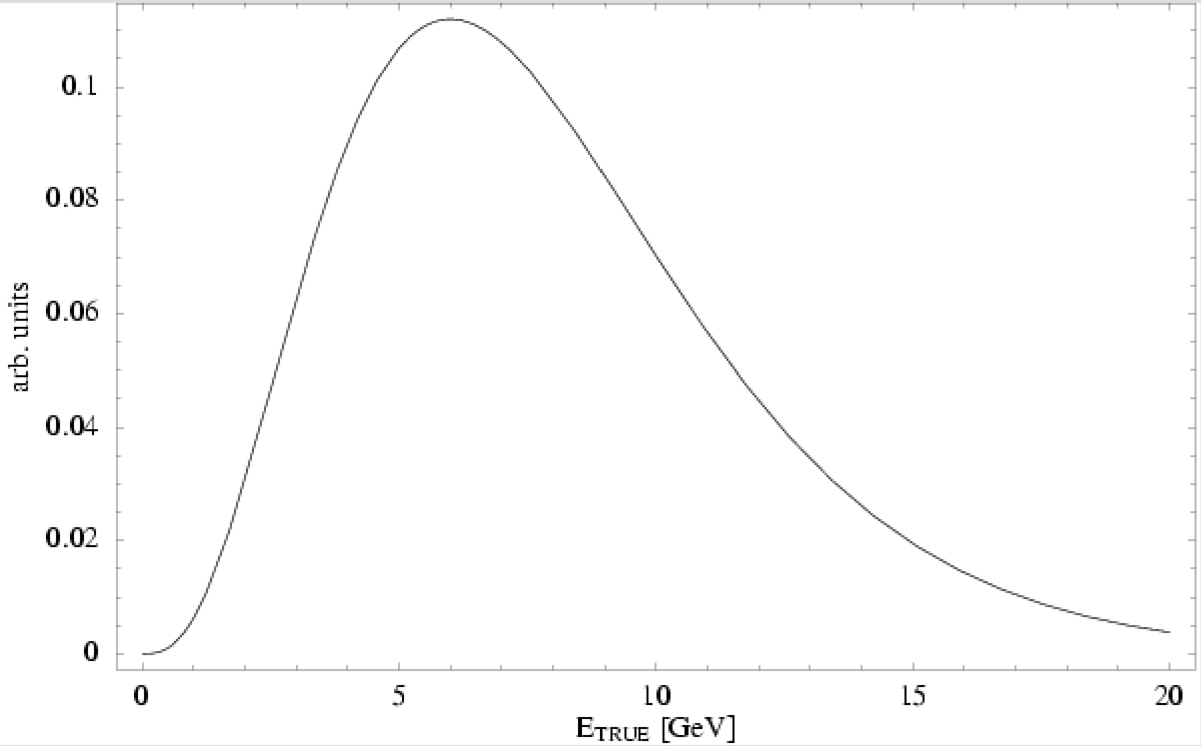
- Different intermediate states may yield a different mapping
- There may be some events cleaner than others, eg. QE
- Different sources for signal and backgrounds
- Different event types, eg. QE vs NC
- Currently up to 32 channels per experiment

Example

- From here on I will present a toy example
- The detector Monte Carlo is a black box
  - I made up a resolution function etc
  - I then “generated” events
  - and arbitrarily tagged them as 'signal' or 'garbage'
- How to put each of the parts into AEDL syntax, is better explained in the Manual

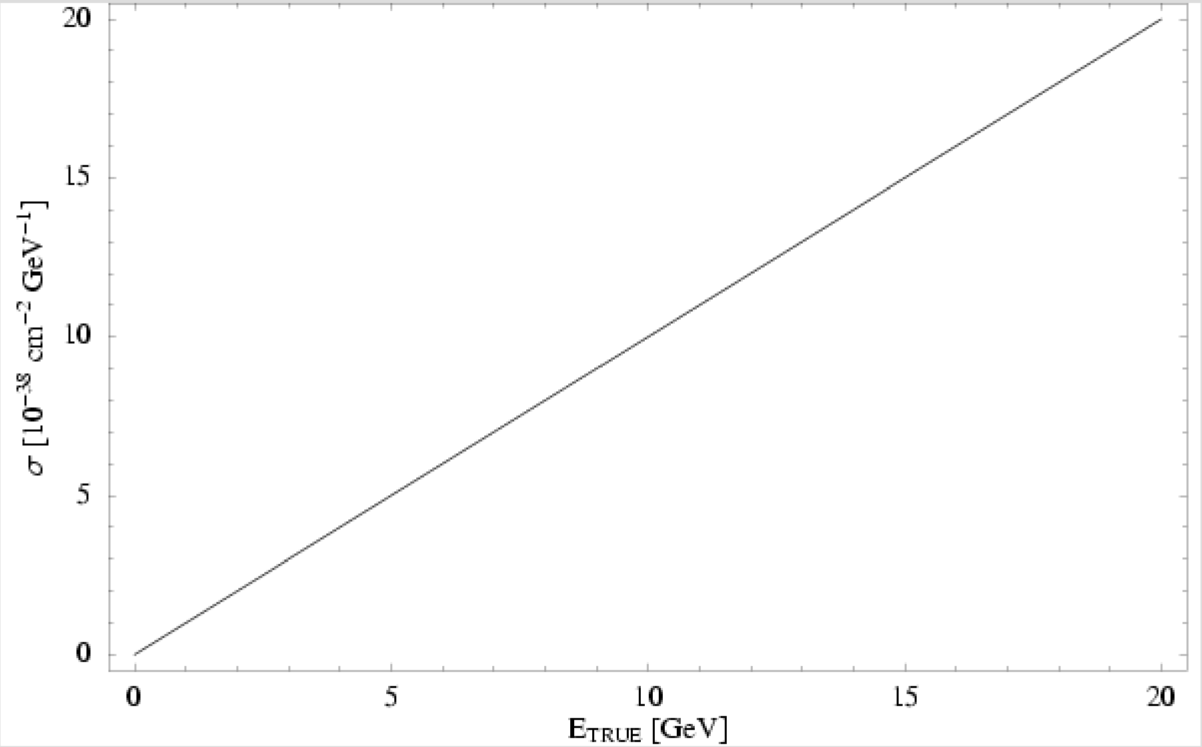
Flux

- Flux either given by formula (NF, Beta-Beam)
- or MonteCarlo (superbeams)



Cross section

- From calculation (rarely)
- From fit to data
- Event generator, eg. NUANCE



Mapping function

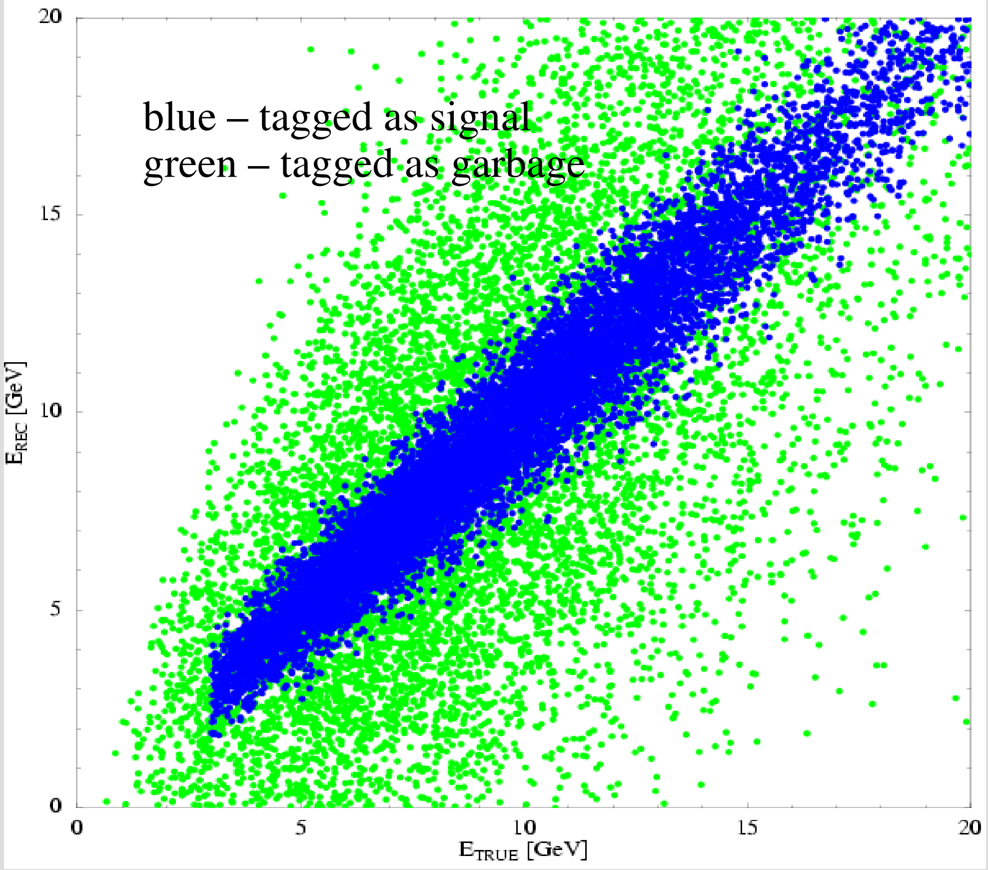
- Has to be derived from MC
- either parametrically, eg. Gaussian resolution
- or in form of migration or smearing matrices

Typical output from MC

Draw true energies from flux times x-section

Follow events through detector

Assign to each event the reconstructed energy and event type

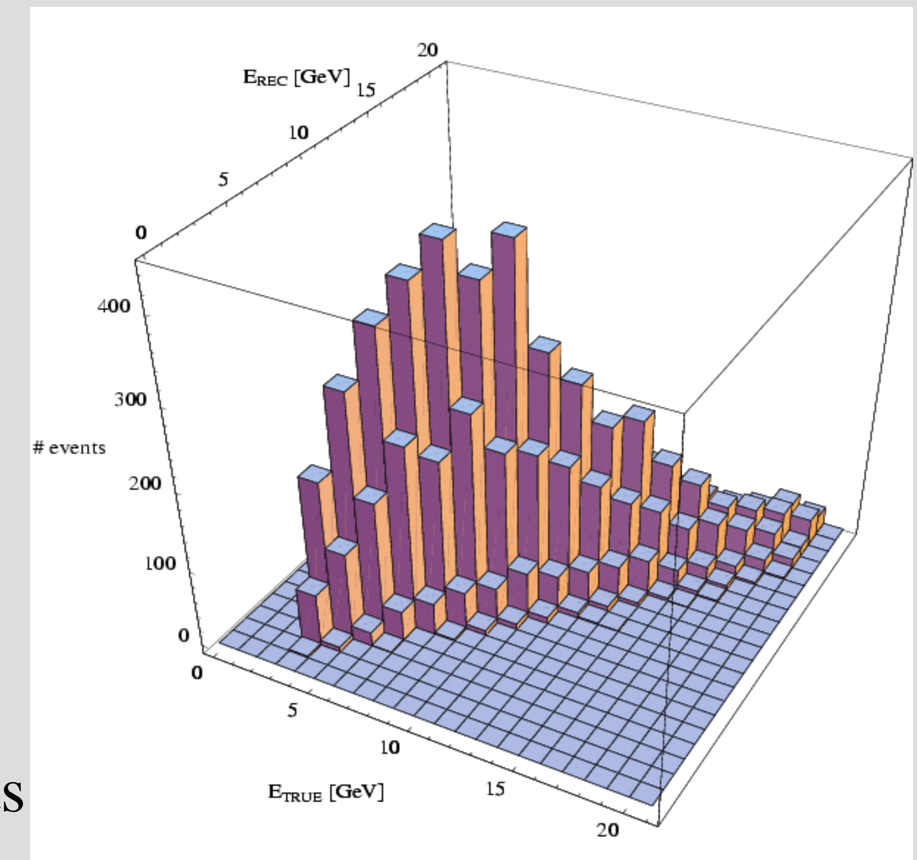


Migration matrix for the signal

- Take the blue points
- Bin them in 2D bins
- Raw matrix

$$
E_{\mathrm{REC}} \quad \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 7 & 9 & 4 & 1 \\ 0 & 0 & 2 & 9 & 7 \end{pmatrix}
$$

$$
E_{\mathrm{TRUE}}
$$



bins in true energy: sampling points
bins in rec. energy: bins

Reweighting

- Raw matrix has to be reweighted
  - flux used for MC will be modulated by oscillations
  - number of events in MC does not correspond to actual number, eg. I generated 20000 events, but the real experiment may have only 500
  - number of generated events usually is not the same than the number of events which are reconstructed as signal (efficiency)

Reweighting cont.

Sum each column

$$n_i = \sum_{j=1}^{N_{\mathrm{bins}}} m_{ij}$$

Divide each column by that sum

$$m_{ij} \rightarrow m_{ij}/n_i$$

Account for 'lost' events, efficiency

$$\epsilon = \frac{\sum_{i=1}^{N_{\mathrm{sampling}}} n_i}{\text{number of generated events}}$$

Computing events

- Event computation reduced to matrix multiplication

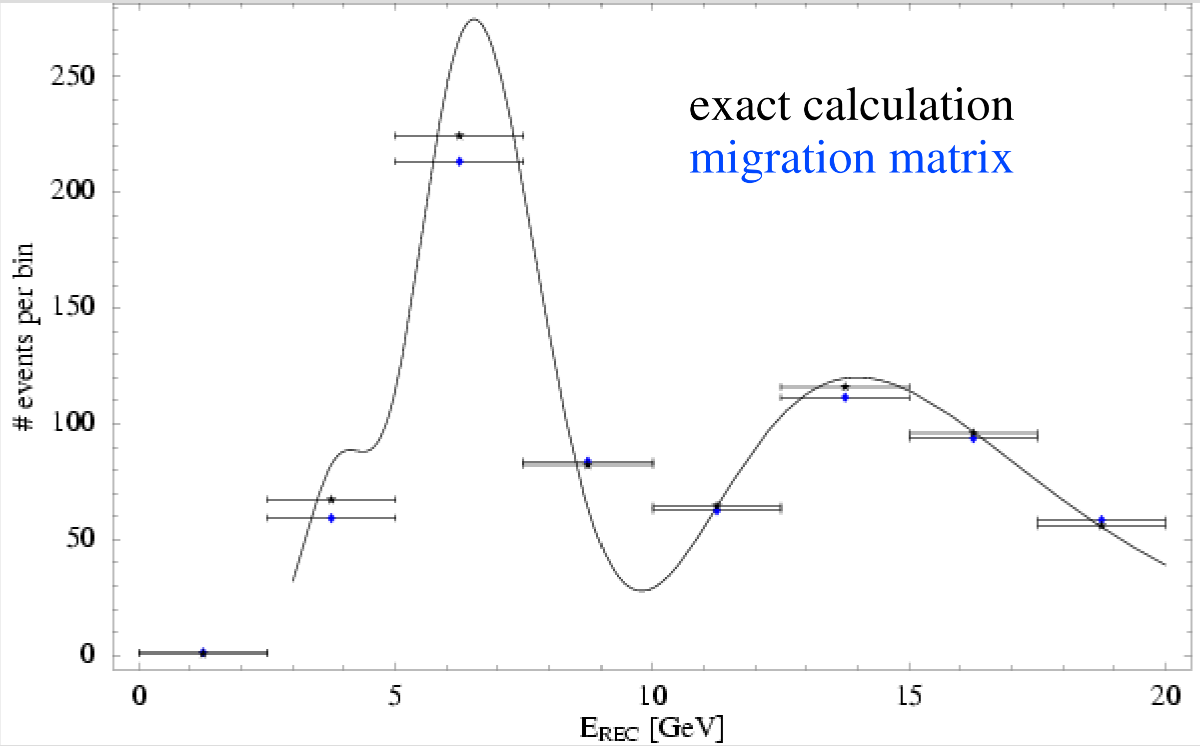
$$\Phi(E) = P(E) \times \sigma(E) \times \phi(E)$$

Can be regarded as vector and thus we can compute the events N

$$N_j = \sum_{i=1}^{N_{\mathrm{sampling}}} m_{ij} \cdot \Phi_i$$

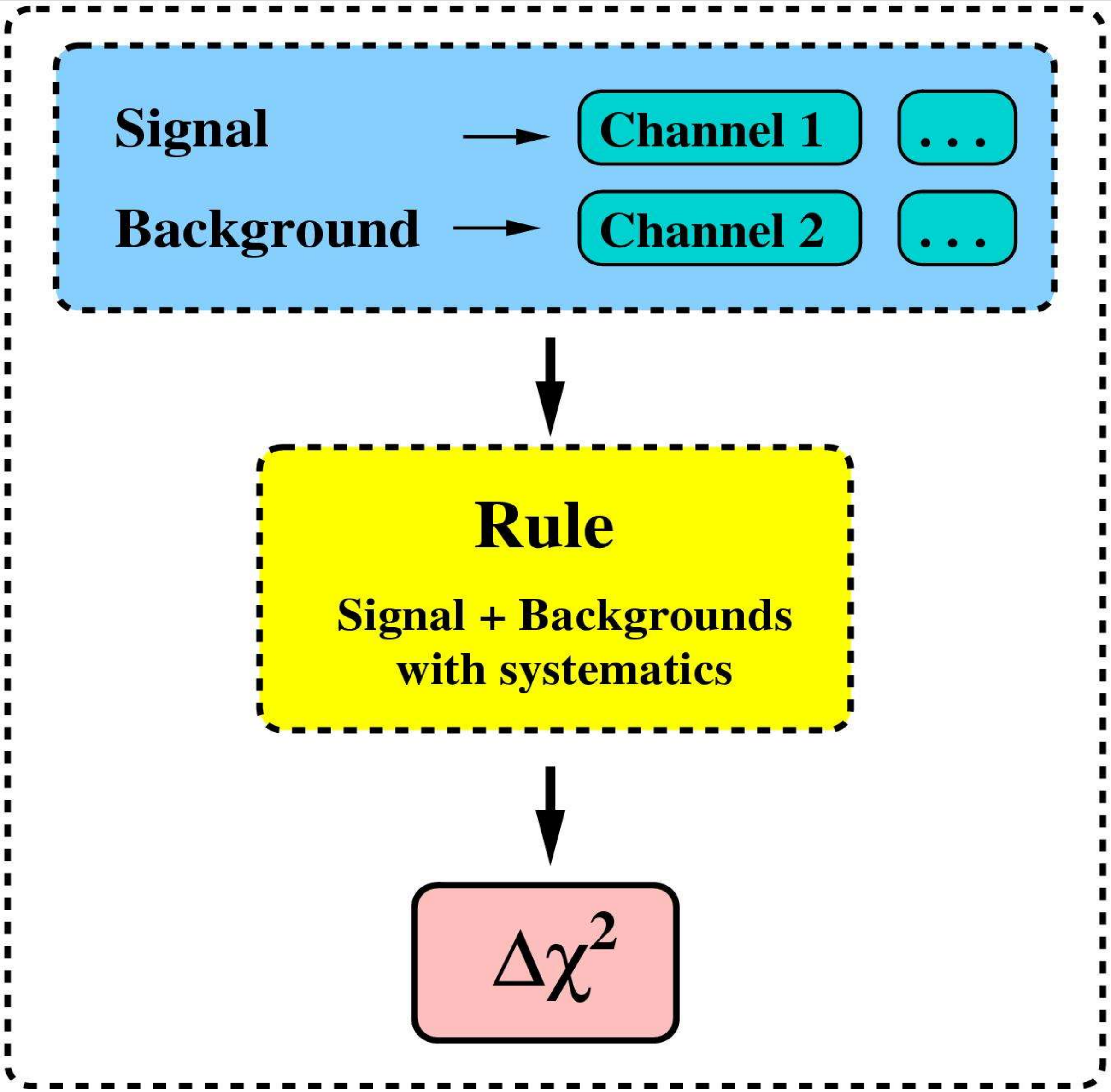
Computing events

Toy oscillation probability $$P(E) = \sin^2\left(\frac{30.0}{E}\right)$$



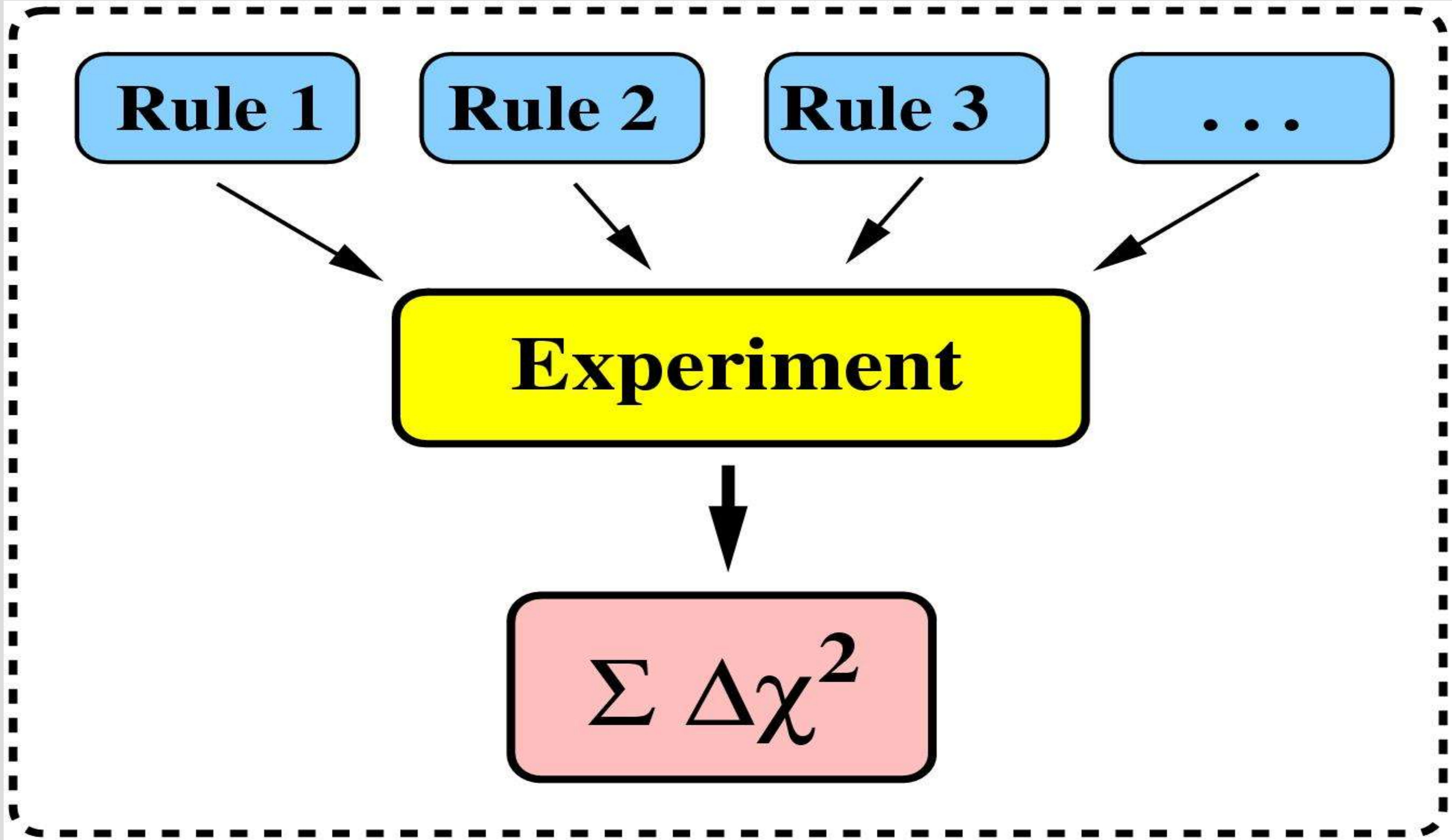
That was only 1 channel – next you would do the same for all the backgrounds

AEDL / Rules



Signal
Background
Channel 1
Channel 2
. . .
. . .
Rule
Signal + Backgrounds
with systematics
$\Delta\chi^2$

AEDL / Experiment



Debugging AEDL

```
> globes -s MINOS.glb

--------- #rule0 ---------
 1.083     0.0603374
 1.25      0.254606
 1.417     0.663695
 1.583     1.35012
 1.75      2.31606
 1.917     3.49245
 2.083     4.76538
 2.25      6.02335
 2.417     7.18871
 2.583     8.21697
 2.75      9.08048
 2.917     9.75821
 3.083     9.44848
 3.25      9.00641
 3.417     8.46126
 3.583     7.84135
 3.75      7.17314
 3.917     6.48066
 4.083     5.7852
 4.25      5.10508
 4.417     4.45543
 4.583     3.8479
 4.75      3.29056
 4.917     2.78792
 5.083     2.3413
 5.25      1.94942
 5.417     1.60913
 5.583     1.31619
 5.75      1.0659
 5.917     0.853586
----------------------
Total:     135.989


..................
.........
....
.
```

Interactive call of globes

=> event rates

globes has many command line options

GLoBES in a nutshell

- GLoBES tries to implement a fast simulation
- Generating migration matrices from MC output is straightforward, but requires care
- Channels offer quite some flexibility
- If all else fails, look into the manual
- There is an AEDL debugging tool, `globes`
- `globes@ph.tum.de`

From chi^2 to sensitivity - I

- Generally the chi^2 function depends on many parameters
- Includes systematics
- Includes external measurements (eg. solar)
- Usually one is only interested in a small subset
- Marginalizing over nuisance parameters

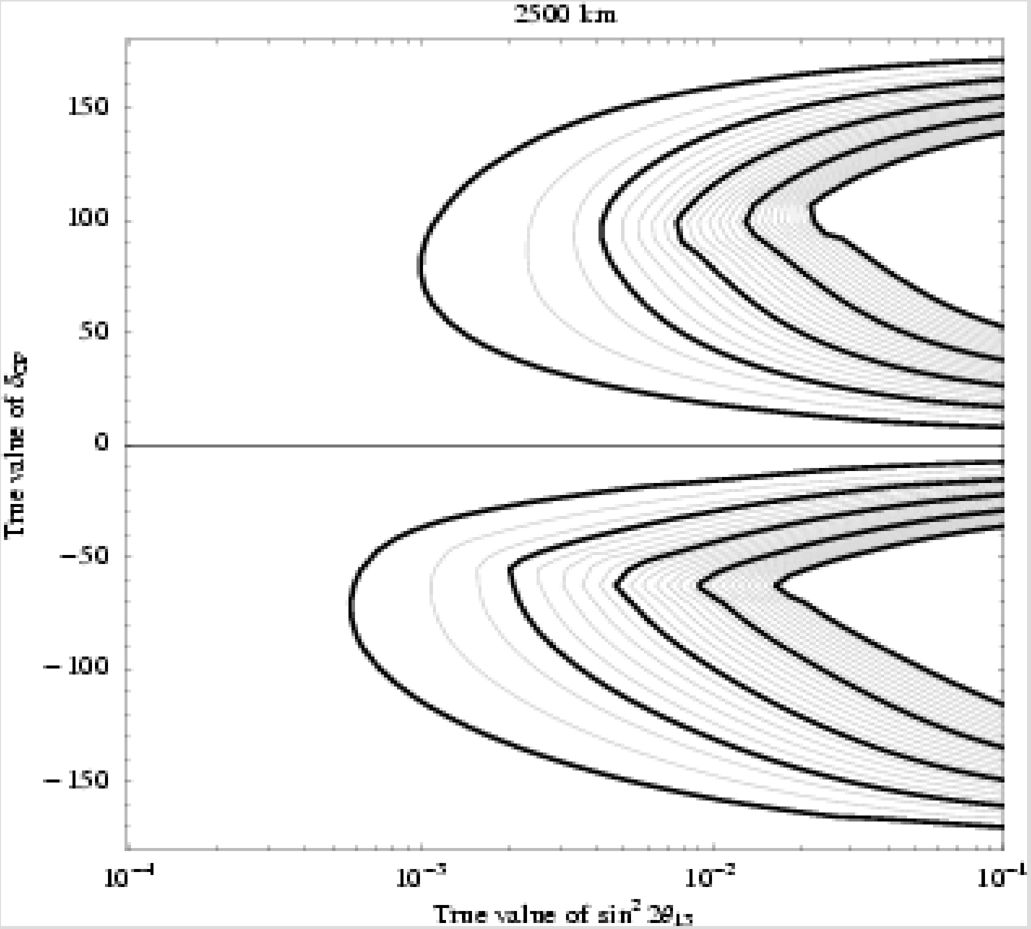
From chi^2 to sensitivity - II

- Discovery reach
- Choose a pair of true values δ and θ_{13}
- Compute event rates and perform a fit including correlations and degeneracies

Case $\sin^2\theta_{13}$ discovery
with $\theta_{13} = 0$
Case CPV discovery
with $\min\{\chi^2(\delta = 0), \chi^2(\delta = \pi)\}$
Case Mass hierarchy
with $min\{\chi^2(-\Delta m^2_{31})\}$

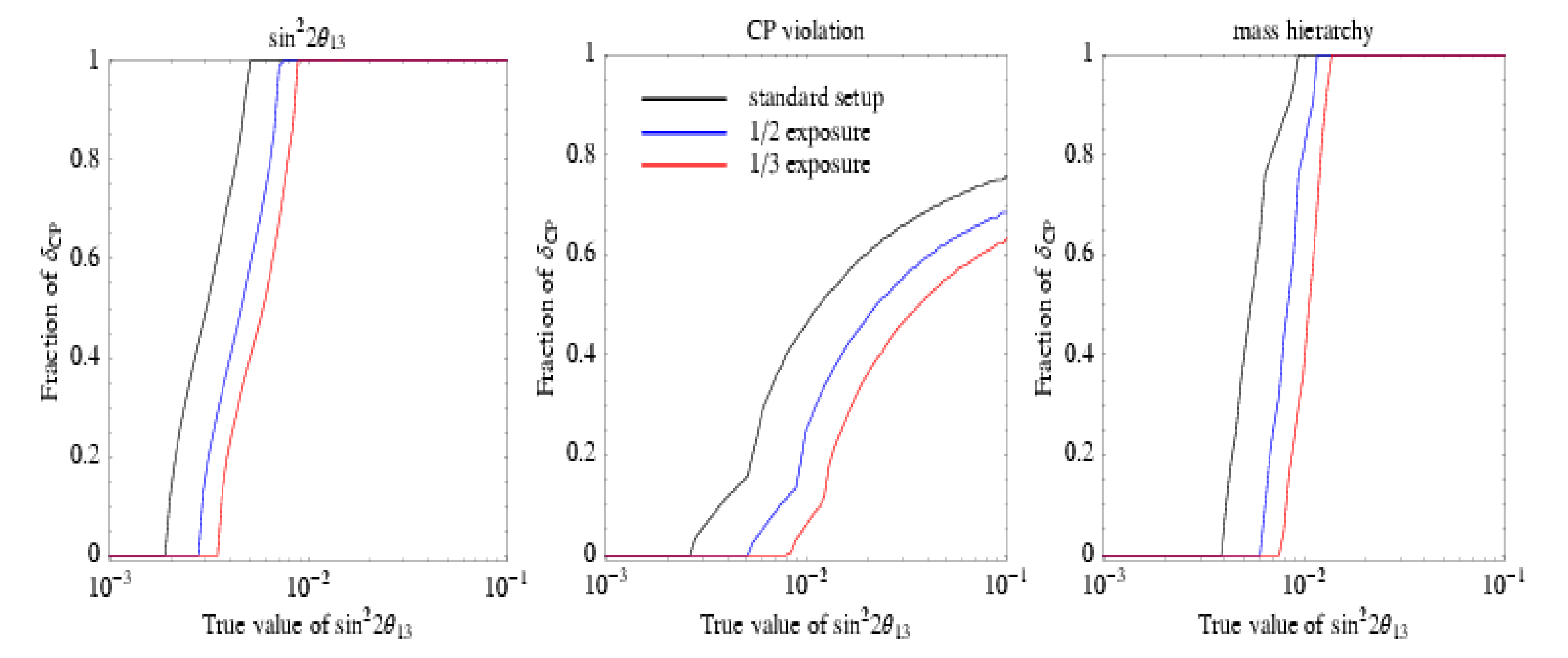
Examples

- CPV



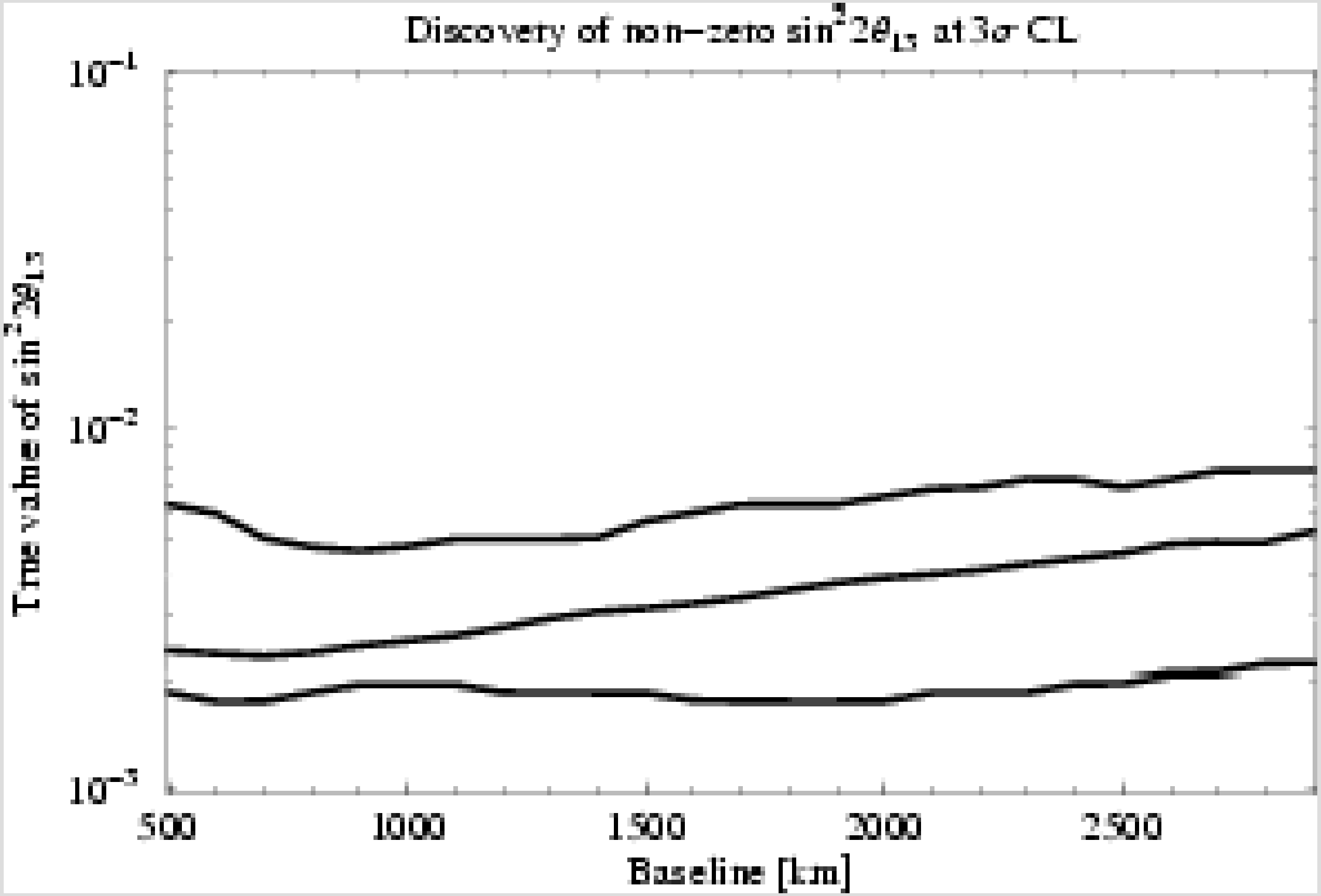
Examples

- Changing exposure



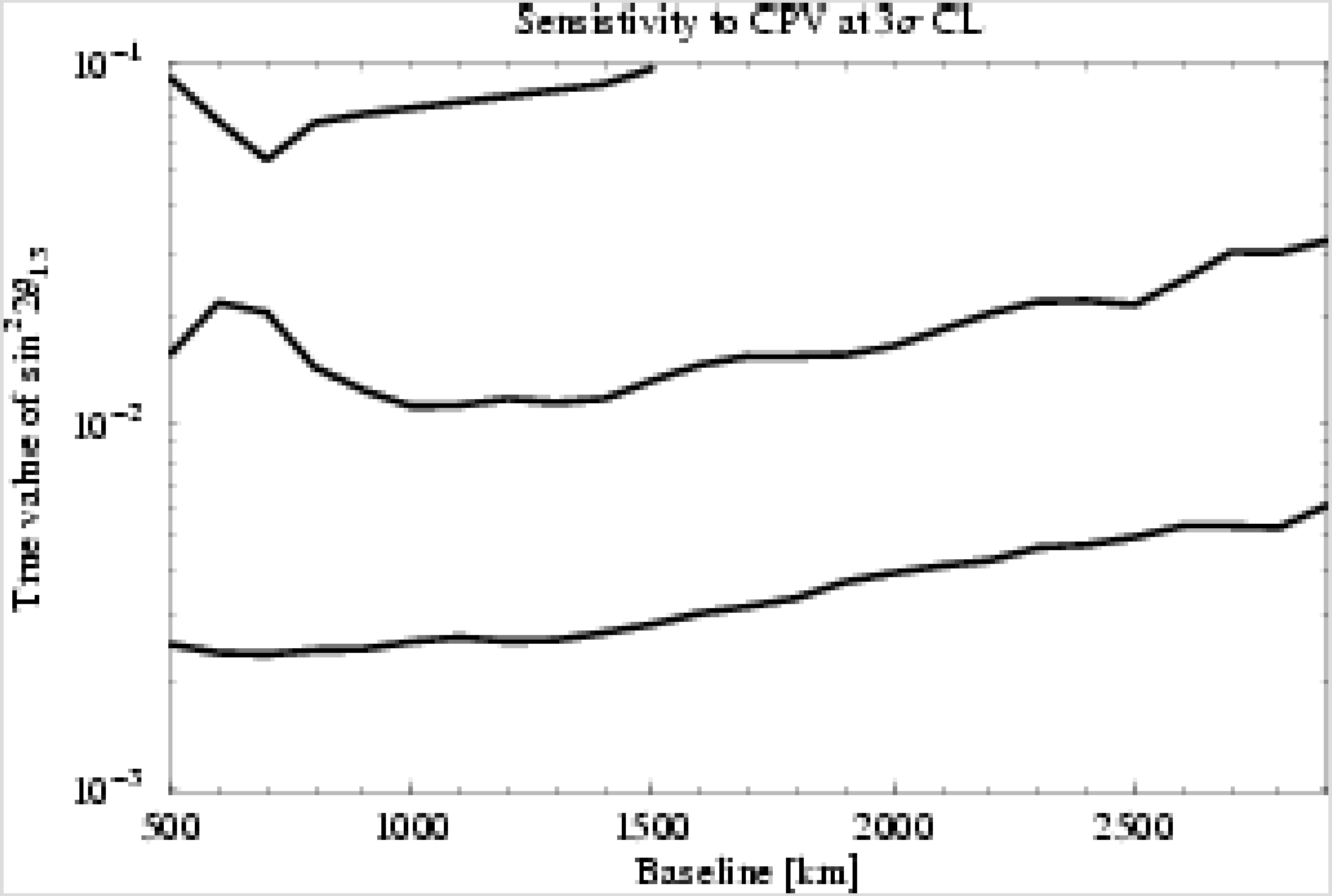
Baselines

- Theta_13



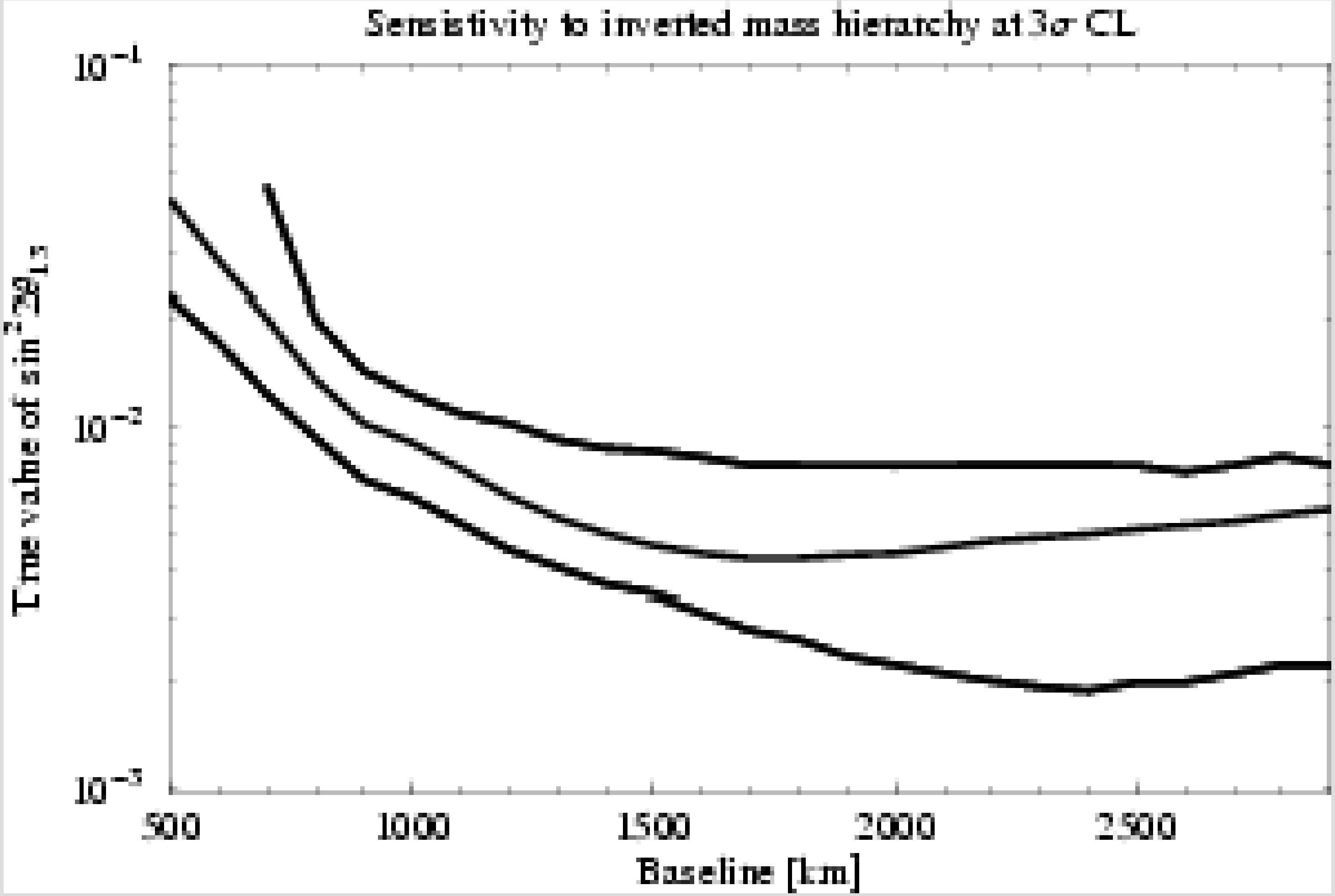
Baselines

- CPV



Baselines

- Mass hierarchy



Summary

- We want to measure physical parameter
- Computing sensitivities requires
  - event rate calculation
  - systematics treatment
  - real fit to data including correlations & degeneracies
- The methods are well understood – but are expert knowledge
- GLoBES can ease that task
- Sensitivity of the wide band approach is quite robust with respect to changes in the baseline