Two Particle Correlations: Saturation and Issues with Universality

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- BX and F. Yuan, arXiv:1003.0482 [hep-ph].

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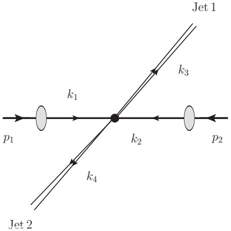
Di-hadron production in hadron-hadron collision

Consider the inclusive production of two high-transverse-momentum back-to-back particles in hadron-hadron collisions, i.e., in the process:

$$H_1 + H_2 \to H_3 + H_4 + X.$$



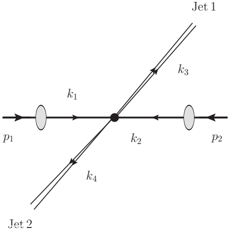
The standard k_t factorization "expectation" is:

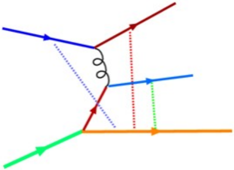
$$\boxed{E_3 E_4 \frac{\mathrm{d}\sigma}{\mathrm{d}^3 p_3 \mathrm{d}^3 p_4} = \sum \int \mathrm{d}\hat{\sigma}_{i+j\to k+l+X} f_{i/1} f_{j/2} d_{3/k} d_{4/l} + \cdots}$$

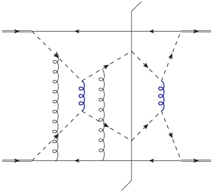
- Convolution of hard cross section $\mathrm{d}\hat{\sigma}$ with TMD parton distribution function $f(x, k_\perp)$ and fragmentation function d.
- Only Drell-Yan process is proved for factorization in hadron-hadron collisions. *[Bodwin; 85, 86], [Collins, Soper, Sterman; 85, 88]*.

Breaking down of the k_t factorization in di-hadron production

- *[Bacchetta, Bomhof, Mulders and Pijlman; 04-06]* Wilson lines approach
  Studies of Wilson-line operators show that the TMD parton distributions are not generally process-independent due to the complicated combinantion of initial and final state interactions. TMD PDFs admit process dependent Wilson lines.
- *[Collins, Qiu; 07], [Collins; 07], [Vogelsang, Yuan; 07] and [Rogers, Mulders; 10]*
  Scalar QED models and its generalization to QCD (Counterexample to Factorization)



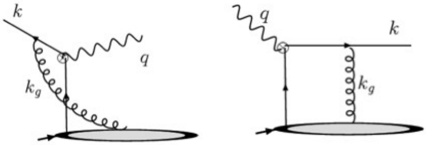




- $\mathcal{O}(g^2)$ calculation shows non-vanishing anomalous terms with respect to standard factorization.
- Remarks: TMD parton distributions are non-universal. k_t factorization is violated in di-jet production.
- Trouble? or Opportunity?
- Resummation up to all order of g including the anomalous terms*[BX, Yuan; 10]*.
- The effect of k_t factorization violation is resummable and calculable. Opportunity!

Why is the di-jet production process special?

Initial state interactions and/or final state interactions



- In Drell-Yan process, there are only initial state interactions.

$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ - i\epsilon} A^+(k_g) = \int_0^{-\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

Eikonal approximation $\Longrightarrow$ gauge links.

- In DIS, there are only final state interactions.

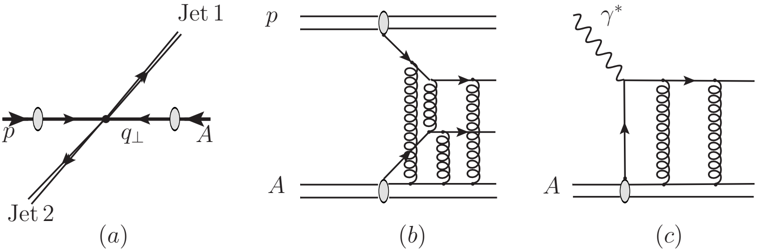
$$\int_{-\infty}^{+\infty} \mathrm{d}k_g^+ \frac{i}{-k_g^+ + i\epsilon} A^+(k_g) = \int_0^{+\infty} \mathrm{d}\zeta^- A^+(\zeta^-)$$

Eikonal approximation $\Longrightarrow$ gauge links.

- However, there are both initial state interactions and final state interactions in the di-jet process.

Breaking down of the k_t factorization at small-x

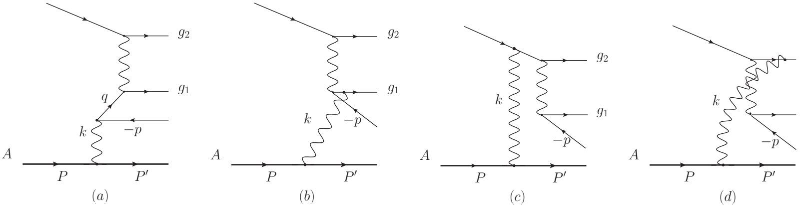
[Brodsky, et al, 02](DIS)
[Collins, Qiu; 07], [Collins; 07], [Vogelsang, Yuan; 07](di-jet)
We employ the same scalar QED model, and study the TMD parton distributions at small-x in di-jet production process in pA collisions.



- This model allows us to calculate the TMD pdfs up to all order in a few of different processes exactly.
- The goal is to compute the TMD pdf in di-jet production and compare it with those calculated in DIS and Drell-Yan process.

The lowest order amplitude

[BX, Yuan; 10] We focus on the TMD parton distributions of the nucleus target at small-x limit, namely $q^+ \ll P^+$.
$qq' \to qq'$ channel:



Remarks:

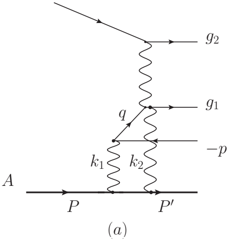
- We assign charge g_1 to the parton from the nucleus target, and g_2 to the scalar quark from the projectile proton. Also assume the nucleus has charge g.
- The lowest order amplitude is found to be

$$A^{(1)}(k,p) = gg_1 \frac{1}{k_\perp^2 + \lambda^2}\left[\frac{1}{D_1} - \frac{1}{D_2}\right],$$ Hard part factorized out.

where $D(p_\perp) = 2xP^+p^- + p_\perp^2 + m^2$ and $D_1 = D(q_\perp)$ and $D_2 = D(p_\perp)$.

The second order amplitude

[BX, Yuan; 10] The second order amplitude:



- There are 20 graphs in total.
- The amplitude starts to show dependence on g_2. Similar with *[Collins; 07]*.

Remarks:

- The second order amplitude is found to be

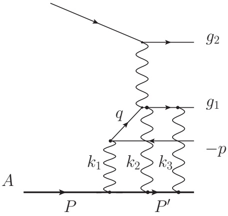
$$A^{(2)}(k,p) = \frac{i}{2}g^2 \int d[1]d[2] \left\{ g_1^2 \left[\frac{1}{D_1} + \frac{1}{D_2} - \frac{1}{D_{21}} - \frac{1}{D_{22}} \right] + g_1 g_2 \left[\frac{2}{D_2} - \frac{2}{D_{21}} \right] \right\},$$

where $D_{1i} = D(q_\perp - k_{i\perp})$, $D_{2i} = D(p_\perp - k_{i\perp})$ and

$$\int d[1]d[2] = \int \frac{d^2k_{1\perp}d^2k_{2\perp}}{(2\pi)^4} \frac{1}{k_{1\perp}^2 + \lambda^2} \frac{1}{k_{2\perp}^2 + \lambda^2} \times (2\pi)^2 \delta^{(2)}(k_\perp - k_{1\perp} - k_{2\perp})$$

The third order amplitude

[BX, Yuan; 10] The third order amplitude:



(b)

- Motivation: $|A^{(1)} + A^{(2)} + A^{(3)} + \cdots|^2$ and resummation.
- There are 120 graphs in total.
- The amplitude also contains dependence on g_2.

Remarks:

- The third order amplitude is found to be

$$A^{(3)}(k,p) = \frac{1}{3!}g^3 \int d[1]d[2]d[3] \left\{ g_1^3 \left[\frac{1}{D_2} - \frac{1}{D_1} + \frac{3}{D_{13}} - \frac{3}{D_{21}}\right] + g_1^2 g_2 \left[\frac{3}{D_2} + \frac{3}{D_{13}} - \frac{3}{D_{21}} - \frac{3}{D_{22}}\right] + g_1 g_2^2 \left[\frac{3}{D_2} - \frac{3}{D_{21}}\right] \right\}.$$

Resummation of the amplitudes up to all order

To do the resummation, one needs to go to the coordinate space via the Fourier transform.

$$A(R,r) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2p_\perp}{(2\pi)^2} e^{-ik_\perp\cdot R_\perp - ip_\perp\cdot r_\perp} A(k,p) \ .$$

- The lowest order amplitude then becomes:

$$A^{(1)}(R,r) = gg_1 V(r_\perp)\left[G(R_\perp + r_\perp) - G(R_\perp)\right]$$

  where $G(R_\perp) = K_0(\lambda R_\perp)/2\pi$ and $V(r_\perp) = K_0(Mr_\perp)/2\pi$ with $M^2 = 2xP^+p^- + m^2$

- The total amplitude exponentiates into a simple form:

$$A^{(\mathrm{tot})}(R,r) = iV(r_\perp)\underbrace{\left\{1 - e^{igg_1[G(R_\perp+r_\perp)-G(R_\perp)]}\right\}}_{\text{Same with DIS}} \underbrace{e^{-igg_2G(R_\perp)}}_{\Rightarrow \text{ process dependent Wilson line.}} \ .$$

Comparison of the TMD pdfs among di-jet production, DIS and Drell-Yan

- The total amplitude ⇒ the TMD pdf for the di-jet process *[Xiao, Yuan, 10]*

$$\begin{aligned}\tilde{q}_{\text{di-jet}}\left(x,q_\perp\right) &= \frac{xP^{+2}}{8\pi^4}\int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp\cdot\left(R_\perp - R'_\perp\right)} \\ &\times V\left(r_\perp\right)V\left(r'_\perp\right) e^{-igg_2\left(G(R_\perp)-G(R'_\perp)\right)} \\ &\times\left\{1-e^{igg_1\left[G\left(R_\perp+r_\perp\right)-G(R_\perp)\right]}\right\}\left\{1-e^{-igg_1\left[G\left(R'_\perp+r'_\perp\right)-G\left(R'_\perp\right)\right]}\right\}.\end{aligned}$$

- TMD pdf in DIS and DY *[Brodsky, et al; 02], [Belitsky, Ji, Yuan; 03], [Peigne; 02]*

$$\begin{aligned}\tilde{q}_{\text{DIS, DY}}\left(x,q_\perp\right) &= \frac{xP^{+2}}{8\pi^4}\int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp\cdot\left(R_\perp - R'_\perp\right)} V\left(r_\perp\right)V\left(r'_\perp\right) \\ &\times\left\{1-e^{igg_1\left[G\left(R_\perp+r_\perp\right)-G(R_\perp)\right]}\right\}\left\{1-e^{-igg_1\left[G\left(R'_\perp+r'_\perp\right)-G\left(R'_\perp\right)\right]}\right\}.\end{aligned}$$

  with $r'_\perp = R_\perp + r_\perp - R'_\perp$.

- Non-Universality and k_t factorization violation! However, calculable!

The Quark Distribution for a large nucleus in DIS

- TMD pdf in DIS *[Brodsky, et al; 02], [Belitsky, Ji, Yuan; 03]*

$$\begin{aligned}\tilde{q}_{\rm DIS}(x,q_\perp) &= \frac{xP^{+2}}{8\pi^4}\int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp\cdot(R_\perp-R'_\perp)} V(r_\perp)\, V(r'_\perp)\\ &\times \left\{1-e^{igg_1[G(R_\perp+r_\perp)-G(R_\perp)]}\right\}\left\{1-e^{-igg_1[G(R'_\perp+r'_\perp)-G(R'_\perp)]}\right\}.\end{aligned}$$

$$\Downarrow$$

$$\begin{aligned}\frac{\mathrm{d}x\tilde{q}(x,q_\perp)}{d^2R_\perp} &= \frac{N_c}{16\pi^6}\int \mathrm{d}y d^2r_\perp d^2r'_\perp e^{-iq_\perp\cdot(r_\perp-r'_\perp)}\, \nabla_{r_\perp} K_0\left(\sqrt{y}r_\perp\right)\cdot\nabla_{r'_\perp} K_0\left(\sqrt{y}r'_\perp\right)\\ &\times \left\{1+\exp\left[-Q_s^2\left(r_\perp-r'_\perp\right)^2/4\right]-\exp\left[-Q_s^2 r_\perp^2/4\right]-\exp\left[-Q_s^2 r'^2_\perp/4\right]\right\},\end{aligned}$$

- Use fermionic quark.
- Perform a replacement as follows:

$$e^{-igg_1[G(x_\perp)]} \Longrightarrow U(x_\perp) = \mathrm{T}\exp\left[-igg_1\int \mathrm{d}z^-\mathrm{d}^2z_\perp G(x_\perp-z_\perp)\,\rho_a(z^-,z_\perp)\,t^a\right].$$

Note that $U(x_\perp) \underbrace{\Longrightarrow}_{\rho_a(z^-,z_\perp)t^a=\delta(z_\perp)\delta(z^-)} e^{-igg_1[G(x_\perp)]}$

- Average the distribution over the gaussian distribution $W[\rho]$.
- Agrees with *[Mueller, 99]*. Drell-Yan also agree with *[Gelis, Jalilian-Marian, 02]*

The Quark Distribution for a large nucleus in di-jet production

- TMD pdf in di-jet production *[Work in progress]*

$$
\begin{aligned}
\tilde{q}_{\text{di-jet}}(x,q_\perp) = & \frac{xP^{+2}}{8\pi^4}\int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp\cdot(R_\perp - R'_\perp)} \\
& \times V(r_\perp)V(r'_\perp)\, e^{-igg_2\left(G(R_\perp)-G(R'_\perp)\right)} \\
& \times\left\{1-e^{igg_1\left[G(R_\perp+r_\perp)-G(R_\perp)\right]}\right\}\left\{1-e^{-igg_1\left[G(R'_\perp+r'_\perp)-G(R'_\perp)\right]}\right\}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
& \frac{\mathrm{d}x\tilde{q}(x,q_\perp)}{d^2R_\perp} \\
= & \frac{N_c}{16\pi^6}\int \mathrm{d}y d^2r_\perp d^2r'_\perp e^{-iq_\perp\cdot(r_\perp - r'_\perp)}\,\nabla_{r_\perp}K_0\left(\sqrt{y}r_\perp\right)\cdot\nabla_{r'_\perp}K_0\left(\sqrt{y}r'_\perp\right) \\
\times & \left\{\begin{array}{c} \exp\left[-Q_s^2\left(r_\perp - r'_\perp\right)^2/4\right]+\exp\left[-Q_s^2\left(r_\perp - r'_\perp\right)^2/2\right] \\ -\exp\left[-Q_s^2\left(\left(r_\perp - r'_\perp\right)^2+r_\perp^2\right)/4\right]-\exp\left[-Q_s^2\left(\left(r_\perp - r'_\perp\right)^2+r'^2_\perp\right)/4\right]\end{array}\right\},
\end{aligned}
$$

- Large N_c approximation.
- Two point functions $\langle U(x_\perp)U^\dagger(x'_\perp)\rangle$(DIS) and Four point functions $\langle U(x_\perp)U^\dagger(x'_\perp)U(y_\perp)U^\dagger(y'_\perp)\rangle$.

Non-Universality

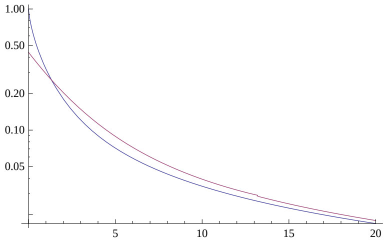


Figure: Comparison of quark distributions in DIS and di-hadron production as functions of $q_\perp^2$. The blue curve stands for the quark distribution in DIS and the red curve represents the distribution involved in di-hadron production.

- Non-Universality (as a result of initial and final state interactions) and k_t factorization violation in CGC. However, they are calculable.
- Integrated quark distributions are universal.
- Similar conclusions with respect to the k_t factorization violation have been reached in sea quark productions *[Blaizot, Gelis, Venugopalan, 04]*, two-gluon production *[Jalilian-Marian, Kovchegov, 04]* and quark-gluon production *[Marquet, 07]*.