Anomalous Transport Processes in Turbulent Nonabelian Plasmas

Berndt Müller

Saturation Workshop

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Work done with
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& S.A. Bass (Duke)

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In this talk…

…we ask the question:

Is strong coupling

really necessary

for small η/s and large $\hat{q}$?

Stellar accretion disks

"A complete theory of accretion disks requires a knowledge of the viscosity. Unfortunately, viscous transport processes are not well understood. Molecular viscosity is so small that disk evolution due to this mechanism of angular momentum transport would be far too slow to be of interest. If the only source of viscosity was molecular, then $v \sim \eta/\rho \sim \lambda\, v_T$, where λ is the particle mean free path and v_T is the thermal velocity. Values appropriate for a disk around a newly formed star might be $r \sim 10^{14}$ cm, $n \sim 10^{15}$ cm^{-3}, $\sigma \sim 10^{-16}$ cm^2, so that $\lambda \sim 10$ cm, and $v_T \sim 10^5$ cm/s . The viscous accretion time scale would then be $r^2/(12v) > 10^{13}$ yr! Longer by a factor of 10^5 - 10^6 than the age conventionally ascribed to such disks. Clearly if viscous accretion explains such objects, there must be an anomalous source of viscosity. The same conclusion holds for all the other astronomical objects for which the action of accretion disks have been invoked."

(From James Graham – Astronomy 202, UC Berkeley)
http://grus.berkeley.edu/~jrg/ay202/lectures.html

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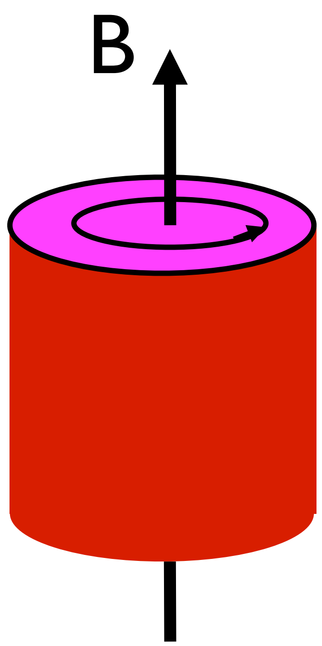
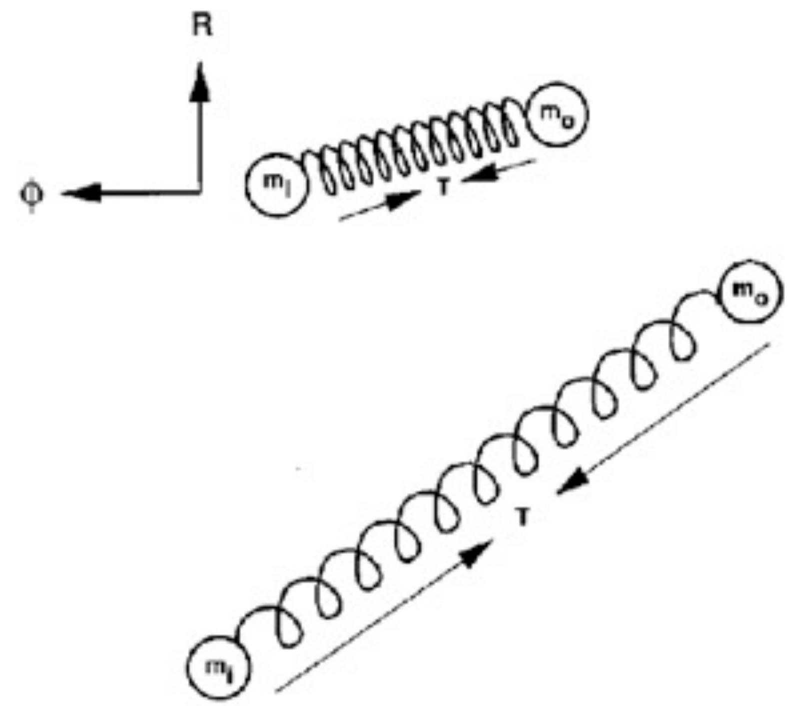
~~The solution is: String theory?~~

Anomalous viscosity



Differentially rotating disc with weak magnetic field B shows an instability (Chandrasekhar)

Magnetic tension $T = \nabla\left(B^2\right)/(4\pi)$



Spontaneous angular momentum transfer from inner mass to outer mass is amplified by interaction with the rotating disk and leads to instability (Balbus & Hawley – 1991).

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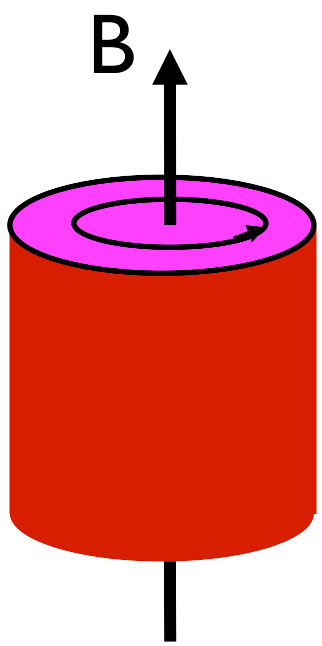

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"Anomalous", i.e. non-collisional viscosity

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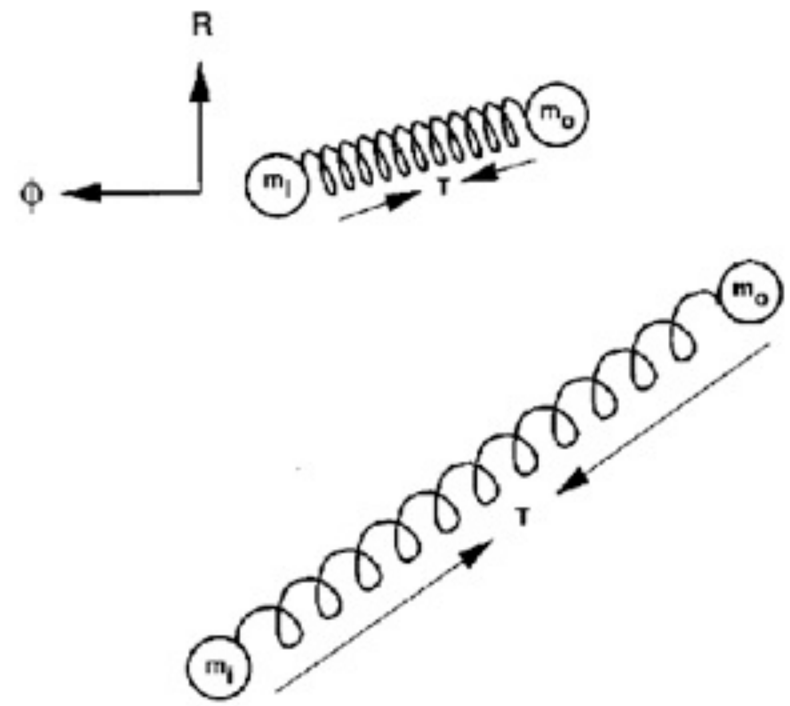

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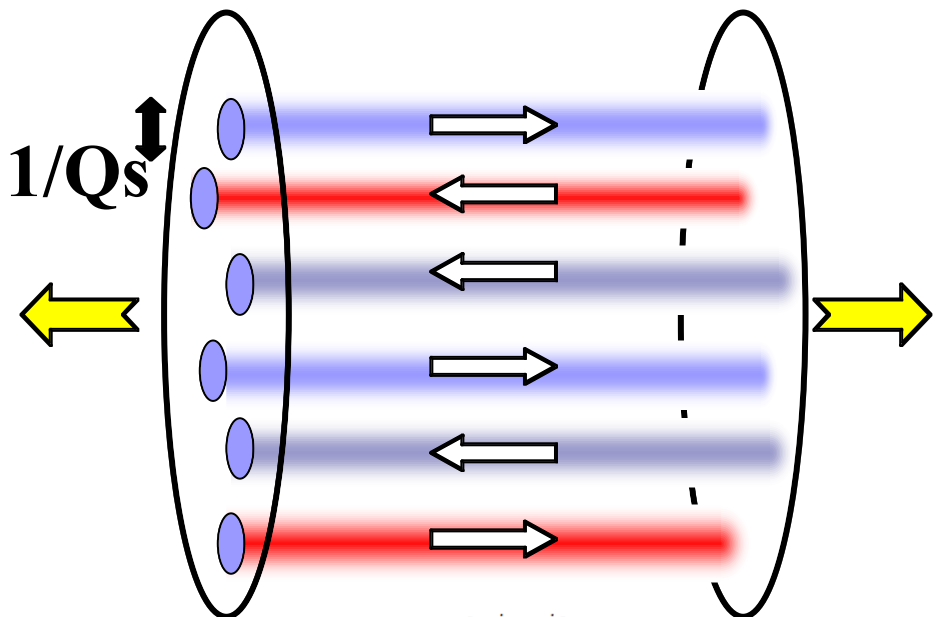
- Strong color fields in the early *glasma* exhibit Sauter and Nielsen-Olesen-type instabilities that create turbulent color fields.
- As we will see, soft color fields generate *anomalous transport coefficients*, which may dominate the transport properties of the plasma even at moderately weak coupling.

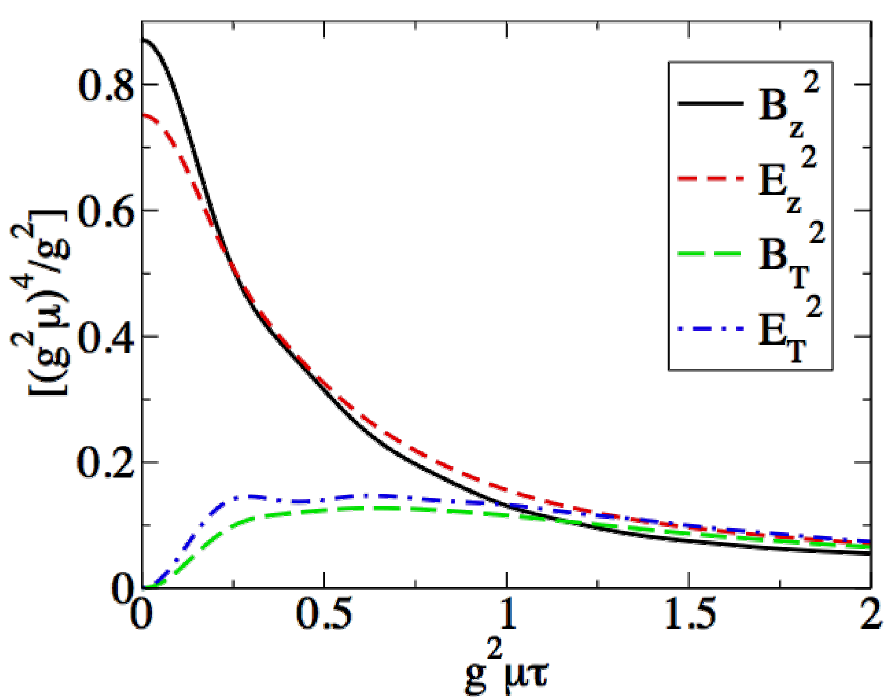
Glasma instabilities

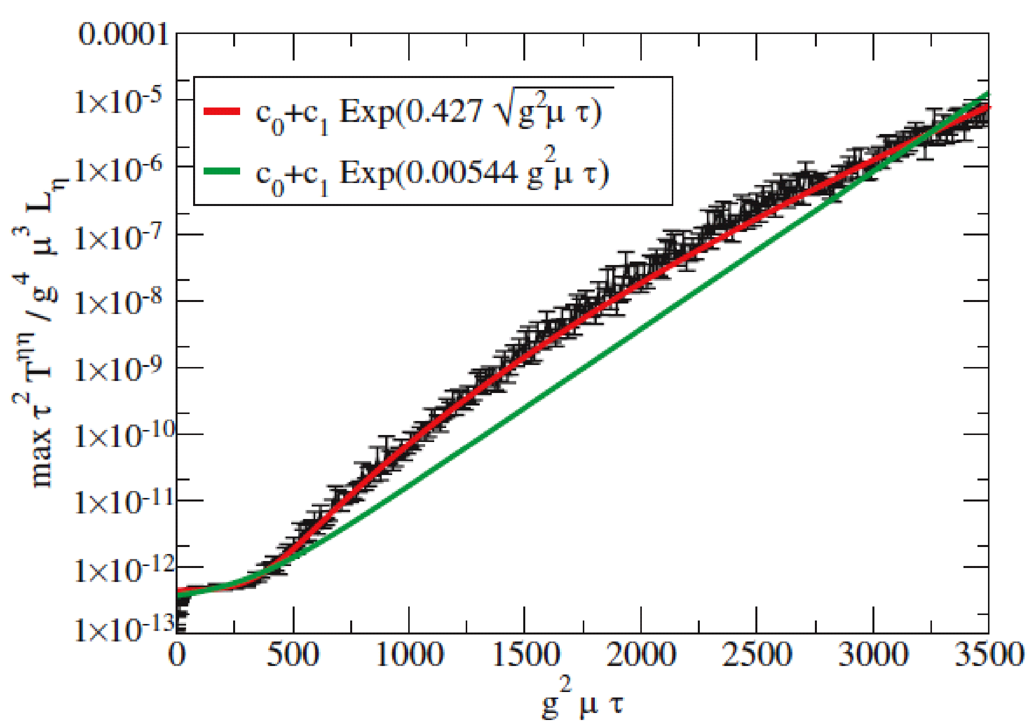


$E^z = ig[\alpha_1^i, \alpha_2^i]$
$B^z = ig\epsilon^{ij}[\alpha_1^i, \alpha_2^j].$

Nielsen-Olesen instability of longitudinal color-magnetic field (Itakura & Fujii, Iwazaki)

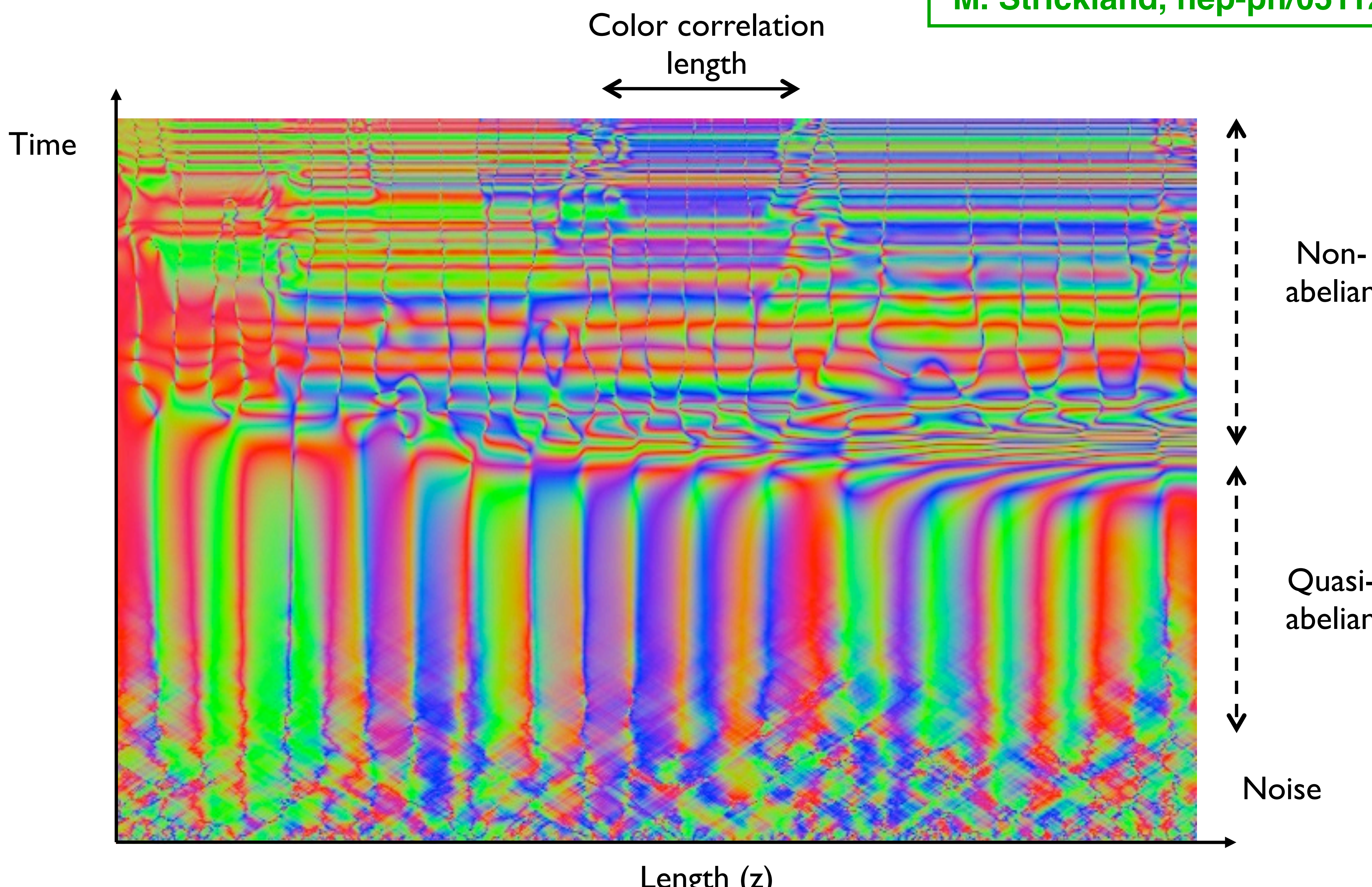
$$\frac{\partial^2\phi}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial\phi}{\partial\tau} + \left(\frac{(k_z - gA_\eta)^2}{\tau^2} - gB_z\right)\phi = 0$$





QGP instabilities

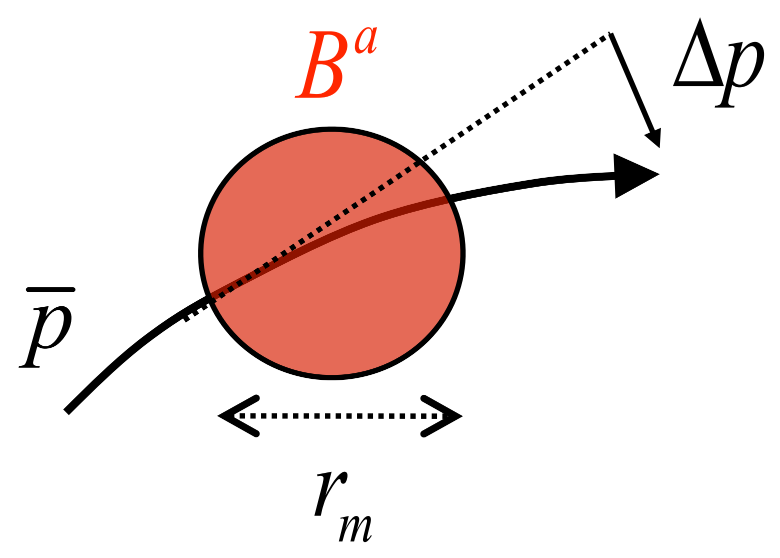
M. Strickland, hep-ph/0511212



Anomalous viscosity

Classical expression for shear viscosity:

$$\eta \approx \tfrac{1}{3} n\bar{p}\lambda_f$$



Momentum change in one coherent domain:

$$\Delta p \approx gQ^a B^a r_m$$

Anomalous mean free path in medium:

$$\lambda_f^{(A)} \approx r_m \left\langle \frac{\bar{p}^2}{(\Delta p)^2} \right\rangle \approx \frac{\bar{p}^2}{g^2 Q^2 \langle B^2 \rangle r_m}$$

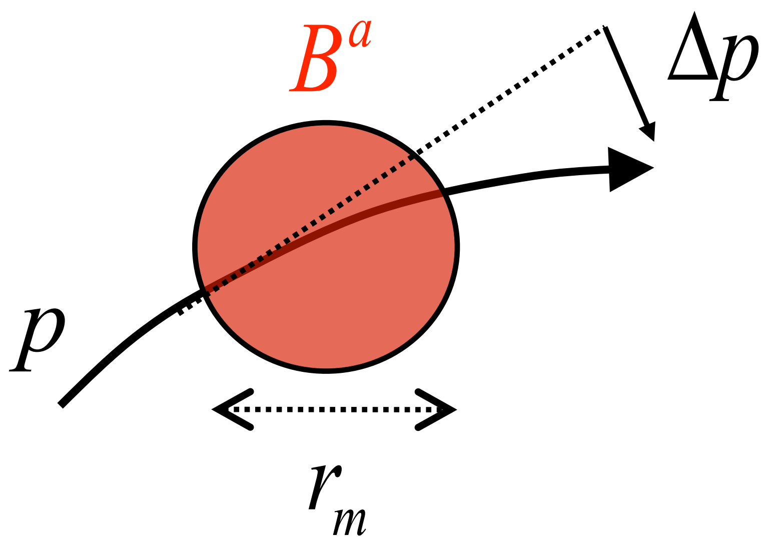
Anomalous viscosity due to random color fields:

$$\eta_A \approx \frac{n\bar{p}^3}{3g^2 Q^2 \langle B^2 \rangle r_m} \approx \frac{\frac{9}{4} sT^3}{g^2 Q^2 \langle B^2 \rangle r_m}$$

Anomalous q-hat

Jet quenching parameter:

$$\hat{q} = \frac{\left\langle \Delta p_T^2(L) \right\rangle}{L}$$



Momentum change in one coherent domain:

$$\Delta p_T = g Q^a B_\perp^a r_m$$

Anomalous jet quenching parameter:

$$\hat{q}_A = \frac{\left\langle \Delta p_T^2 \right\rangle}{r_m} = g^2 Q^2 \left\langle B_\perp^2 \right\rangle r_m$$

Relation to anomalous shear viscosity:

$$\frac{\eta_A}{s} \approx \frac{T^3}{\hat{q}_A}$$

Special case of general relation between η/s and q^ (A. Majumder, BM & Wang, PRL 99, 192301 ('07).

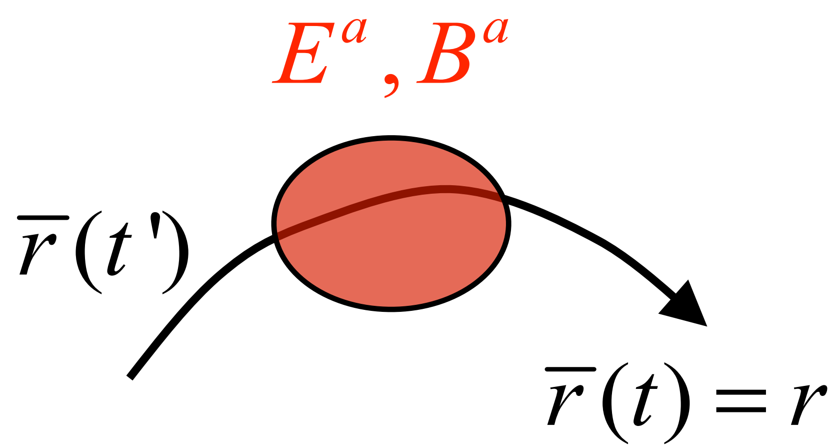
Turbulence $\Leftrightarrow$ p-Diffusion

Vlasov-Boltzmann transport of thermal partons:

$$\left[\frac{\partial}{\partial t} + \frac{p}{E_p}\cdot\nabla_r + F\cdot\nabla_p\right] f(r,p,t) = C[f]$$

with Lorentz force

$$F = gQ^a\left(E^a + \mathrm{v}\times B^a\right)$$



Assuming E, B random $\Rightarrow$ Fokker-Planck eq:

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with diffusion coefficient

$$D_{ij}(p) = \int_{-\infty}^{t} dt'\left\langle F_i\left(\overline{r}(t'),t'\right)F_j\left(r,t\right)\right\rangle.$$

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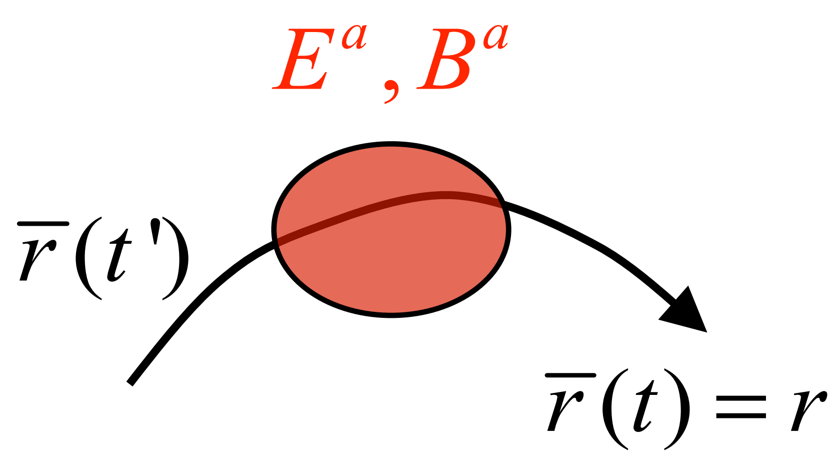

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Diffusion is dominated by chromo-magnetic fields:

$$\int dt'\left\langle B(t')B(t)\right\rangle \equiv \left\langle B^2\right\rangle\tau_m$$

Turbulent fields

Iterated Vlasov force term:

$$\left\langle F_i^a(x)U_{ab}(x,x')F_j^b(x')f(\mathbf{p})\right\rangle = \left\langle F_i^a(x)U_{ab}(x,x')F_j^b(x')\right\rangle \bar{f}(\mathbf{p})$$

Random force assumption (with finite correlation length / time):

$$\left\langle \mathcal{E}_i^a(x)U_{ab}(x,x')\mathcal{E}_j^b(x')\right\rangle = \left\langle \mathcal{E}_i^a\mathcal{E}_j^a\right\rangle \Phi_\tau^{(\mathrm{el})}\left(|t-t'|\right)\tilde{\Phi}_\sigma^{(\mathrm{el})}\left(|\mathbf{x}-\mathbf{x}'|\right)$$

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$$\left\langle g\mathbf{F}^a(x)\cdot\nabla_p f^a\right\rangle = -\frac{g^2C_2}{N_c^2-1}\left[\tau_m^{\mathrm{el}}\left\langle \mathcal{E}_i^a\mathcal{E}_j^a\right\rangle\frac{\partial^2}{\partial p_i\partial p_j} + \tau_m^{\mathrm{mag}}\left\langle \mathcal{B}_i^a\mathcal{B}_j^a\right\rangle(\mathbf{v}\times\nabla_p)_i(\mathbf{v}\times\nabla_p)_j\right]\bar{f}(\mathbf{p})$$

$$\equiv -\nabla_p\cdot D(\mathbf{p})\nabla_p\bar{f}(\mathbf{p})$$

Memory time: $$\tau_m^{\mathrm{el/mag}} = \frac{1}{2}\int_{-\infty}^{\infty} dt'\,\Phi_\tau^{(\mathrm{el/mag})}\left(|t-t'|\right)\tilde{\Phi}_\sigma^{(\mathrm{el/mag})}\left(|\mathbf{v}(t-t')|\right)$$

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Weibel regime

Take moments of $\left[\frac{\partial}{\partial t}+\frac{p}{E_p}\cdot\nabla_r-\nabla_p\cdot D(p)\cdot\nabla_p\right]\bar{f}(r,p,t)=C\left[\bar{f}\right]$ with p_z^2

$$\frac{1}{\eta}=O(1)\frac{N_c}{N_c^2-1}\frac{g^2\left\langle B^2\right\rangle\tau_m}{sT^3}+O\left(10^{-2}\right)\frac{g^4\ln g^{-1}}{T^3}\equiv\frac{1}{\eta_A}+\frac{1}{\eta_C}$$

Self-consistency $\Rightarrow$ $\boxed{\frac{\eta_A}{s}\sim\left(\frac{T}{g^3\left|\nabla u\right|}\right)^{1/2}}$ compare with $\boxed{\frac{\eta_C}{s}\sim\frac{1}{g^4\ln g^{-1}}}$

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Anomalous shear viscosity dominates over *collisional shear viscosity* at fixed ∇u in the limit $g \to 0$.

Glasma regime

In the *glasma*, most of the energy density is in the form of *color fields*:

Anomalous transport dominates over Boltzmann (collision) transport.

$$\eta_A \approx \frac{N_c^2 - 1}{15\pi^2 C_2 g^2 \left\langle \mathcal{E}^2 + \mathcal{B}^2 \right\rangle \tau_m} \int_0^\infty dp\, p^5 f(p) \approx \frac{\left(N_c^2 - 1\right) Q_s^2}{C_2 g^2 \tau_m} \cdot \frac{\varepsilon_{\text{part}}}{\varepsilon_{\text{field}}}$$

Anomalous jet quenching:

$$\hat{q}_A \approx \frac{C_2 g^2 \left\langle \mathcal{E}^2 + \mathcal{B}^2 \right\rangle \tau_m}{N_c^2 - 1} \approx \frac{g^2 \varepsilon_{\text{field}}}{Q_s} \approx \frac{Q_s^3}{(Q_s \tau)} \approx \frac{10\ \text{GeV}^2/\text{fm}}{Q_s \tau}$$

In line with estimates of q^ ~ 2 - 4 GeV^2/fm from fits to data.

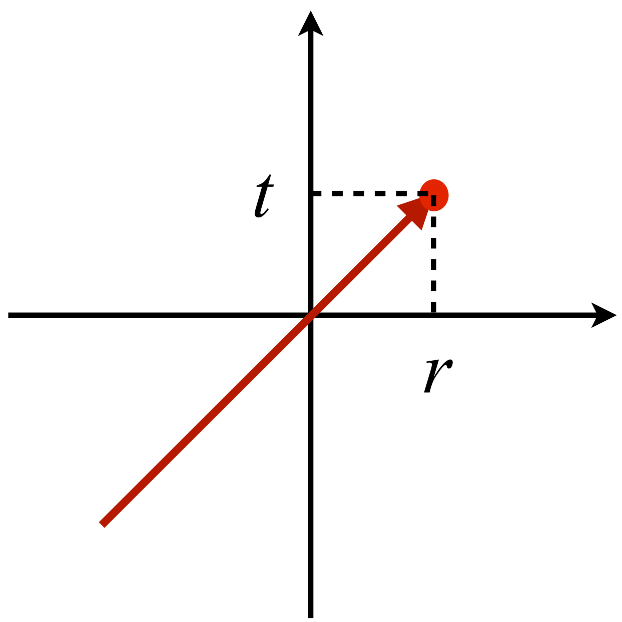
What's to be done

Time to get quantitative! The quantity to be calculated is:

$$D_{ij}(\mathrm{v}) = \int_{-\infty}^{0} d\tau \left\langle F_i(r + \mathrm{v}\tau, t + \tau) F_j(r,t) \right\rangle$$

where *F* is the color Lorentz force on a parton.

Note that $C_2 g^2 D_{\perp\perp} = \hat{q} \quad \Rightarrow \quad \eta \propto 1/\hat{q}$



D_{ij}(v) is the color force autocorrelation function along the light-cone.

Numerical evaluation by real-time lattice simulations are urgently needed, both in the stationary regime for fixed momentum anisotropy and in the CGC-seeded glasma.

The END