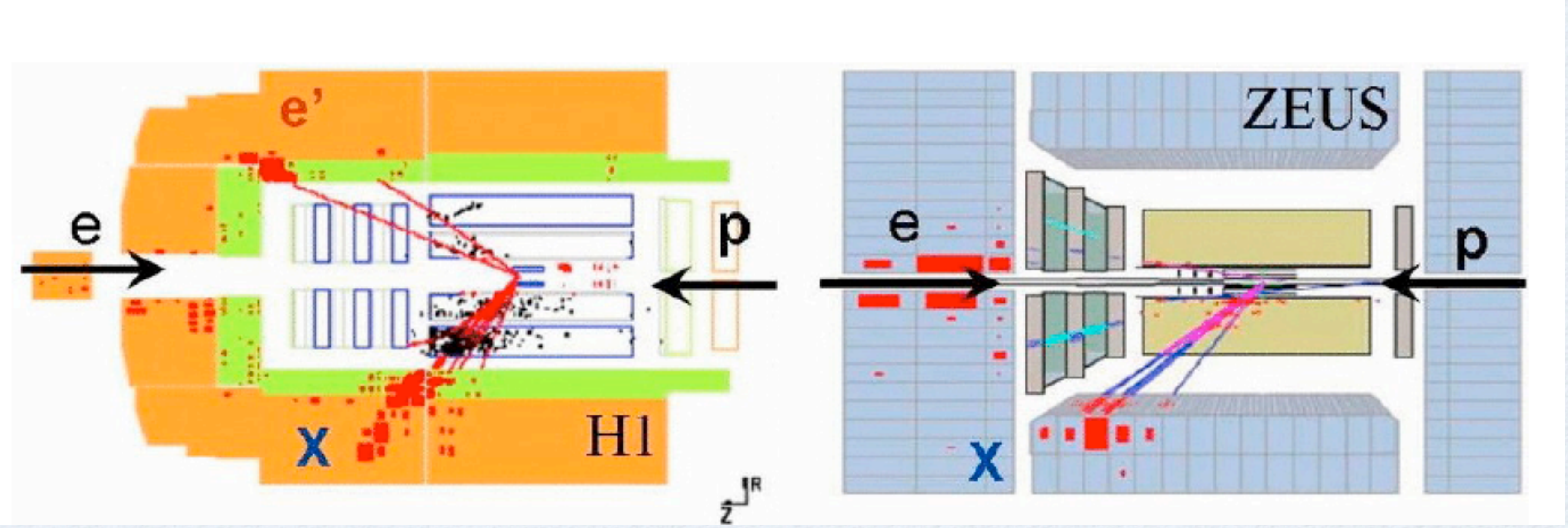
The New HERA F_2 data
and
the Determination of the Infrared Behaviour of the BFKL Amplitude

talk by Henri Kowalski,
based
on the H1 and ZEUS Combination paper, JHEP 1001:109,2010
and
the paper with L.N. Lipatov, D.A. Ross and G. Watt
arXiv 1005.0355

Workshop on Saturation, the Color Glass Condensate and Glasma

BNL, 10th of May 2010

Combining ZEUS and H1 F_2 data



H1 and ZEUS collected similar amount of data: 100 pb^{-1}

➥ improved statistical precision by ~ $1/\sqrt{2}$

Improved systematic precision

H1 and ZEUS detectors and data analysis are quite different.

➥ The H1 and ZEUS cross-sections have different sensitivities to similar sources of correlated systematic uncertainties

Combination Method

Swim H1 and ZEUS data to the same grid points:

$$\sigma_{\rm H1}\left(x_{\rm H1}, Q^2_{\rm H1}\right) \rightarrow \sigma_{\rm H1}\left(x, Q^2\right) \quad ; \quad \sigma_{\rm ZEUS}\left(x_{\rm ZEUS}, Q^2_{\rm ZEUS}\right) \rightarrow \sigma_{\rm ZEUS}\left(x, Q^2\right)$$

New measurements are obtained by building the χ^2 estimate:

Combination at point i
*[Estimate of 1 **true cross section**]*

Measurement at point i

$$\chi^2_{\rm exp}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[\boxed{m^i} - \sum_j \boxed{\gamma^i_j} m^i \boxed{b_j} - \boxed{\mu^i}\right]^2}{\delta^2_{i,\rm stat}\,\mu^i\left(m^i - \sum_j \gamma^i_j m^i b_j\right) + \left(\delta_{i,\rm uncor}\, m^i\right)^2} + \sum_j b_j^2$$

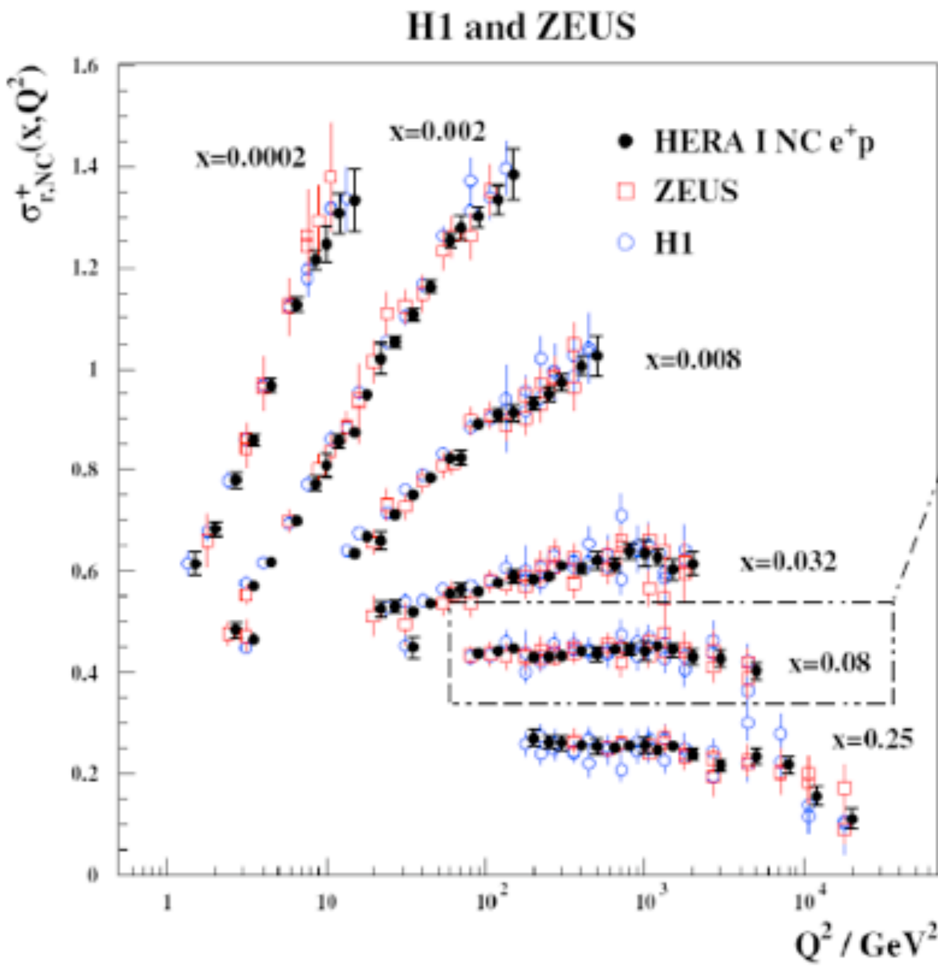
***Sensitivity** of the cross section to the j^{th} source of **correlated** uncertainty.*

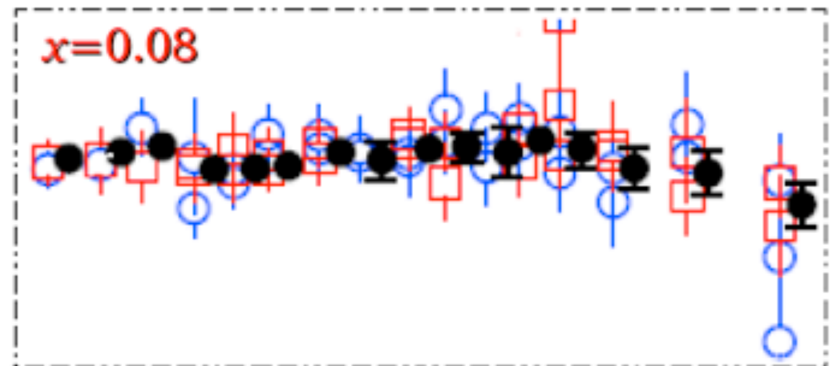
*Shift of the j^{th} source of **correlated** uncertainty*

γ^i_j defined as the relative change of the measurement for a 1 sigma shift of the error source

$\delta_{i,\rm stat} / \delta_{i,\rm uncor}$ *Relative stat. / syst. error on the measurement*

- **1402** measurements with **110** correlated sources of uncertainty combined to **741** cross sections.
- χ^2 / dof = **636.5 / 656** ; No tension in Pulls ; $|b_j| < 2 \Rightarrow$ **H1 and ZEUS Agree!**





Systematic Uncertainty:

- $\delta_{\text{H1 LAR}} \rightarrow 0.45\ \delta_{\text{H1 LAR}}$
- $\delta_{\text{ZEUS BG}} \rightarrow 0.35\ \delta_{\text{ZEUS BG}}$

Overall Precision:

- 2% for $3 < Q^2 < 500$ GeV^2
- 1% for $2 < Q^2 < 100$ GeV^2

Study of the gluon-gluon amplitude (above the saturation region)

One of the major results from HERA is that, at low x, F_2 is dominated by the gluon density. The study of the gluon dynamics is very interesting because of its importance to other physics reactions, like Higgs production at LHC, but also because it is a fundamental quantity, which is comparable to black body radiation in QED.

The dynamics of the gluon distribution at low x is determined by the amplitude for the scattering of a gluon on a gluon, described by the BFKL eq.

$$\frac{\partial}{\partial \ln s}\mathcal{A}(s,\mathbf{k},\mathbf{k}') = \delta(k^2 - k'^2) + \int dq^2 \mathcal{K}(\mathbf{k},\mathbf{q})\mathcal{A}(s,\mathbf{q},\mathbf{k}'),$$

which can be solved in terms of the eigenfunctions of the kernel

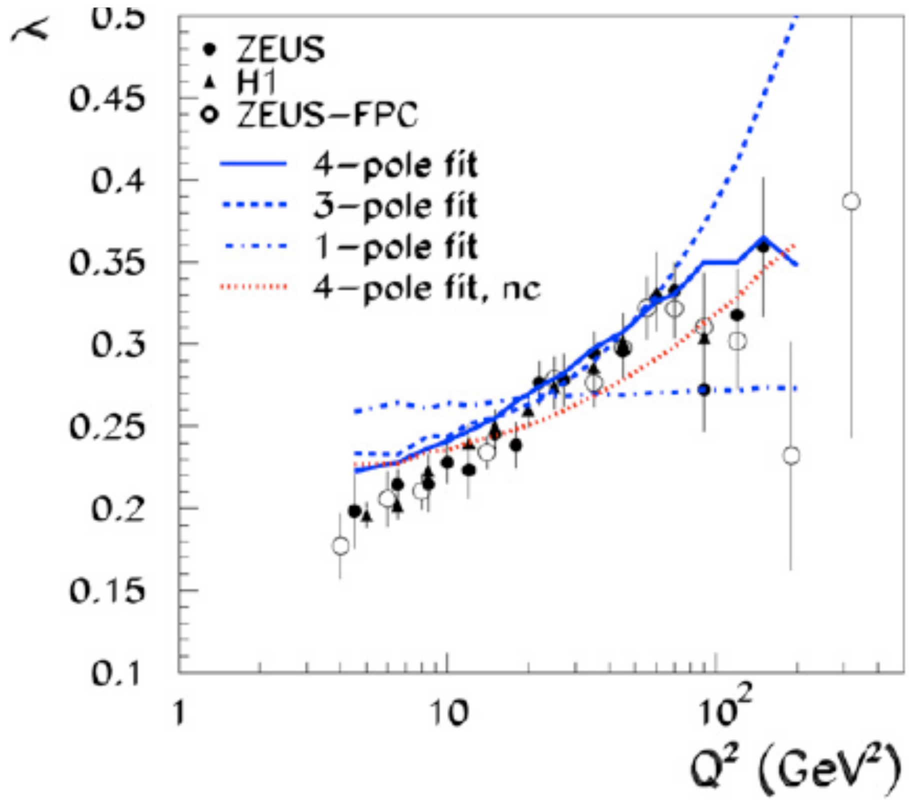
$$\int dk'^2 \mathcal{K}(\mathbf{k},\mathbf{k}') f_\omega(\mathbf{k}') = \omega f_\omega(\mathbf{k}).$$

in LO $f_\omega(\mathbf{k}) = (k^2)^{i\nu - 1/2}$ with $\omega = \alpha_s \chi_0(\nu)$ for fixed α_s

The BFKL eq., with the fixed α_s predicts that the rate of rise λ is only slowly varying with Q^2 , $\lambda \sim 0.3$ (in NLO). Therefore, the prevailing opinion was that the BFKL analysis was not applicable to HERA data.

The rate of rise λ

$F_2 \sim (1/x)^\lambda$



First hints that λ can be substantially varying with Q^2 in BFKL was given in PL 668 (2008) 51 by EKR

Lipatov 86 & EKR 2008: BFKL solutions with the running α_s are substantially different from solutions with the fixed α_s.

in NLO, with running α_s, the BFKL frequency becomes k-dependent, $\nu(k)$ and is determined, in the semiclassical approximation for slowly varying $\nu(k)$,

from
$$\alpha_s(k^2)\chi_0(\nu(\mathrm{k})) + \alpha_s^2(k^2)\chi_1(\nu(\mathrm{k})) \;=\; \omega,$$

ν has to become a function of k because ω cannot depend on k.

For sufficiently large k, the above equation no longer has a real solution for ν. The transition from real to imaginary values of $\nu(k)$ singles out a special value of $k=k_{crit}$, such that $\nu(k_{crit})=0$. The solutions below and above this critical momentum k_{crit} have to match which fixes the phase of ef's.

Near $k=k_{crit}$, the BFKL eq. becomes the Airy eq. which is solved by the Airy eigenfunctions

$$k\, f_\omega(k) = \bar{f}_\omega(k) = \mathrm{Ai}\left(-(\frac{3}{2}\phi_\omega(k))^{\frac{2}{3}}\right)$$

with

$$\phi_\omega(k) = 2\int_k^{k_{\rm crit}} \frac{d\,k'}{k'} |\nu_\omega(k')|$$

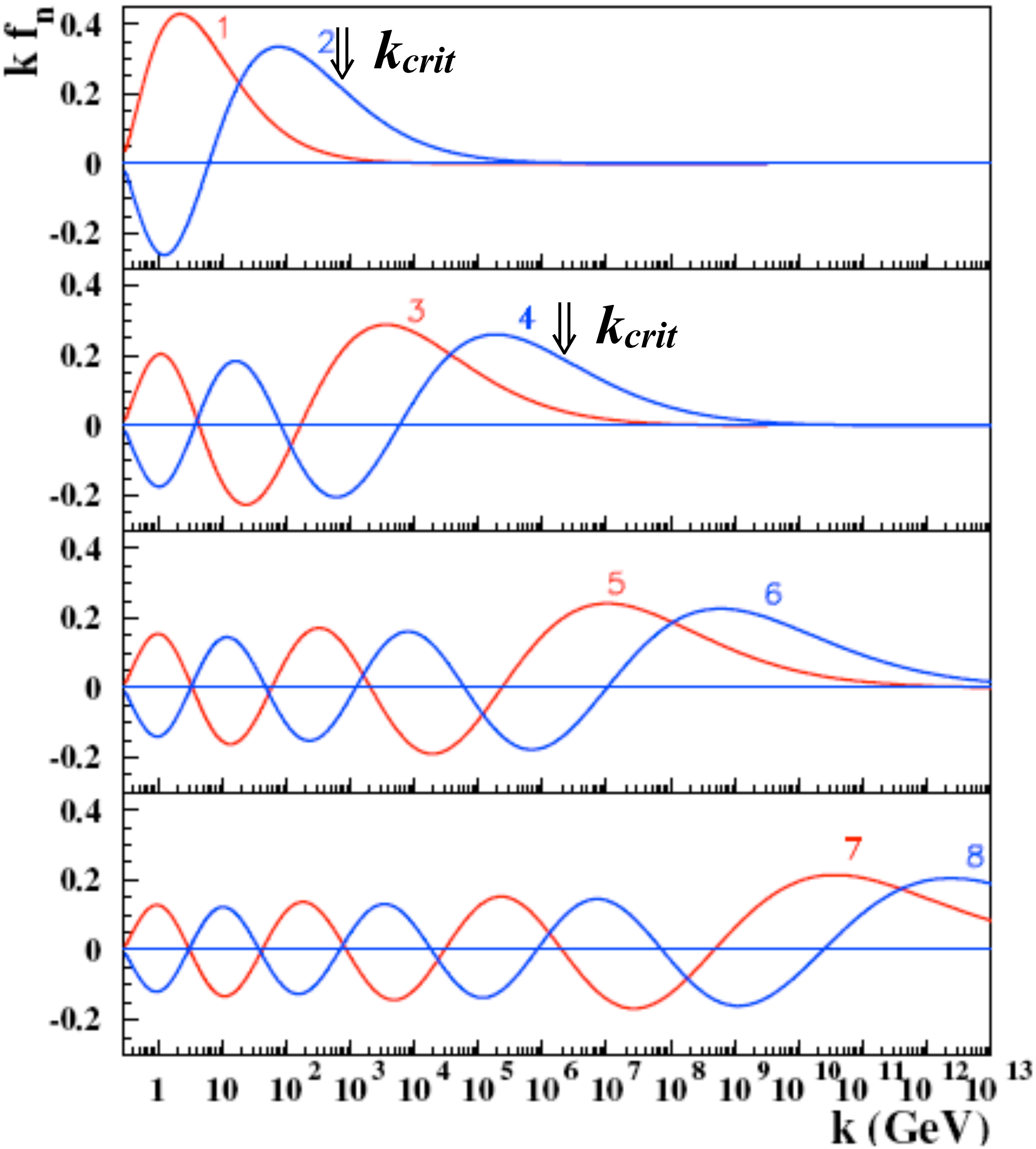
for $k \ll k_{crit}$ the Airy function has the asymptotic behaviour

$$k\, f_\omega(k) \sim \sin\left(\phi_\omega(k) + \frac{\pi}{4}\right)$$

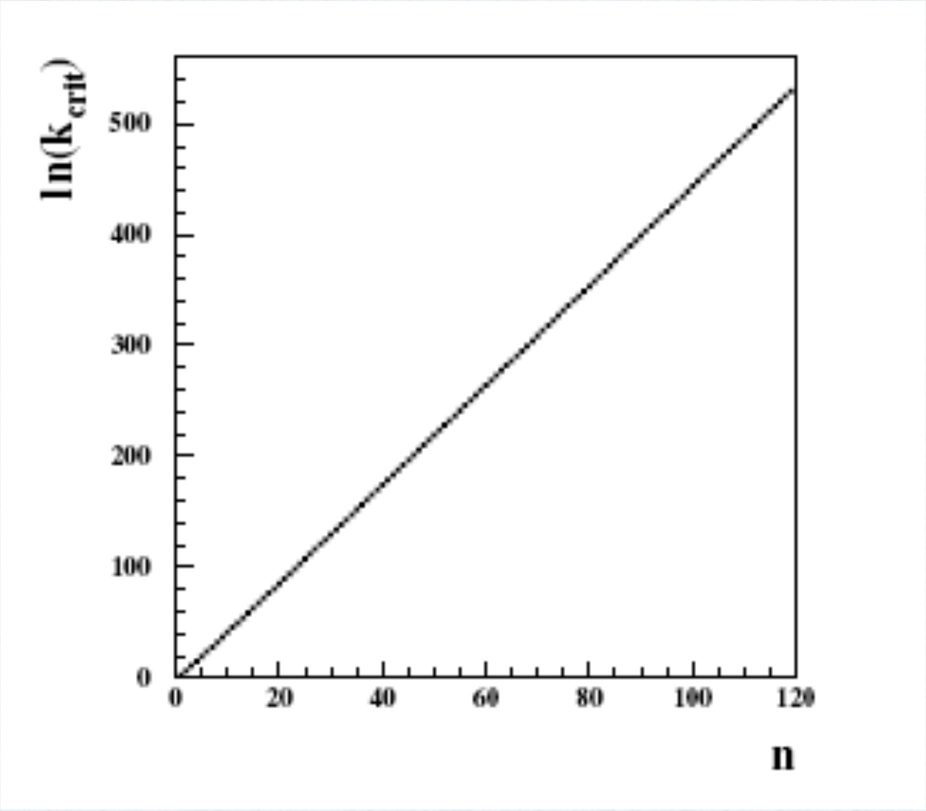
This gives rise to two bounday conditions at $k=k_{crit}$ and at $k=k_0$ and leads to the quantization condition

$$\phi_\omega(k_0) = \left(n - \frac{1}{4}\right)\pi + \eta\,\pi.$$

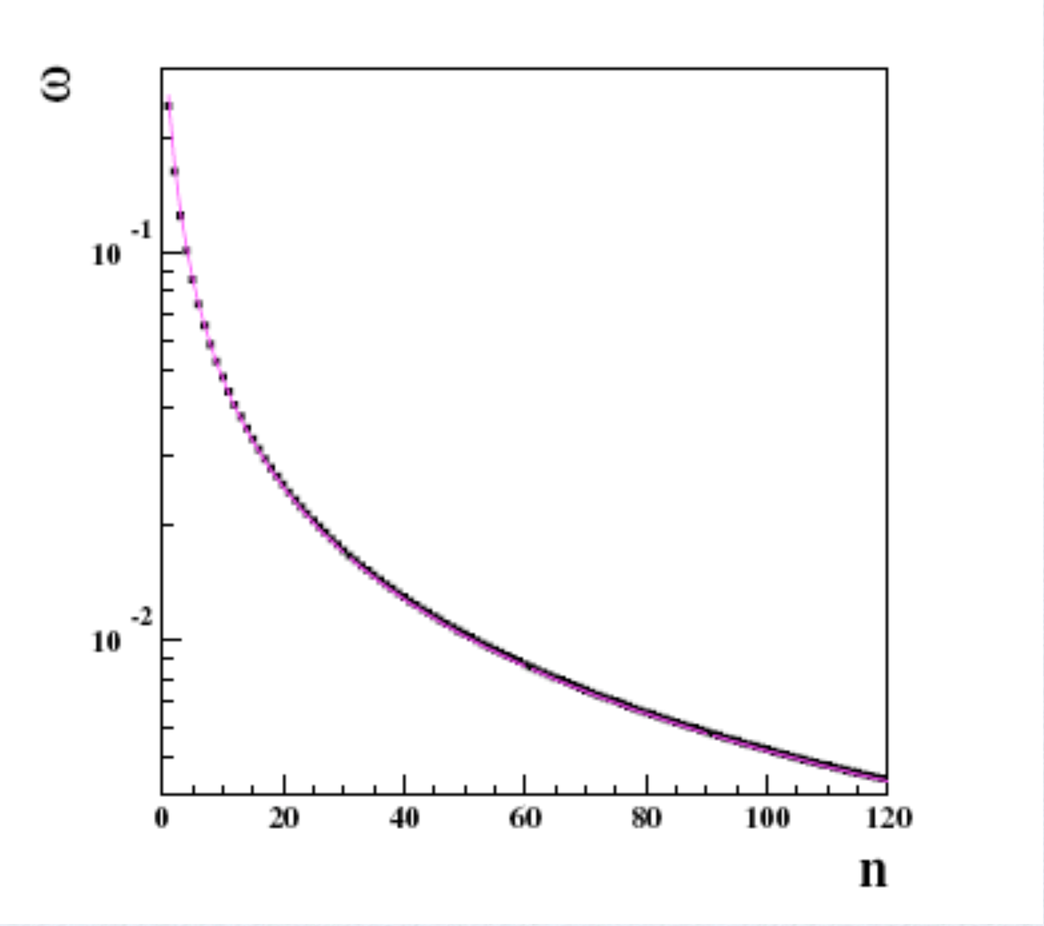
The first eight eigenfunctions determined at η=0



Logarithms of the critical momenta



Eigenvalues ω



$$\omega_n \approx \frac{0.5}{1 + 0.95\, n}.$$

Non-Hermitian kernel

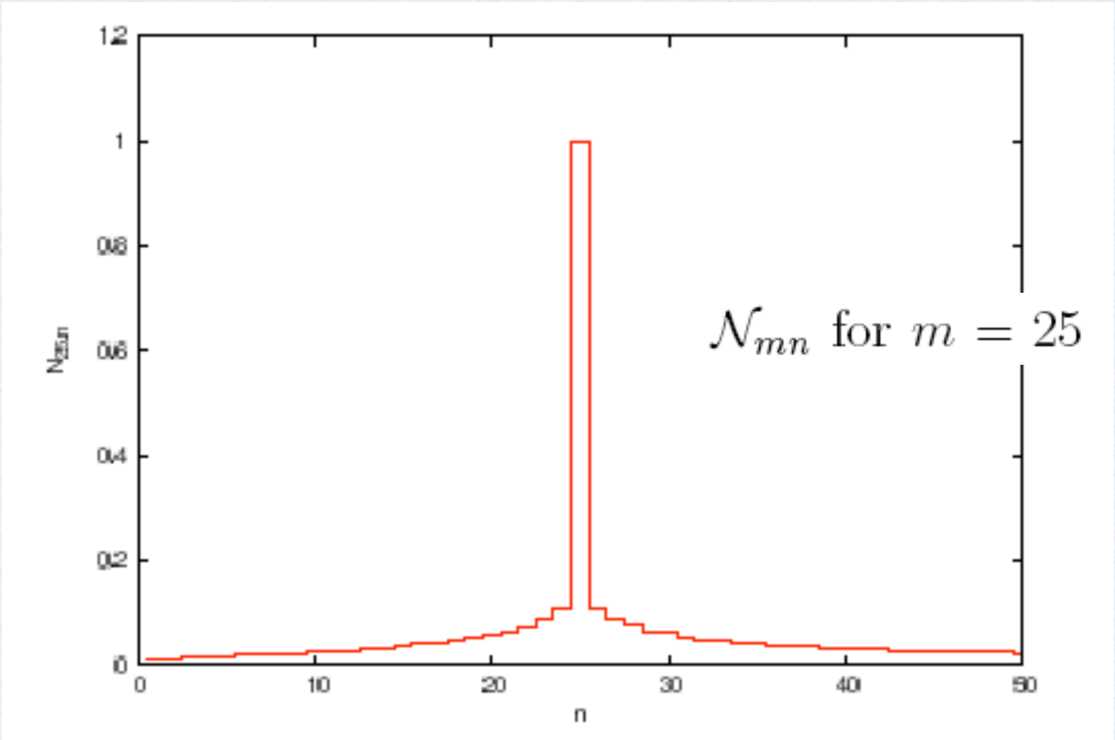
$$\mathcal{K}(\mathbf{k},\mathbf{k}') = \alpha_s \mathcal{K}_0(\mathbf{k},\mathbf{k}') + \alpha_s^2 \mathcal{K}_1(\mathbf{k},\mathbf{k}') + \cdots$$

k **and** ***k'*** **are not entering the kernel in a symmetric way**

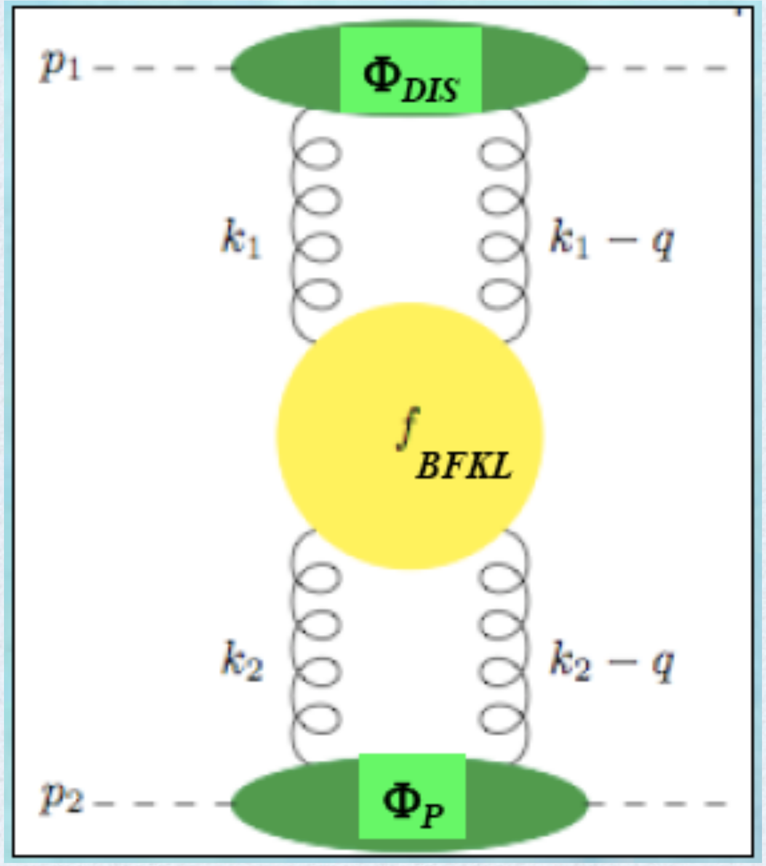
$$\Downarrow$$

$$\int d^2\mathbf{k}\, f_m(\mathbf{k}) f_n^*(\mathbf{k}) = \mathcal{N}_{mn} \neq \delta_{mn},$$

$$\delta^2(\mathbf{k}-\mathbf{k}') = \sum_{n,m} f_n(\mathbf{k}) \mathcal{N}_{nm}^{-1} f_m^*(\mathbf{k}').$$



Comparison with HERA data



Discreet Pomeron Green function

$$\mathcal{A}(\mathbf{k},\mathbf{k}') = \sum_{m,n} f_m(\mathbf{k}) \mathcal{N}^{-1}_{mn} f_n(\mathbf{k}') \left(\frac{s}{kk'}\right)^{\omega_n}.$$

Integrate with the photon and proton impact factors

$$\mathcal{A}_n^{(U)} \equiv \int_x^1 \frac{d\xi}{\xi} \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(Q^2,k,\xi) \left(\frac{\xi k}{x}\right)^{\omega_n} f_n(\mathbf{k}),$$

$$\mathcal{A}_m^{(D)} \equiv \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{1}{k'}\right)^{\omega_m} f_m(\mathbf{k}').$$

$$F_2(x,Q^2) = \sum_{m,n} \mathcal{A}_n^{(U)} \mathcal{N}^{-1}_{nm} \mathcal{A}_m^{(D)}.$$

Proton impact factor

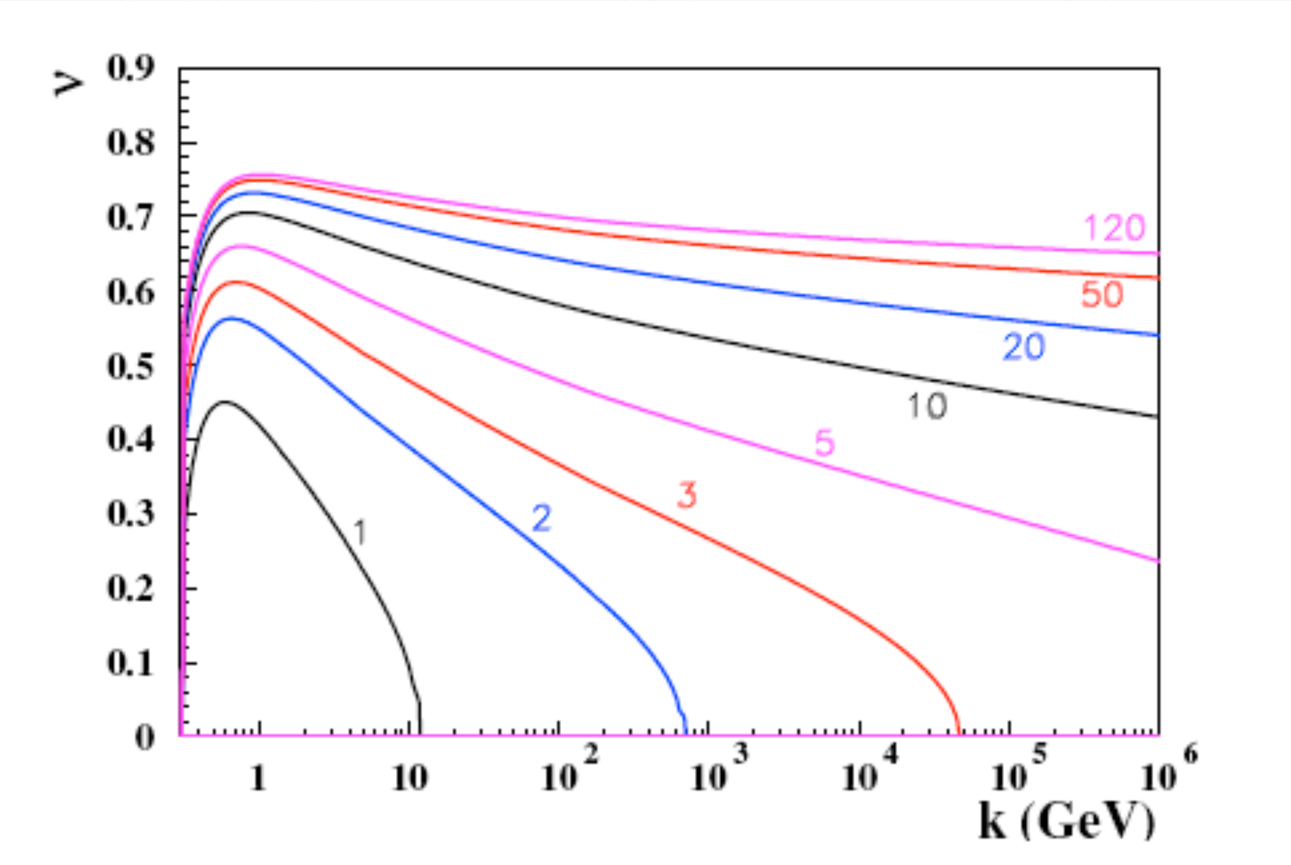
$$\Phi_p(\mathbf{k}) = A k^2 e^{-bk^2},$$

The fit is not sensitive to the particular form of the impact factor as long as it is positive and $k < O(1)$ GeV. The support of the proton impact factor is much smaller than the oscillation period of f_n and because the frequencies ν have a limited range

➤ many eigenfunctions have to contribute and η has to be a function of n

$$\eta = \eta_0 \left(\frac{n-1}{n_{\max}-1} \right)^{\kappa}$$

The frequencies *ν(k)*



The qualities of fits for various numbers of eigenfunctions and overlaps

$n_{\max}$	χ^2/N_{df}	κ	A	b
40	193.3 /125	0.84	2315	23.2
60	163.3 /125	0.78	3647	25.6
80	156.5 /125	0.73	3081	24.4
100	149.1 /125	0.69	2414	22.8
120	143.7 /125	0.66	2041	21.8

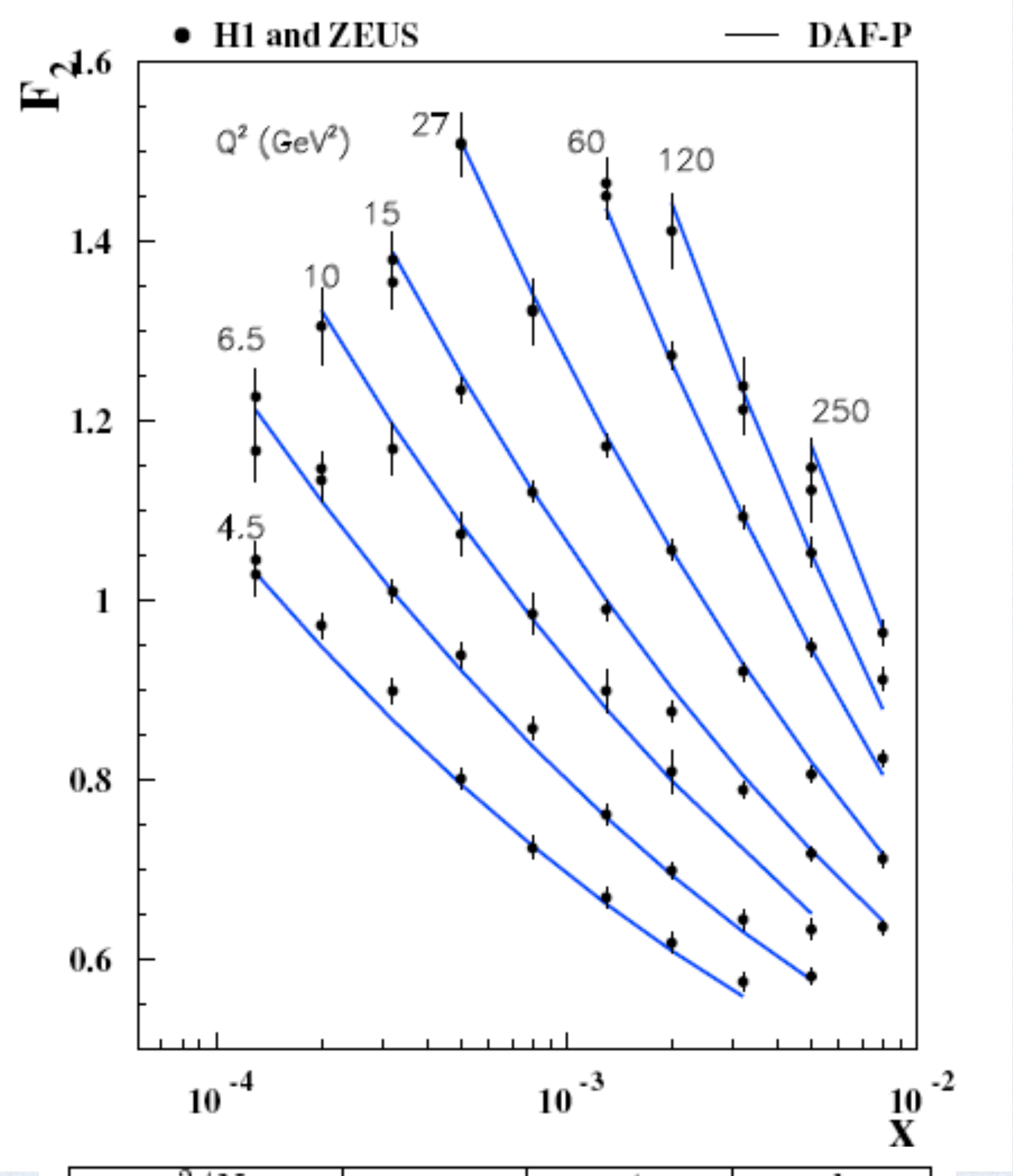
Table 1: The qualities of fits using up to $n_{\max}$ eigenfunctions, and the corresponding parameters of the fits, with $\eta_0 = -0.9$ and 4 flavours in the photon impact factor. The parameters A and b are both given in units of GeV^{-2}.

n_{overl}	χ^2/N_{df}	κ	A	b
0	354.6 /125	0.41	7.80	1.40
10	206.9 /125	0.50	69.1	5.83
20	150.8 /125	0.60	444.4	13.5
30	143.7 /125	0.66	2041	21.8

Note that the differences in the fit qualities would be negligible if the errors where more than 2-times larger

➤ new data are crucial for finding the right solution

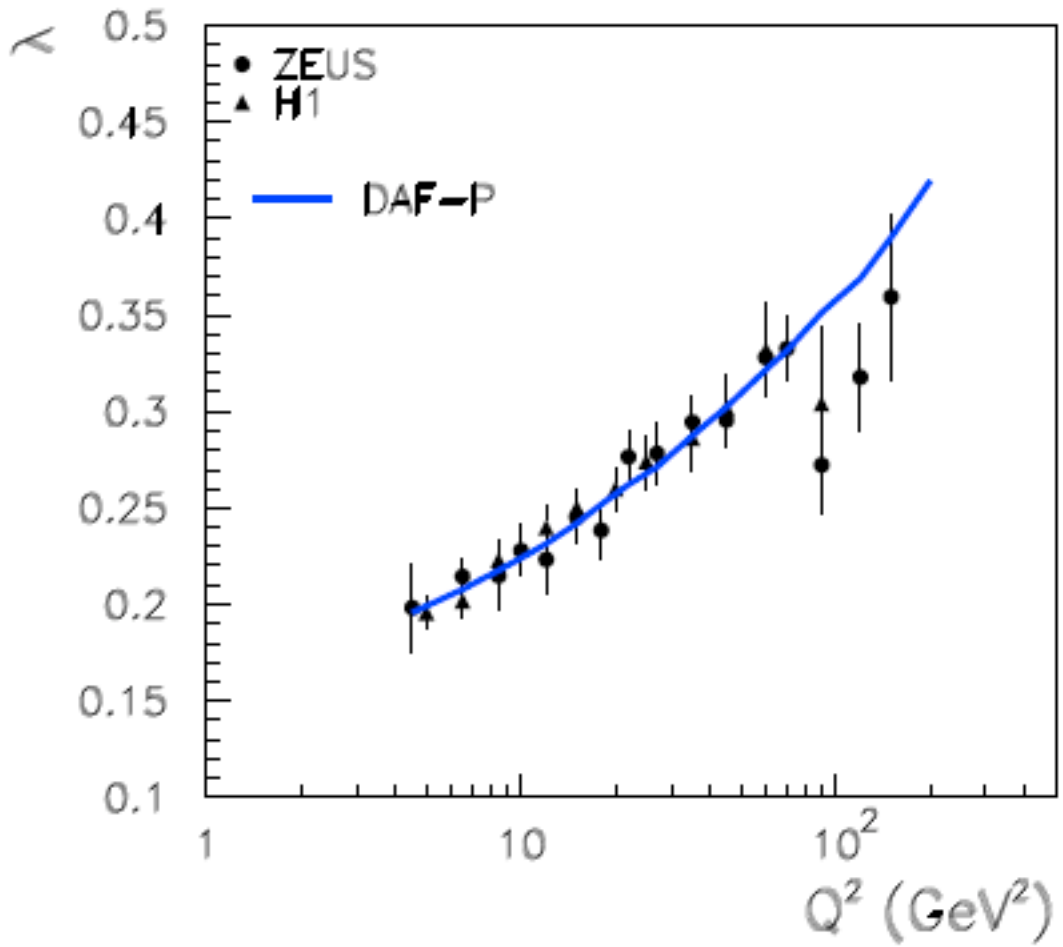
The final fit performed with 120 ef's and 30 overlaps and 5 flavours



χ^2/N_{df}	κ	A	b
154.7 /125	0.65	1660	20.6

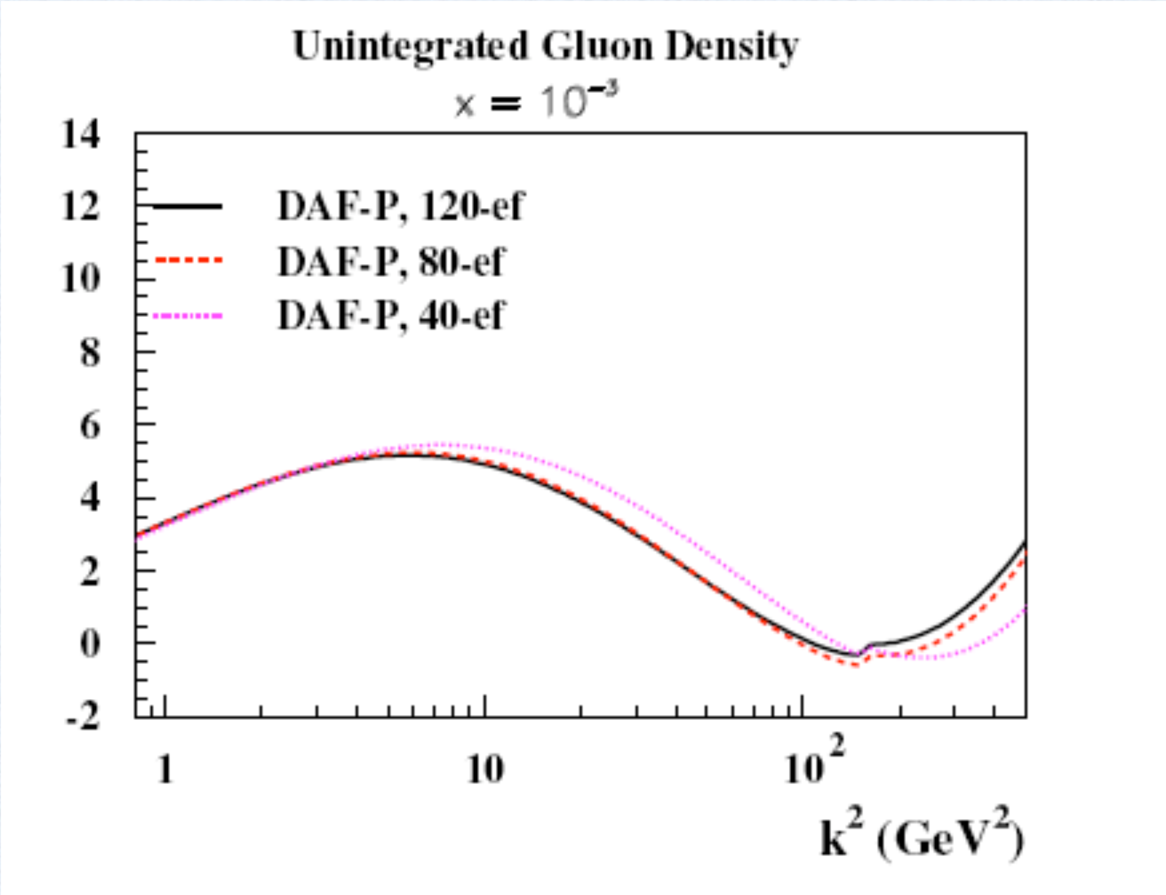
The rate of rise λ

$F_2 \sim (1/x)^\lambda$



The first successful pure BFKL desciption of the λ plot.

For many years it was claimed that BFKL analysis was not applicable to HERA data because of the observed substantial variation of λ with Q^2



Unintegrated Gluon Density
x = 10^-3
DAF-P, 120-ef
DAF-P, 80-ef
DAF-P, 40-ef
14
12
10
8
6
4
2
0
-2
1
10
10^2
k^2 (GeV^2)

η - ω relation

$$\phi_\omega(k) = 2\int_k^{k_{\rm crit}} \frac{dk'}{k'}|\nu_\omega(k')| \;=\; -2\nu_\omega(k)\ln(k) + 2\int_0^{\nu_\omega(k)} \nu_\omega^{-1}(\nu')d\nu',$$

In LO $$\int_0^{\nu_0} \nu_\omega^{-1}(\nu')d\nu' = \frac{4\pi}{\beta_0\omega}a - \frac{\pi}{4}$$ with $\beta_0 = 11 - 2n_f/3$ and $a = \int_0^{\nu_0}\chi_0(\nu')d\nu' \approx 0.92.$

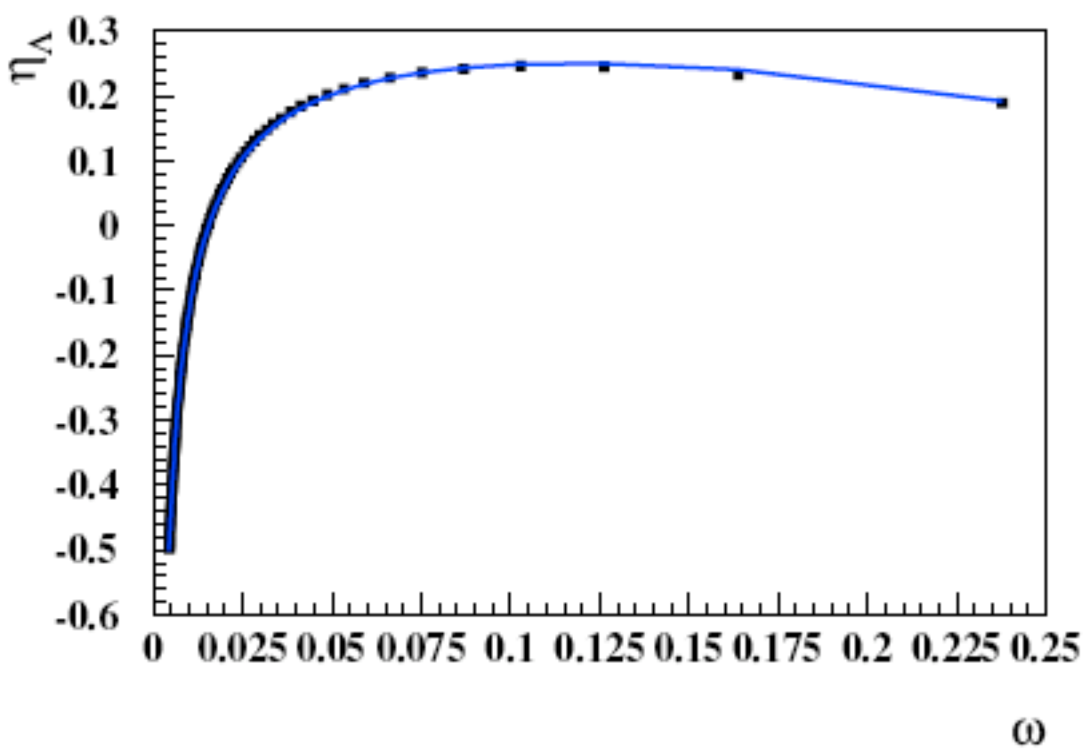
This led to the original assumption of a constant phase (Lipatov 86), up to $n\pi$, and gave the relation

$$\omega_n = \frac{4a}{\beta_0}\,\frac{1}{n+\eta+1/4}.$$

In NLO

$$\eta = -2\nu(k_0)\ln\left(\frac{k_0}{\Lambda_{\rm QCD}}\right) + \eta_\Lambda,$$

η_Λ - ω relation



$$\eta = \eta_0 \left(\frac{n-1}{n_{\max}-1} \right)^{\kappa}$$

$$\eta_\Lambda = 0.4 - 0.14\omega - 1.9\omega^2 - \frac{0.0265}{\omega^{0.65}}.$$

physical interpretation of the η_Λ - ω relation

$$\eta_\Lambda = 0.4 - 0.14\omega - 1.9\omega^2 - \frac{0.0265}{\omega^{0.65}}.$$

The polynomial term contains information about the non-perturbative gluonic dynamics inside the pomeron because the BFKL equation can be considered to be analogous to the Schroedinger equation of the interacting two gluon system. This analogy suggests that the perturbative wave functions can be smoothly extended to very low virtualities, i.e. into the non-perturbative region.

➤ Universal Pomeron?

Can be checked experimentally at EIC by determining the η_Λ - ω relation eg in diffractive reaction on the nuclei.

physical interpretation of the η_Λ - ω relation

$$\eta_\Lambda = 0.4 - 0.14\omega - 1.9\omega^2 - \frac{0.0265}{\omega^{0.65}}.$$

The singular term is generated by the perturbative effects which were not fully taken into account in our evaluation. This term is sensitive to the high k behaviour of the glue-glue amplitude, much beyond the virtualities which are actually tested in the experiment. This is due to the fact that the values of k_{crit} are growing quickly with the increase of the eigenfunction number and that HERA data indicate clearly that a lot of ef's give significant contributions. The high k region is not suppressed in the phase so that already the phase of the 10th ef is due to the integration over the k region which is above the thresholds of BSM particles.

$$\phi_\omega(k) = 2\int_k^{k_{\rm crit}} \frac{d\,k'}{k'} |\nu_\omega(k')|$$

Back up slides

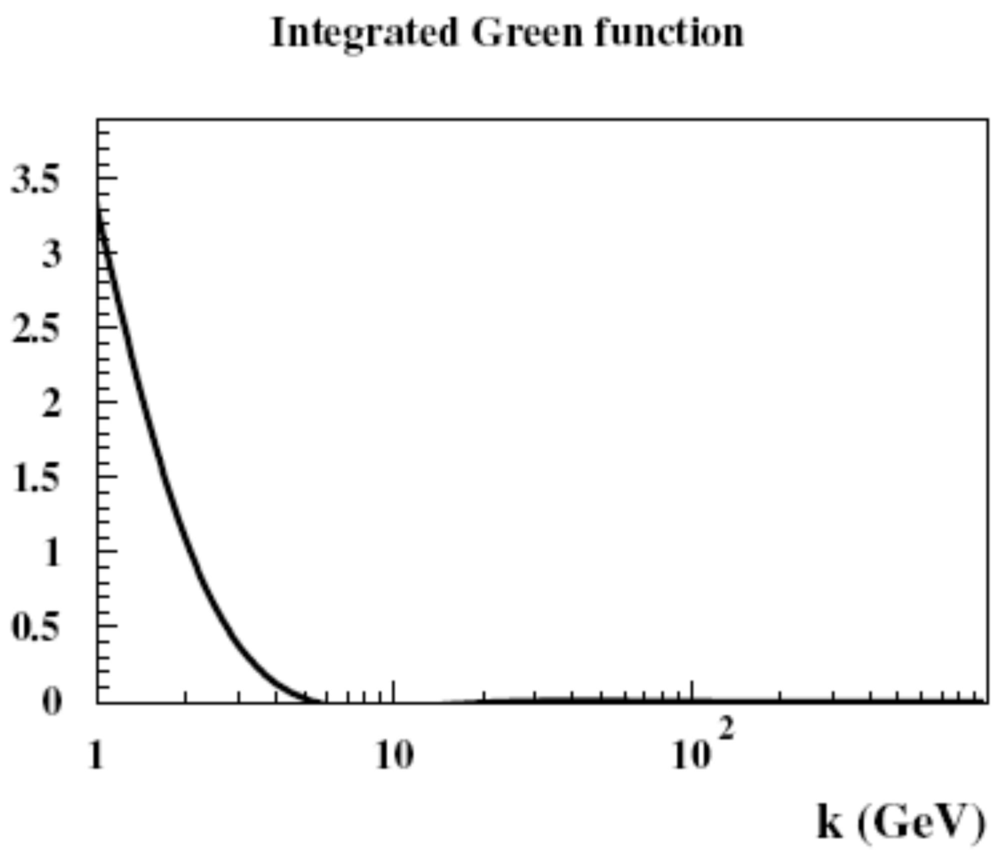


Figure 6: Distribution of the momentum k in the Green function, $G_y(\mathrm{k}, \mathrm{k}')$, integrated over k' with the proton impact factor at $y = \ln(s/k^2) = \ln(1/x = 10^3)$.

Using HERA Data to Determine the Infrared Behaviour of the BFKL Amplitude

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