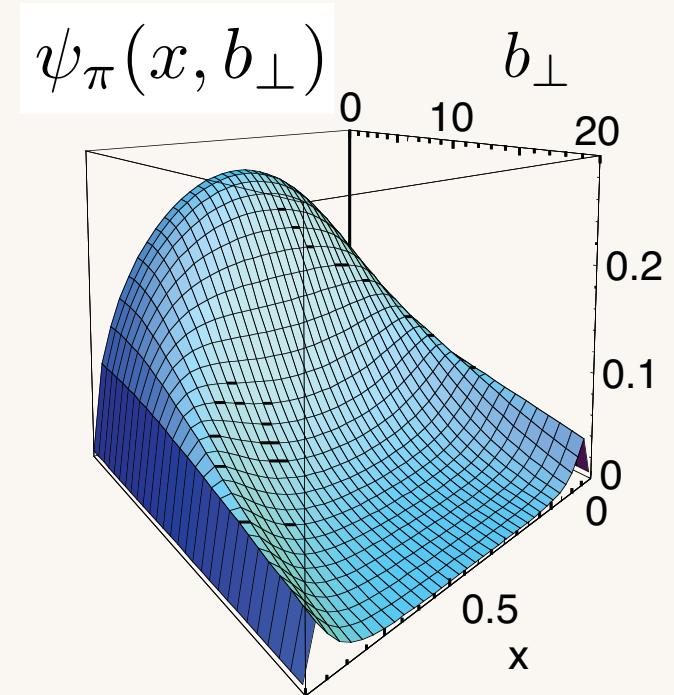
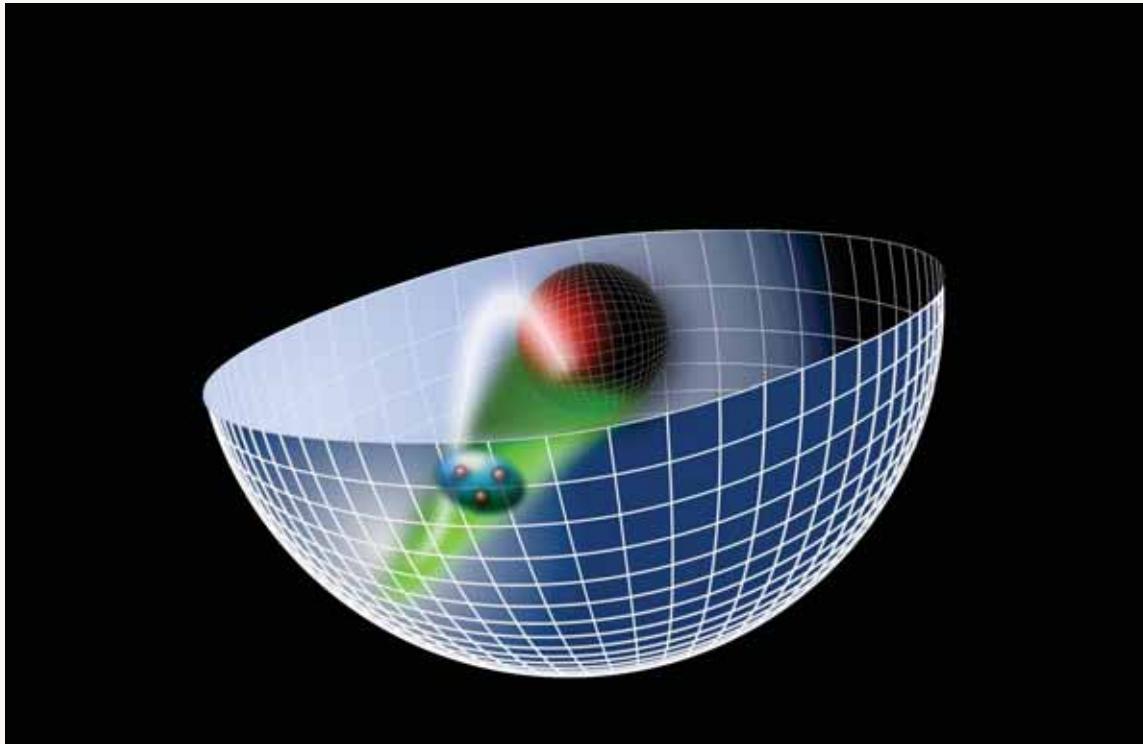


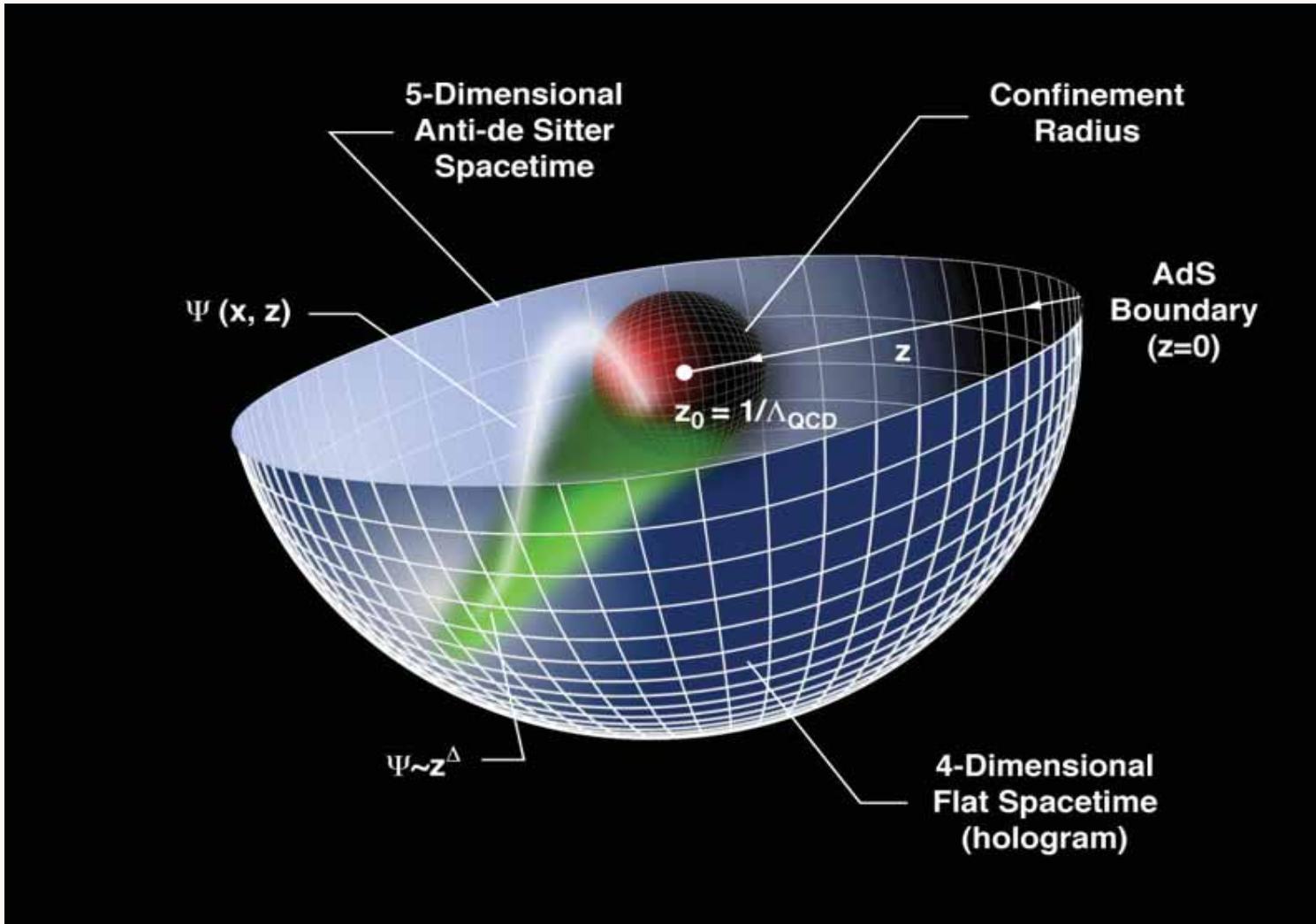
Hadronic Phenomenology from AdS/CFT for Experimentalists



BNL Physics Seminar
March 7, 2008

Stan Brodsky
SLAC, Stanford University

Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

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AdS/QCD
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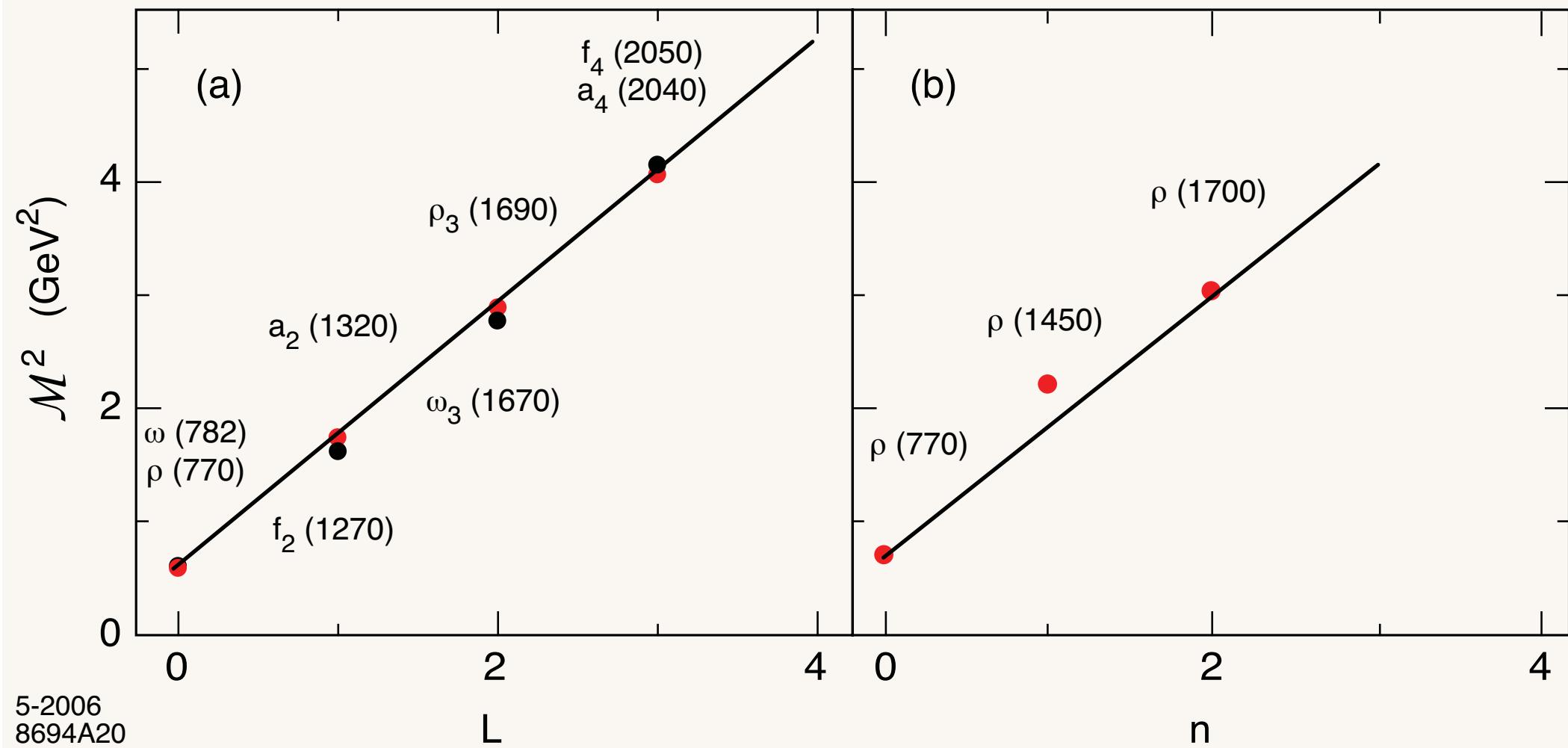
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Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Coulomb Equation for Atomic Physics**
- *AdS/QCD Holographic Model*

$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

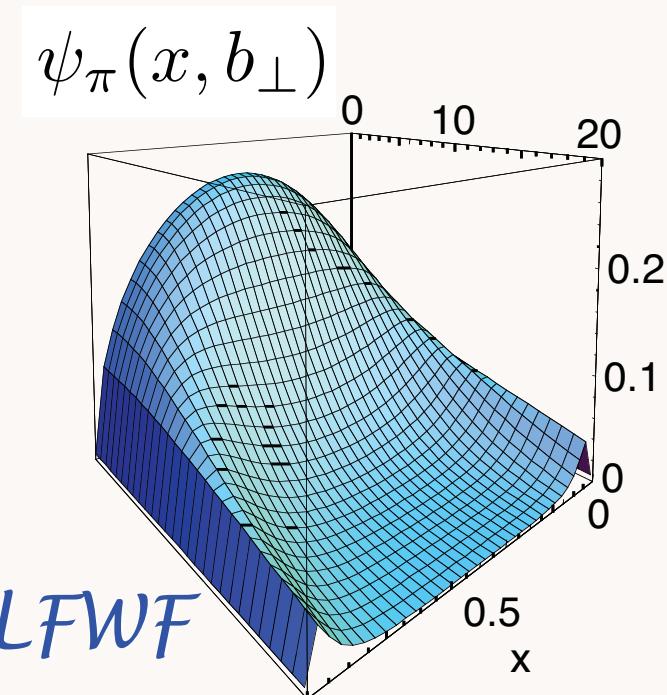
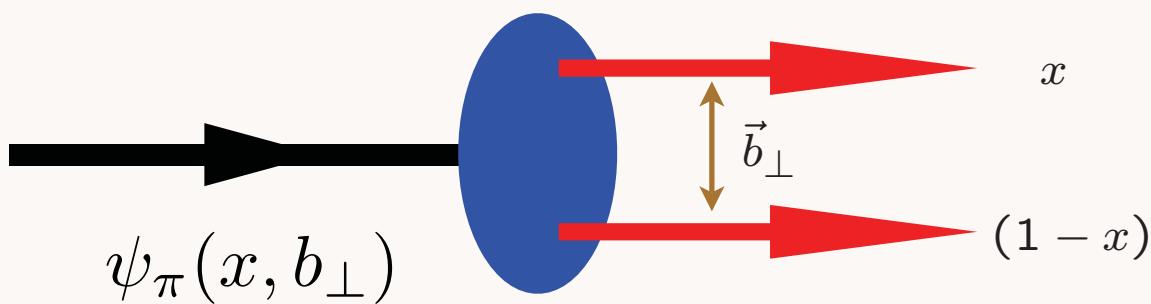
$$S = 1$$



Relativistic Light Front Equation Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta) \quad S = 0$$

$$V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 \quad \zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Prediction from AdS/CFT: Meson LFWF

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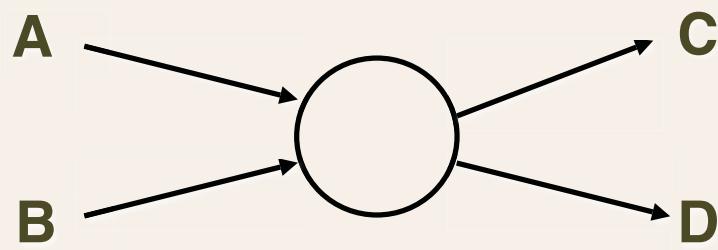
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal;** however, it has manifestations of a scale-invariant theory:
Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const}$ at small Q^2
- **Use mathematical mapping of the conformal group $SO(4,2)$ to AdS₅ space**

Constituent Counting Rules



$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos \theta_{cm}$

$$\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s^{[n_{tot}-2]}} \quad s = E_{cm}^2$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H-1}$$

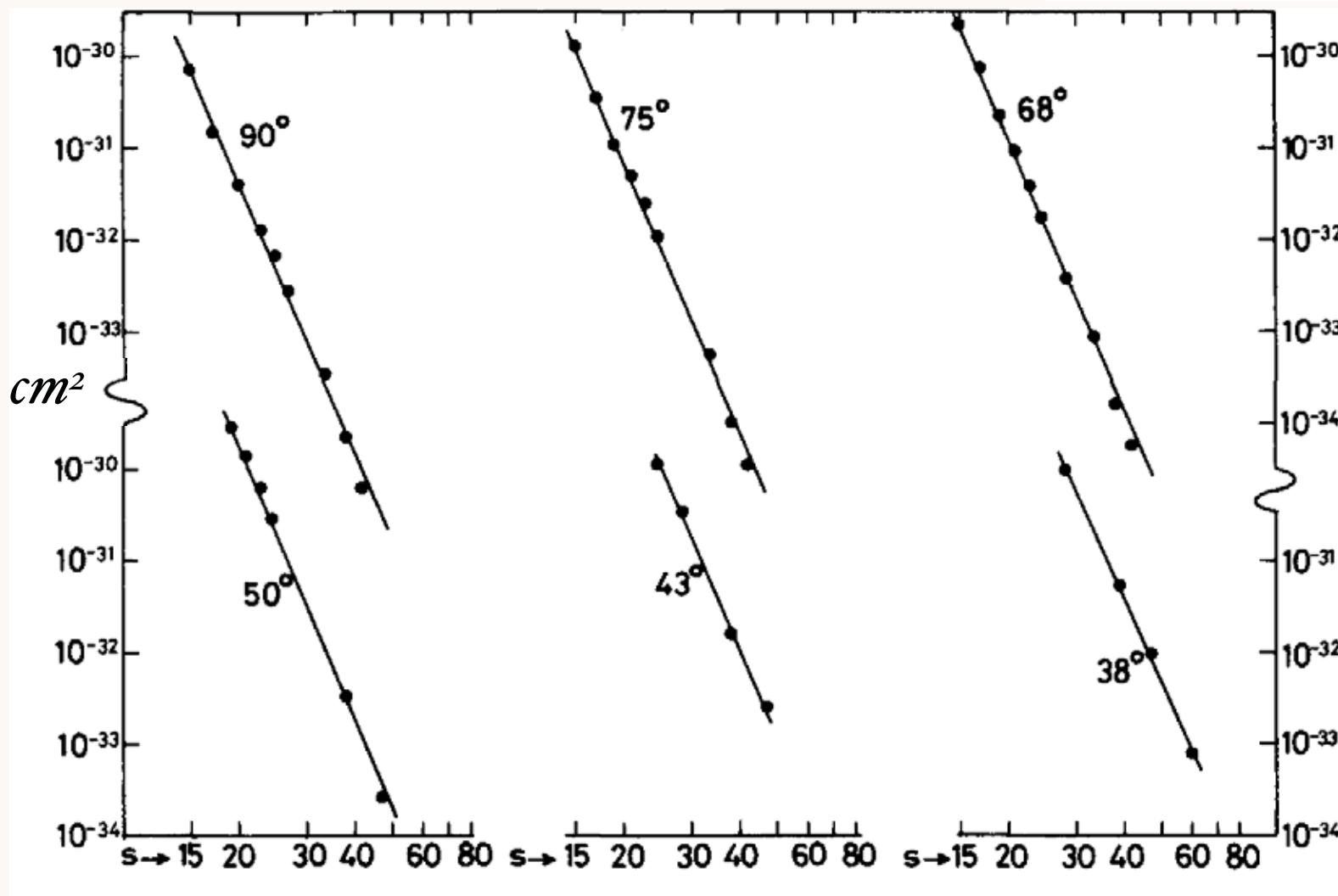
Farrar & sjb; Matveev, Muradyan,
Tavkhelidze

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Quark-Counting: $\frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}}$

$$n = 4 \times 3 - 2 = 10$$

P.V. LANDSHOFF and J.C. POLKINGHORNE



Angular distribution \sim quark interchange

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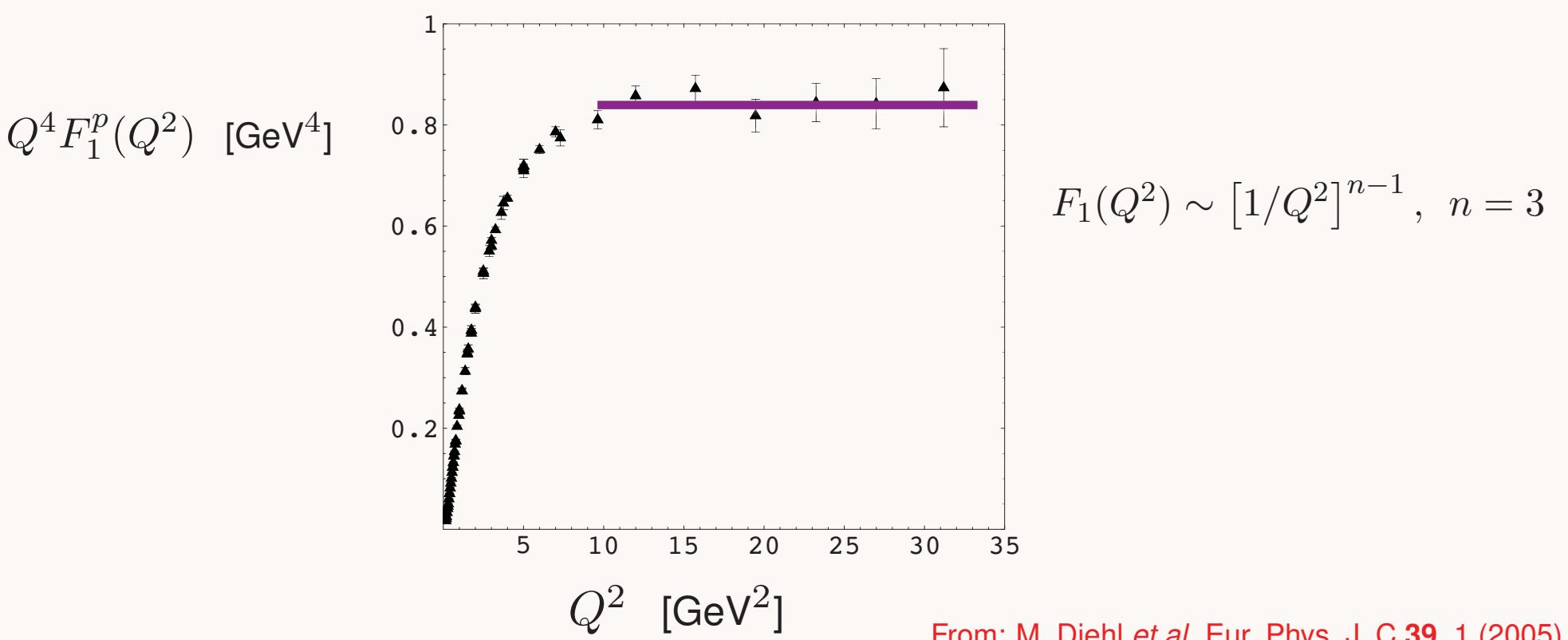
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Best Fit

$$n = 9.7 \pm 0.5$$

Reflects
underlying
conformal
scale-free
interactions



From: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

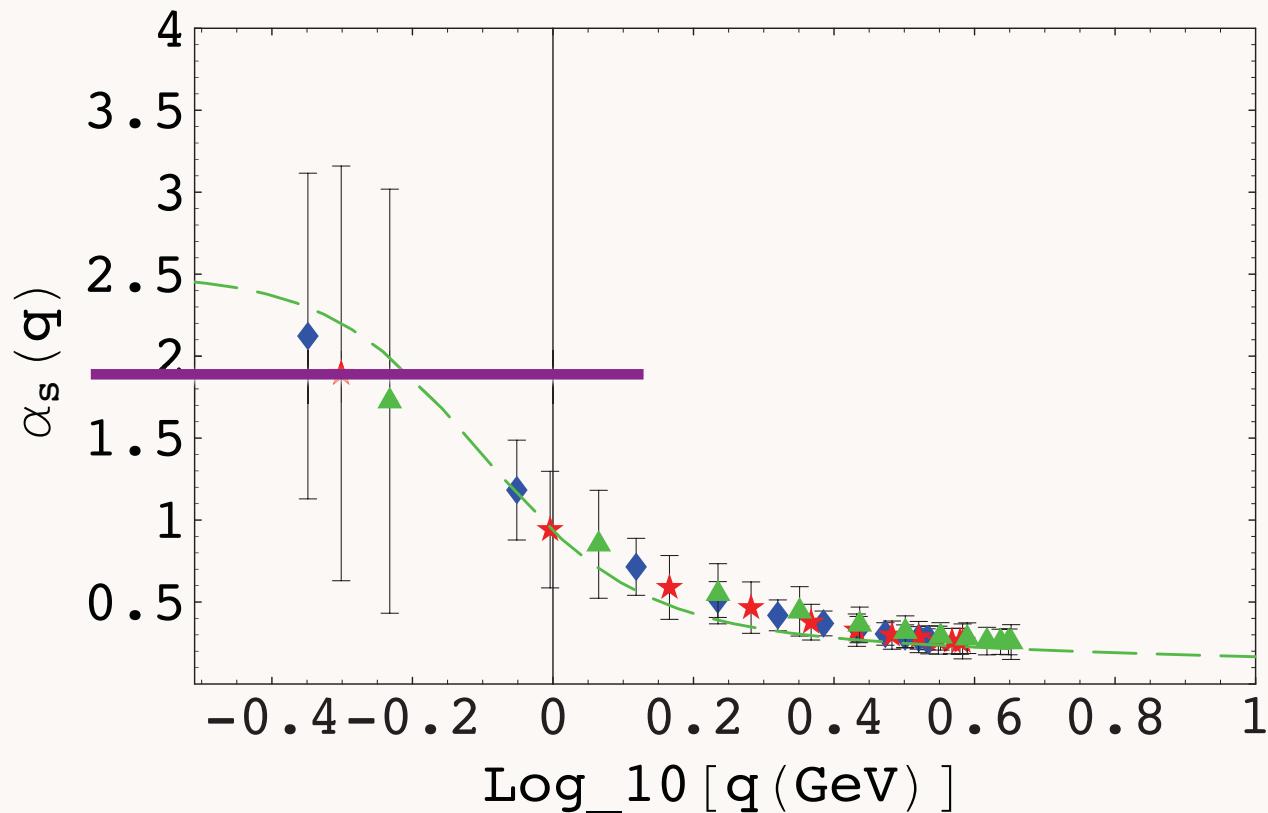
implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

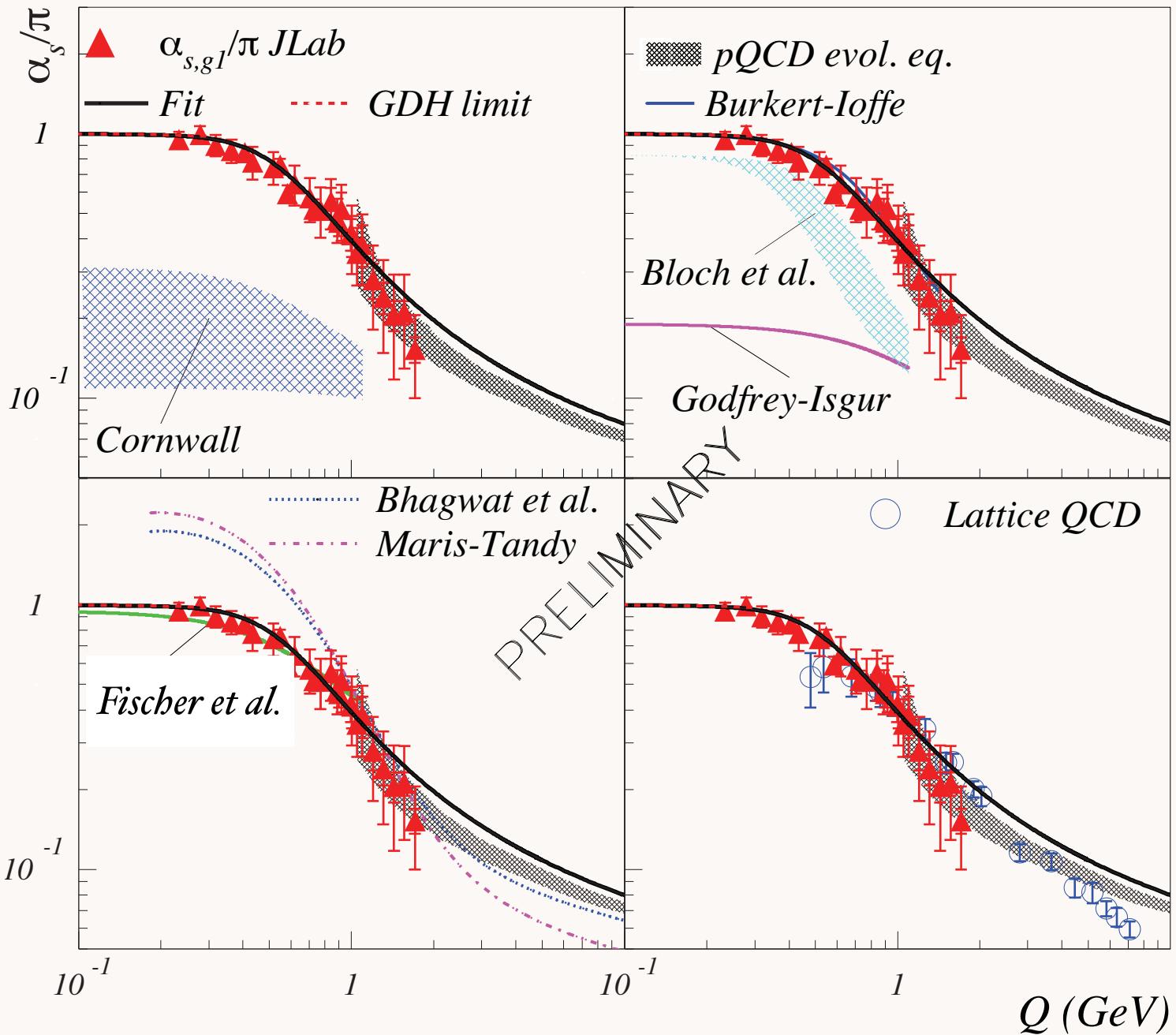
Farrar and sjb (1973); Matveev *et al.* (1973).

- Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).

Conformal QCD Window in Exclusive Processes

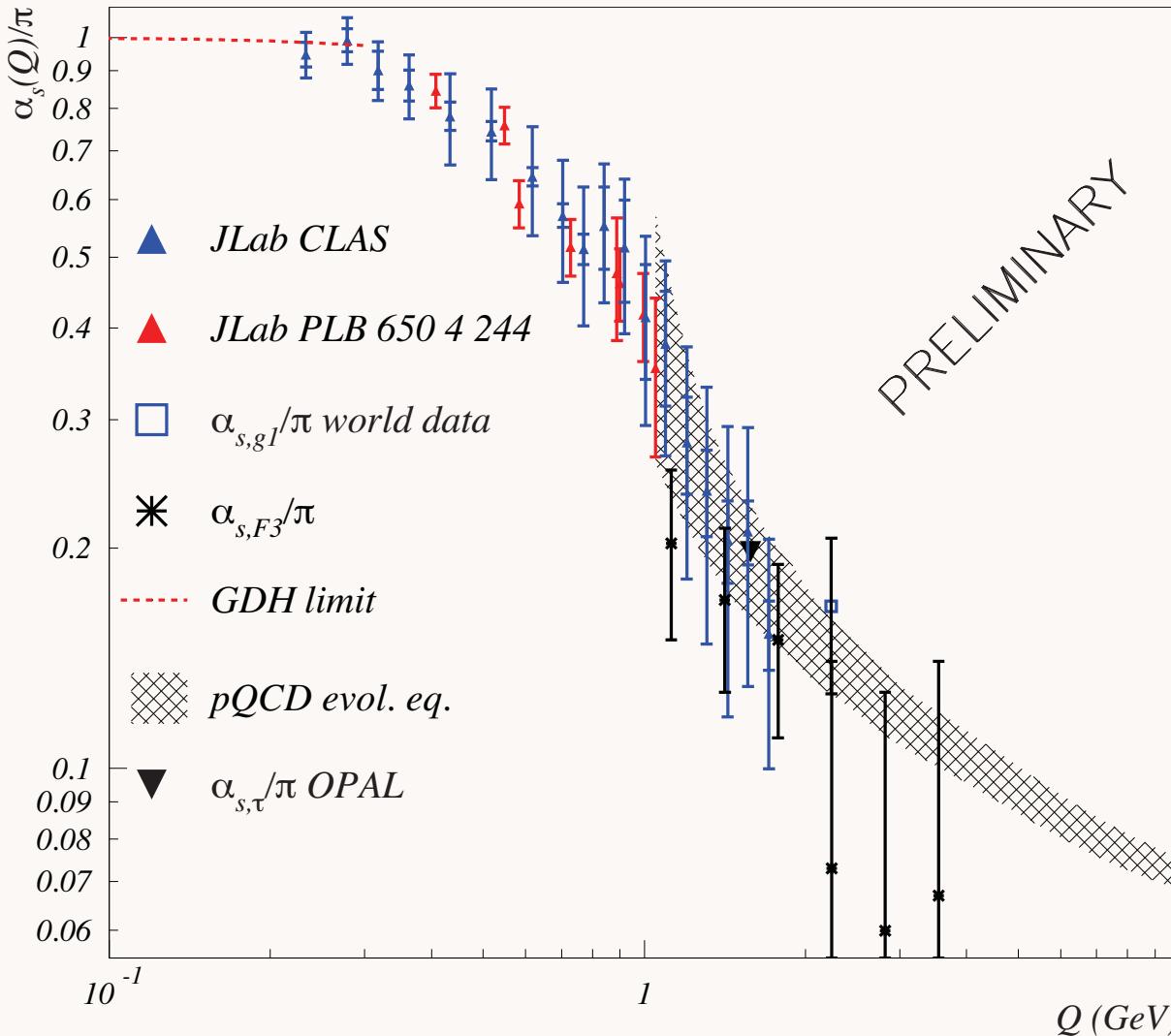
- Does α_s develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur ...
- Recent lattice simulations: evidence that α_s becomes constant and is not small in the infrared Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).





Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[1 - \frac{\alpha_s^{g_1}(Q^2)}{\pi} \right]$$



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AdS/QCD
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New Perspectives on QCD Phenomena from AdS/CFT

- **AdS/CFT:** Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions
- AdS/CFT provides novel insights into the quark structure of hadrons

Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function, confinement, and quark masses
- Eigensolutions of ERBL evolution equation for distribution amplitudes V. Braun et al;
Frishman, Lepage, Sachrajda, sjb
- Commensurate scale relations: relate observables at corresponding scales:
Generalized Crewther Relation
- Fix Renormalization Scale (BLM)
- Use AdS/CFT

Conformal Theories are invariant under the Poincare and conformal transformations with

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$

the generators of $SO(4,2)$

$SO(4,2)$ has a mathematical representation on AdS_5

Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad \text{invariant measure}$$

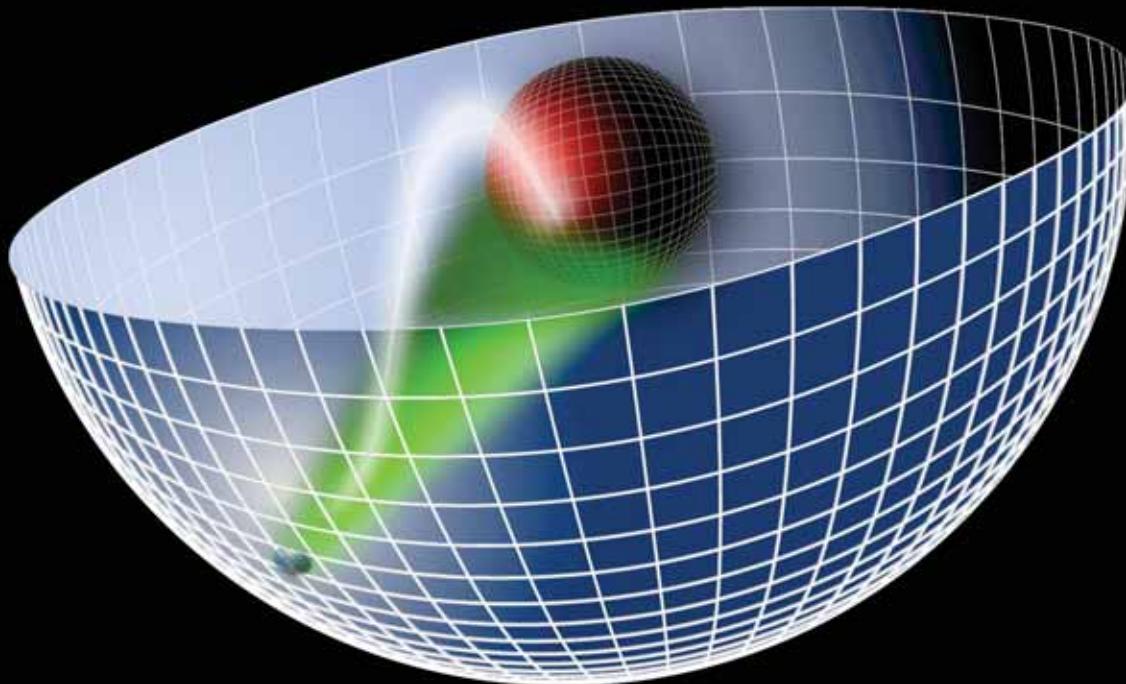
$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

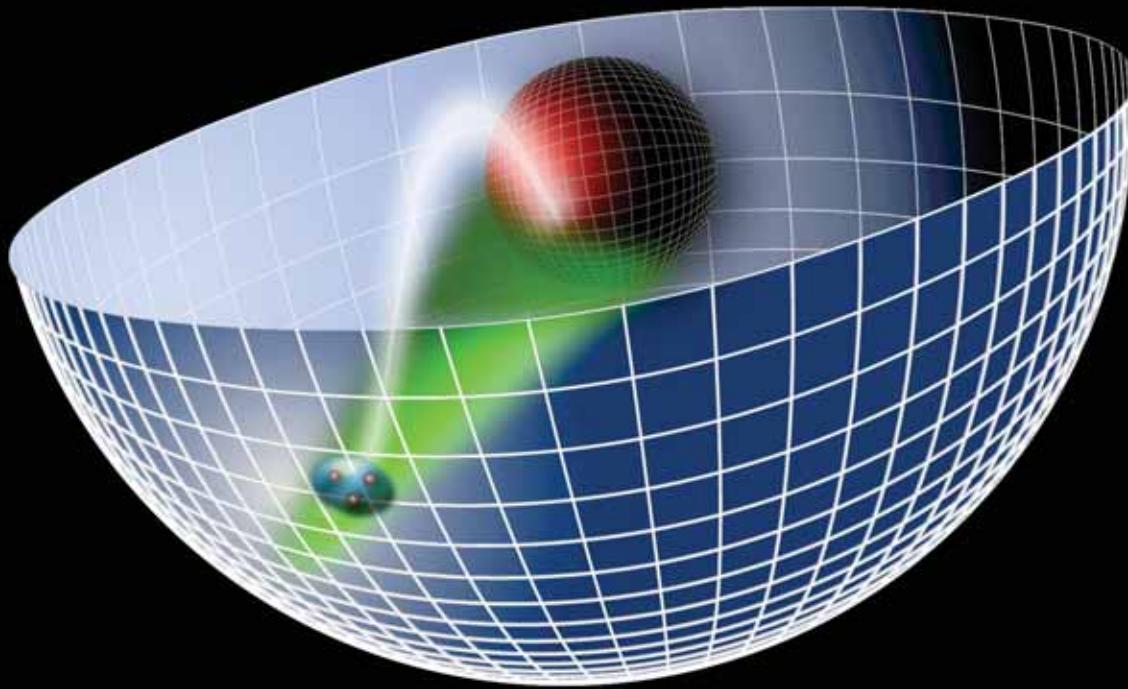
- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.



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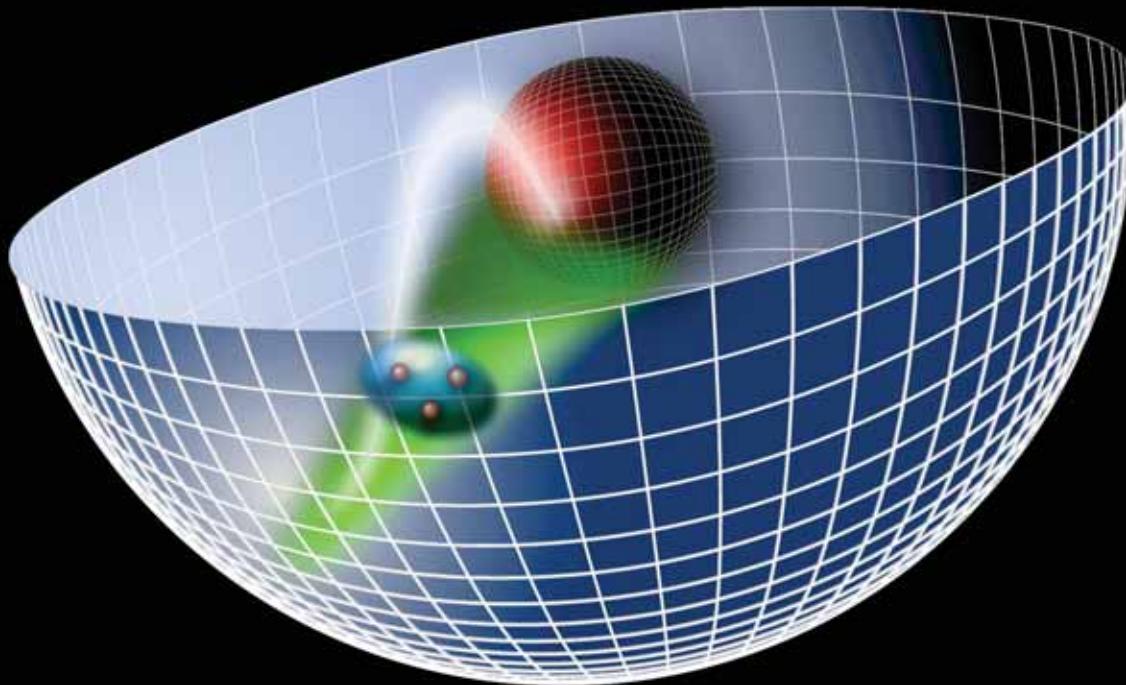
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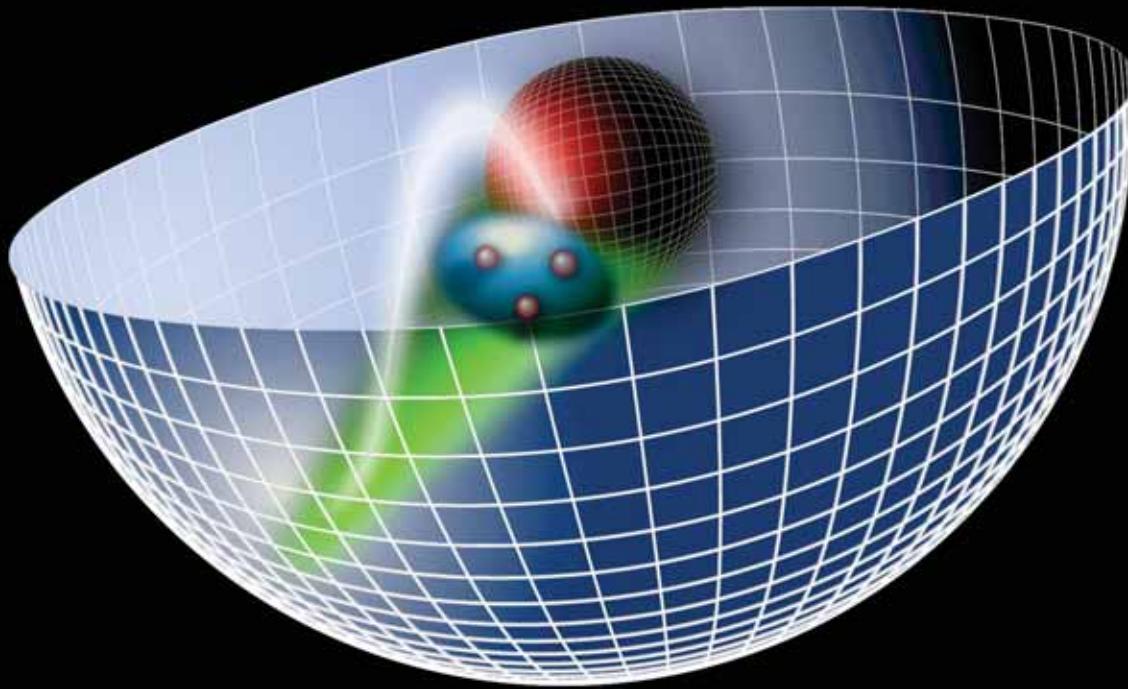
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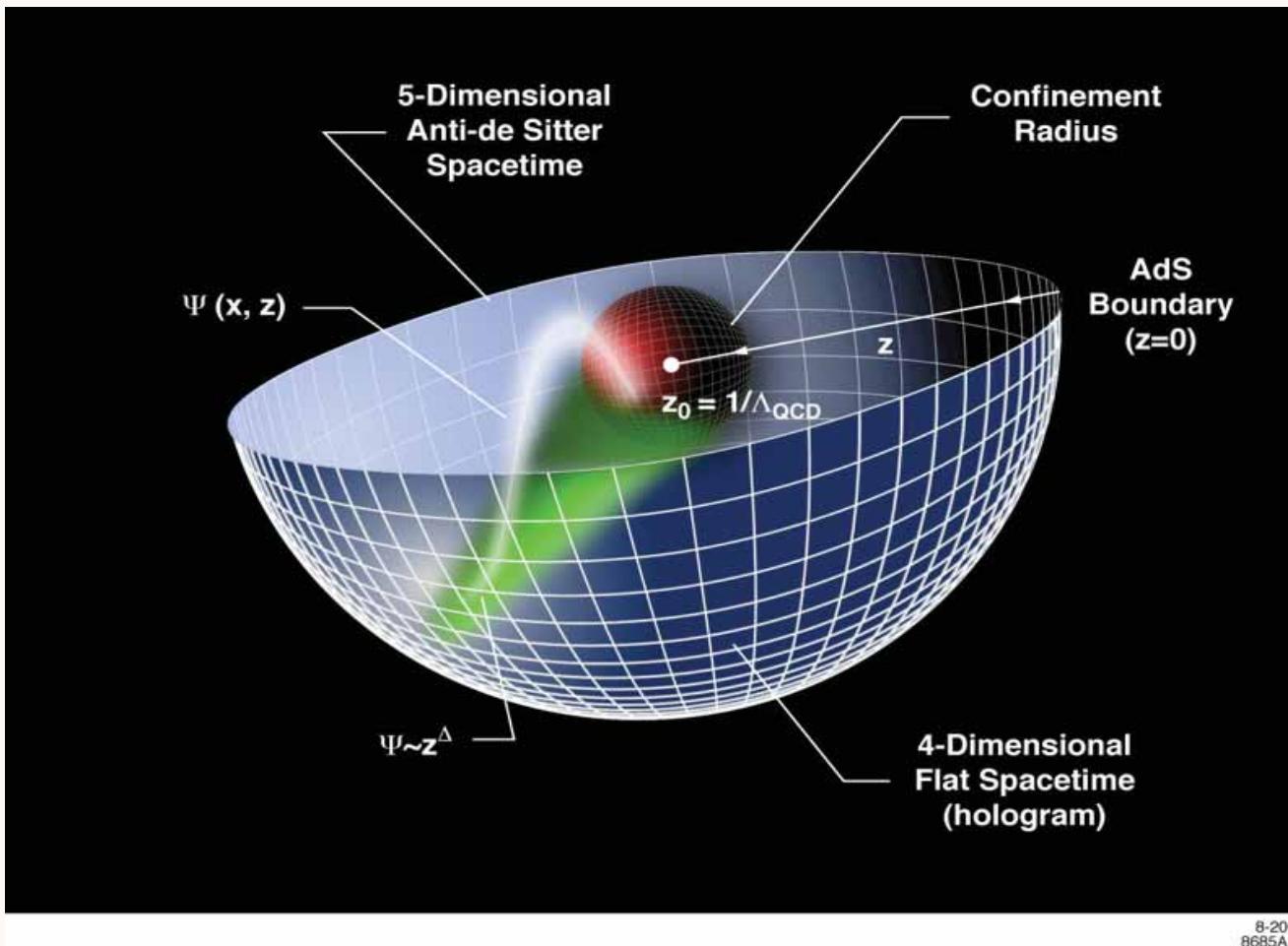
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- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) [Polchinski and Strassler \(2001\)](#).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) [Karch, Katz, Son and Stephanov \(2006\)](#).

We will consider both holographic models

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- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation
- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances
- de Teramond, sjb: AdS/QCD Holographic Model: Initial “semi-classical” approximation to QCD. Predict light-quark hadron spectroscopy, form factors.
- Karch, Katz, Son, Stephanov: Linear Confinement
- Mapping of AdS amplitudes to $3+1$ Light-Front equations, wavefunctions
- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H_{\text{LF}}^{\text{QCD}}$; variational methods

AdS/CFT

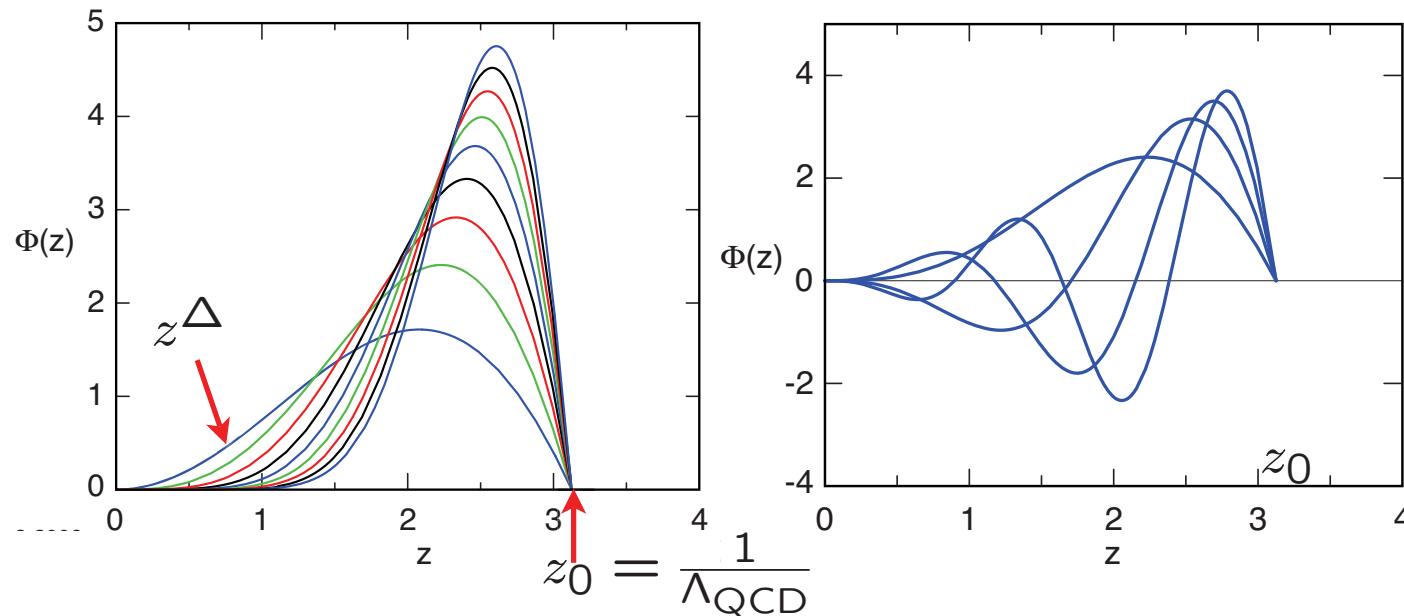
- Use mapping of conformal group $\text{SO}(4,2)$ to AdS_5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \rightarrow \lambda^2 x_\mu^2$ $z \rightarrow \lambda z$
- Hard wall model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$
- Truncated space simulates “bag” boundary conditions

$$\psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$

Match fall-off at small z to conformal twist-dimension at short distances

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.
- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates , $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

*AdS Schrodinger Equation for bound state
of two scalar constituents:*

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

$$V(z) = -\frac{1-4L^2}{4z^2}$$

**Interpret L
as orbital angular
momentum**

Derived from variation of Action in AdS₅

Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

*AdS Schrodinger Equation for bound state
of two scalar constituents:*

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Hard wall model: truncated space

$$V(z) = -\frac{1-4L^2}{4z^2} \quad \phi(z = z_0 = 1/\Lambda_0) = 0$$

Soft wall model: Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Derived from variation of Action in AdS₅

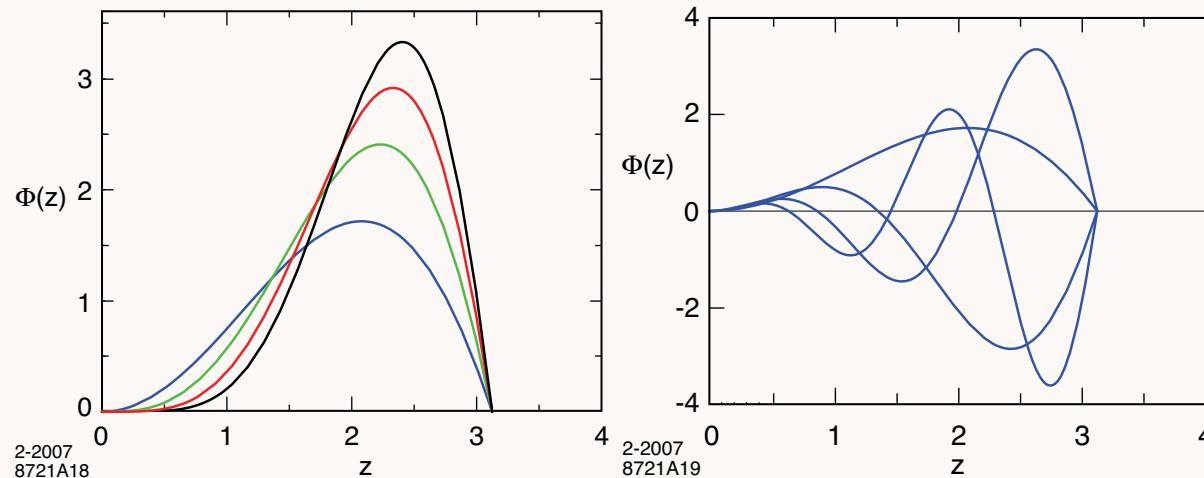


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32 \text{ GeV}$.

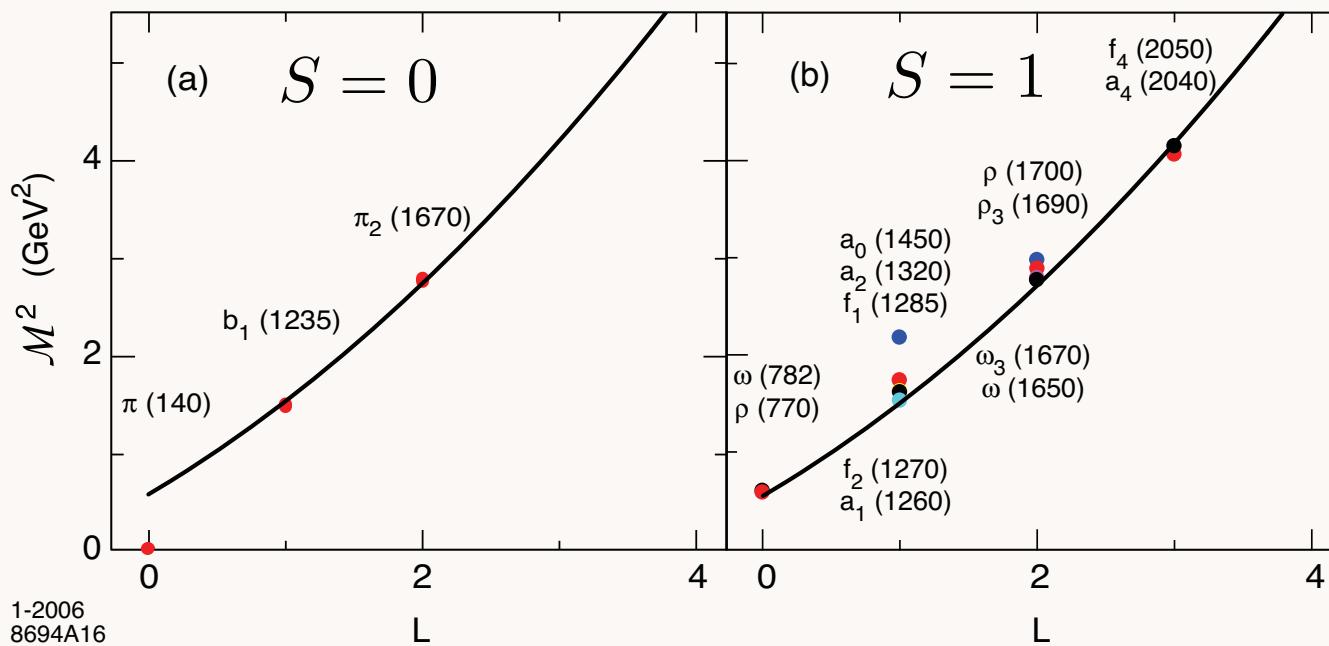


Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

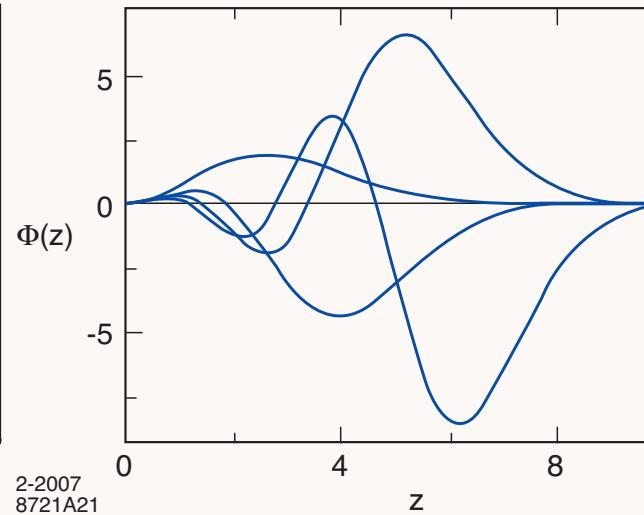
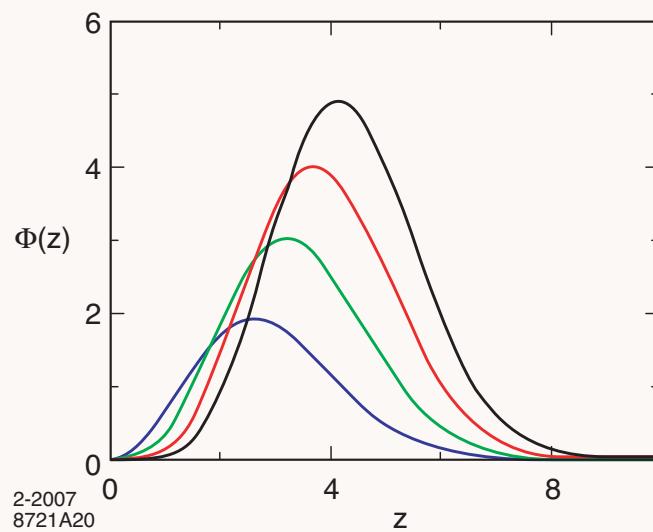
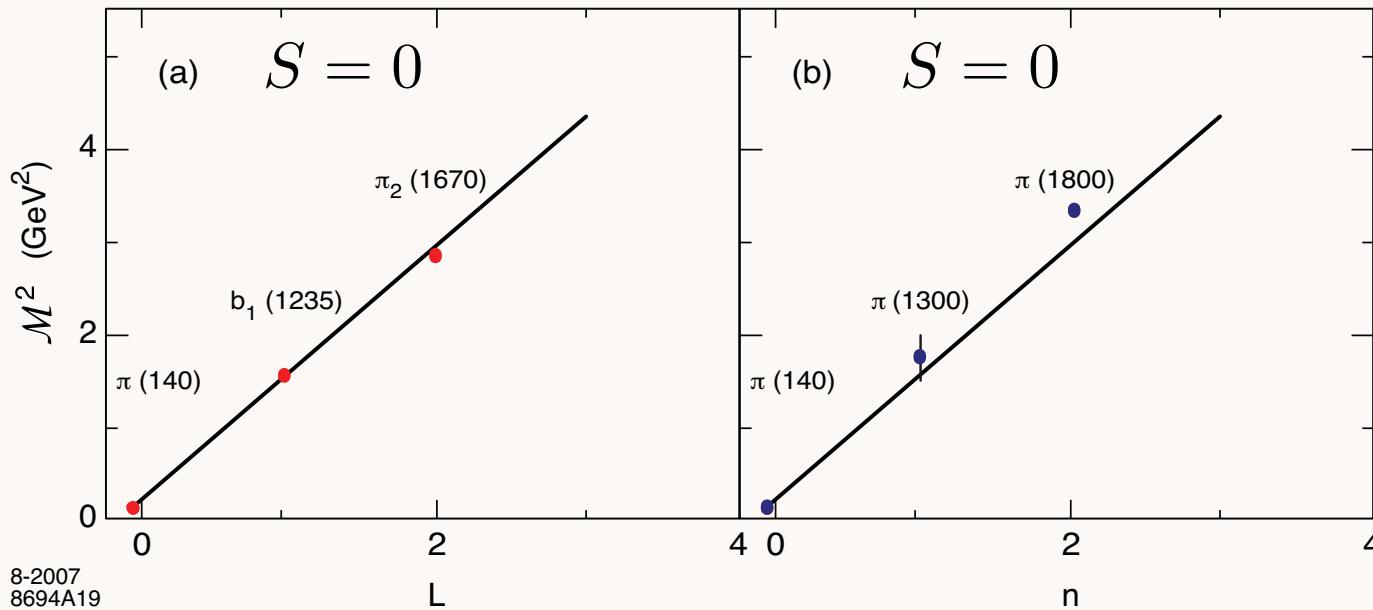


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .



Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

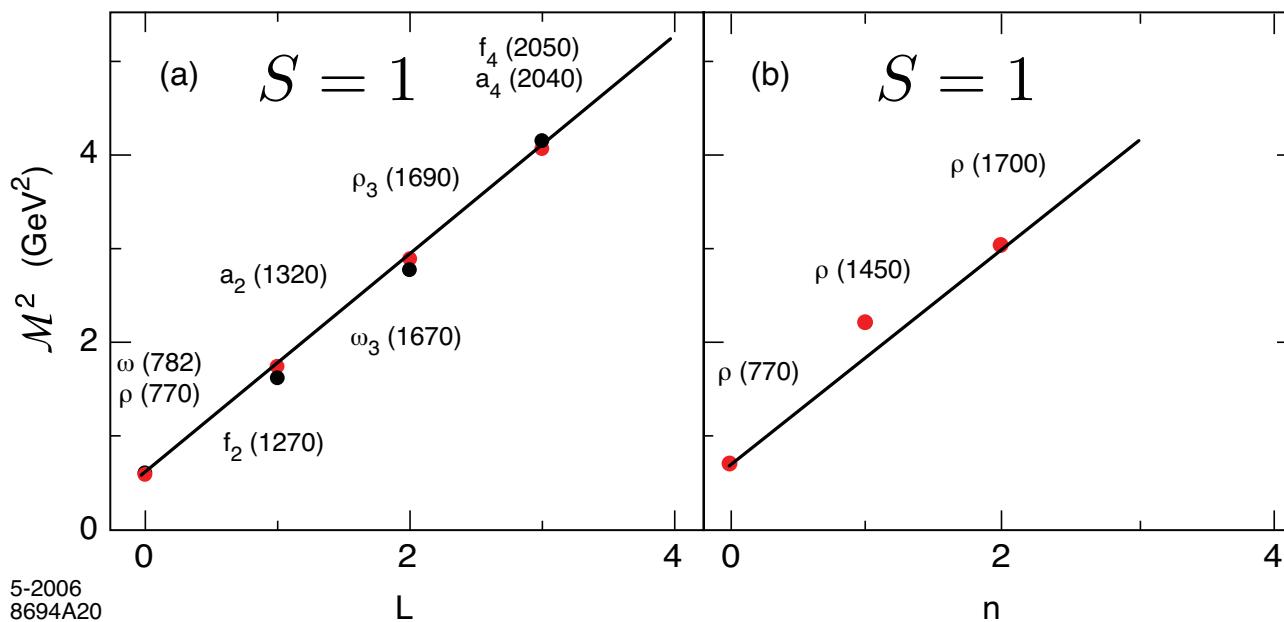
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L + S - 1) \right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)$$

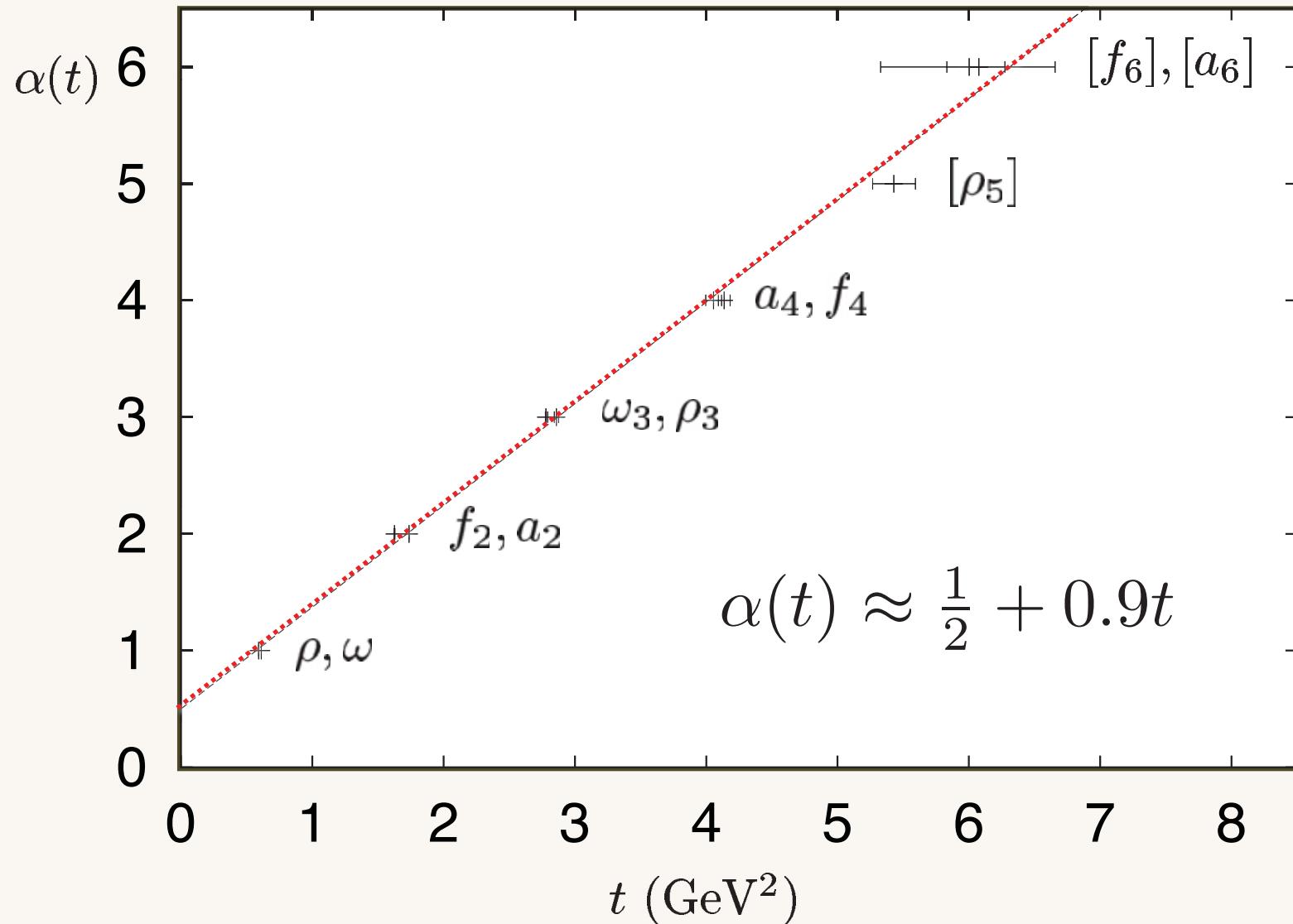
with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$.

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma(n + L + 1/2)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Fazio, Jugeau and Nicotri (2007).



AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

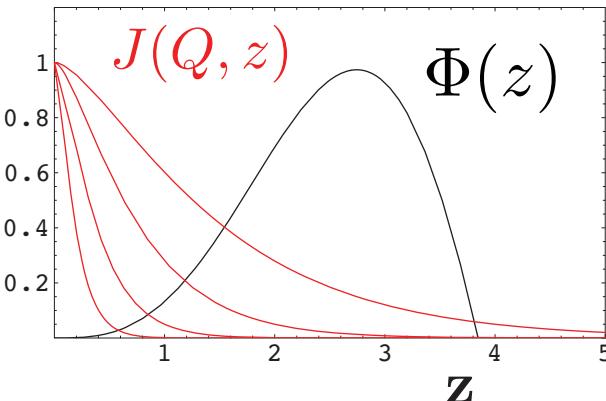
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQ K_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



Polchinski, Strassler
de Teramond, sjb

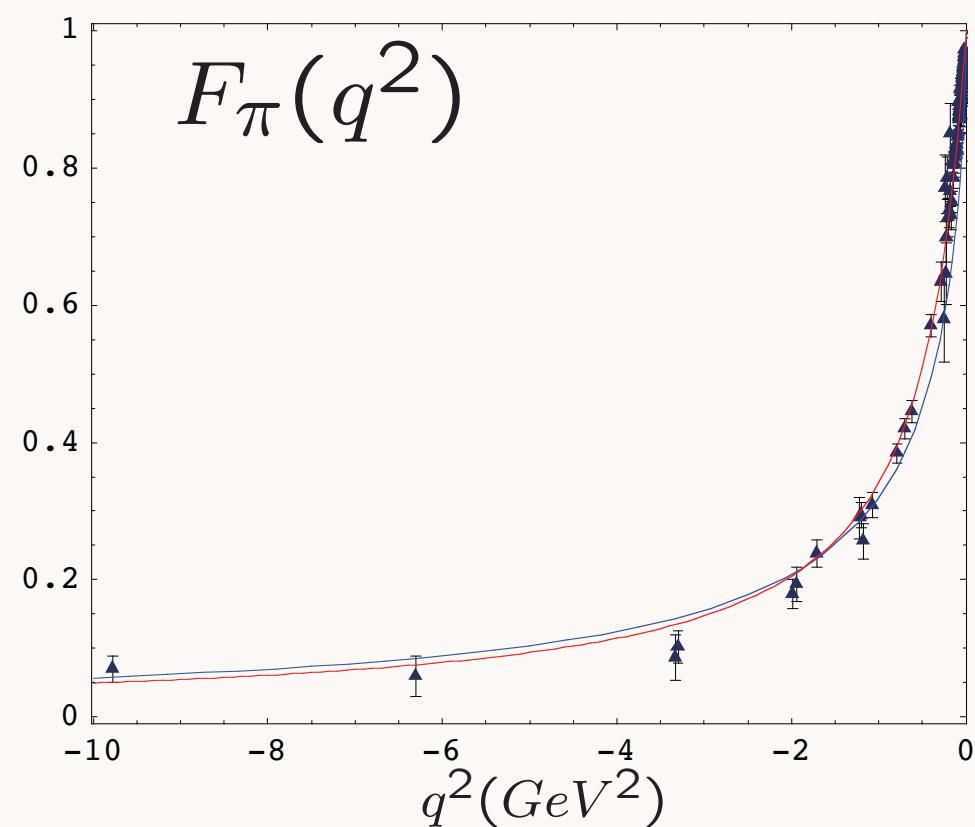
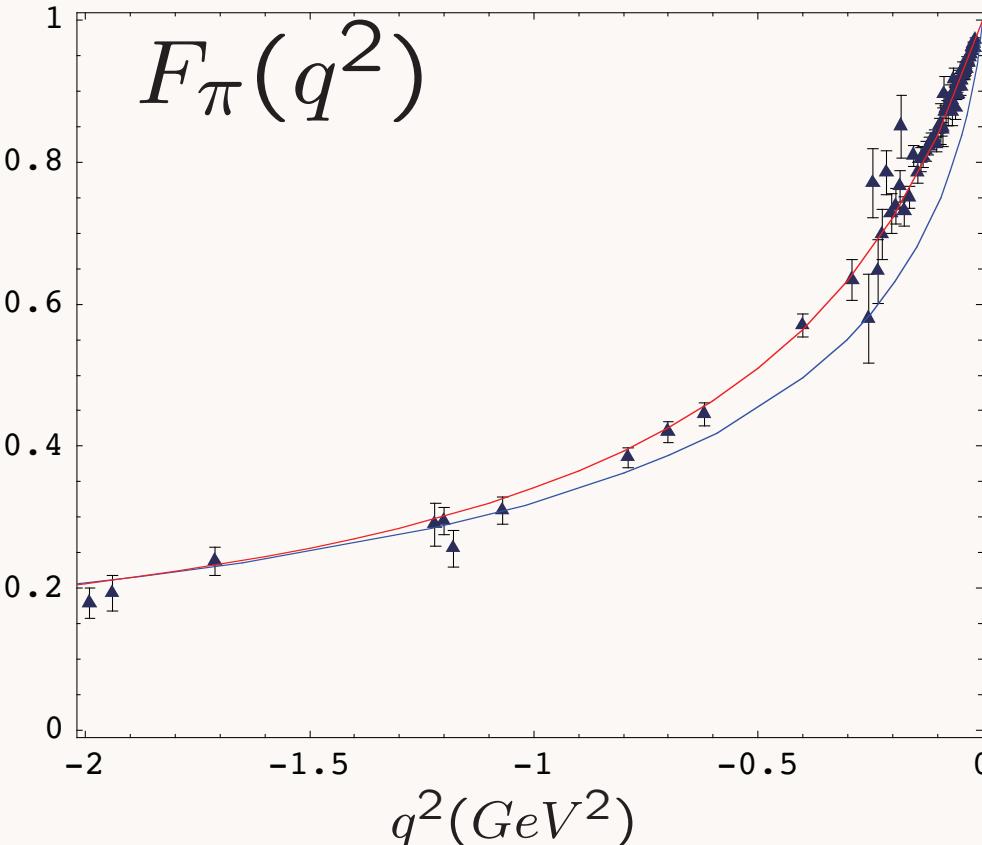
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rule
General result from
AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer



SW: Harmonic Oscillator Confinement



HW: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb

Note: Contributions to Mesons Form Factors at Large Q in AdS/QCD

- Write form factor in terms of an effective partonic transverse density in impact space \mathbf{b}_\perp

$$F_\pi(q^2) = \int_0^1 dx \int db^2 \tilde{\rho}(x, b, Q),$$

with $\tilde{\rho}(x, b, Q) = \pi J_0[b Q(1-x)] |\tilde{\psi}(x, b)|^2$ and $b = |\mathbf{b}_\perp|$.

- Contribution from $\rho(x, b, Q)$ is shifted towards small $|\mathbf{b}_\perp|$ and large $x \rightarrow 1$ as Q increases.

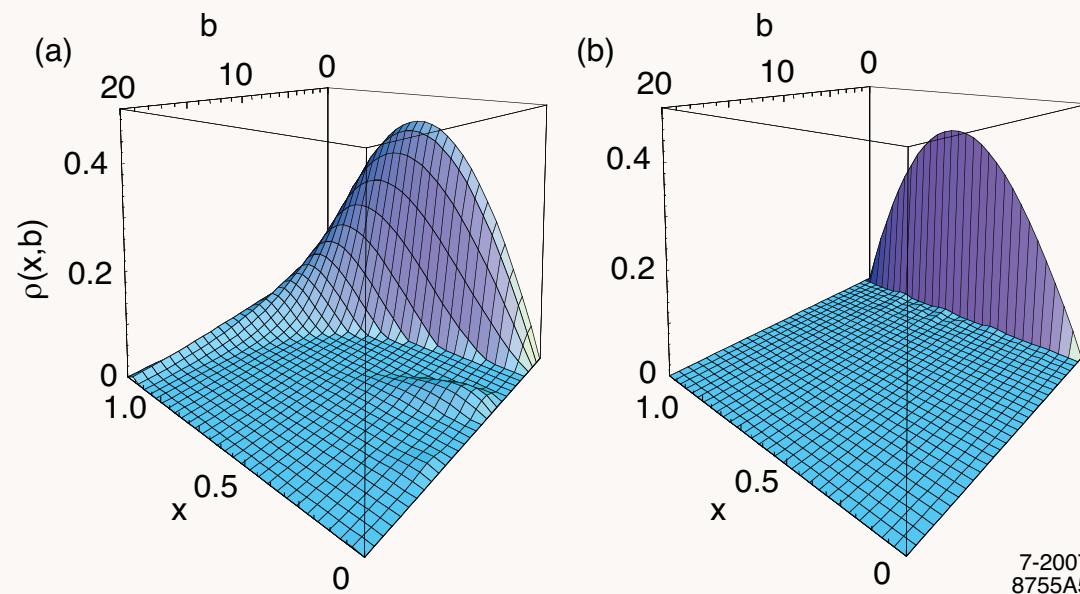


Fig: LF partonic density $\rho(x, b, Q)$: (a) $Q = 1$ GeV/c, (b) very large Q .

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ , Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\Gamma(N+z) = (N-1+z)(N-2+z)\dots(1+z)\Gamma(1+z)$.
- Form factor expressed as $N-1$ product of poles

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3,$$

...

$$F(Q^2) = \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)\dots\left(N-1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.$$

- For large Q^2 :

$$F(Q^2) \rightarrow (N-1)! \left[\frac{4\kappa^2}{Q^2} \right]^{(N-1)}.$$

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x \vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space \vec{b}_\perp

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i \vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix \vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 && \text{Soper} \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

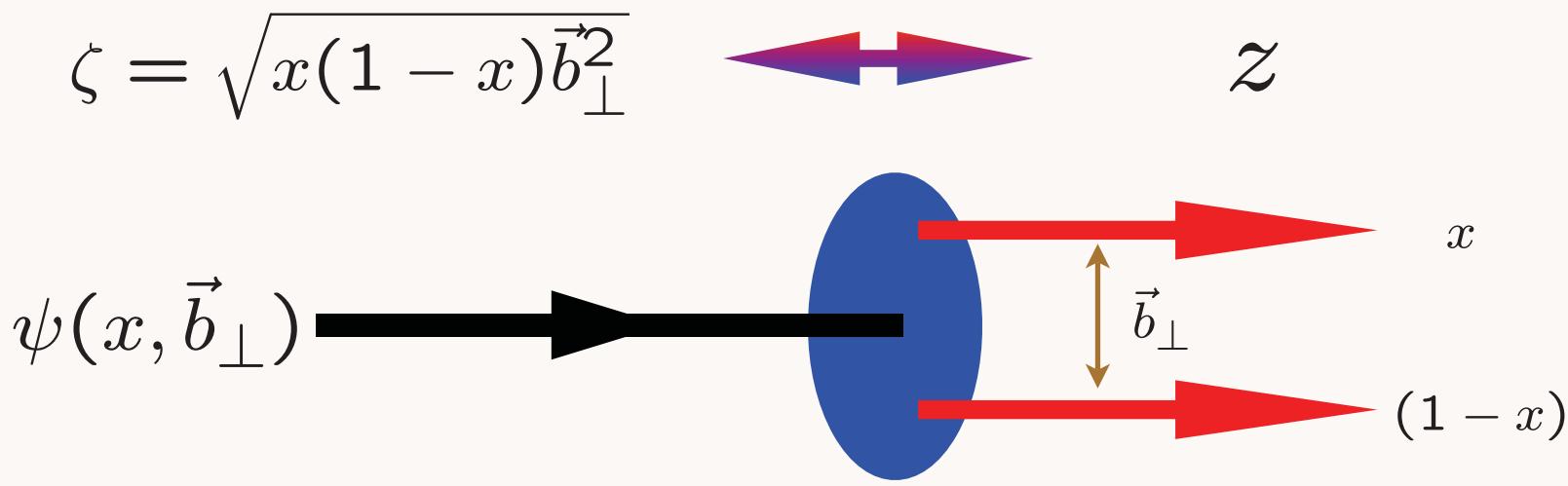
$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

LF(3+1)

AdS₅

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

Holography: Map AdS/CFT to $3+1$ LF Theory

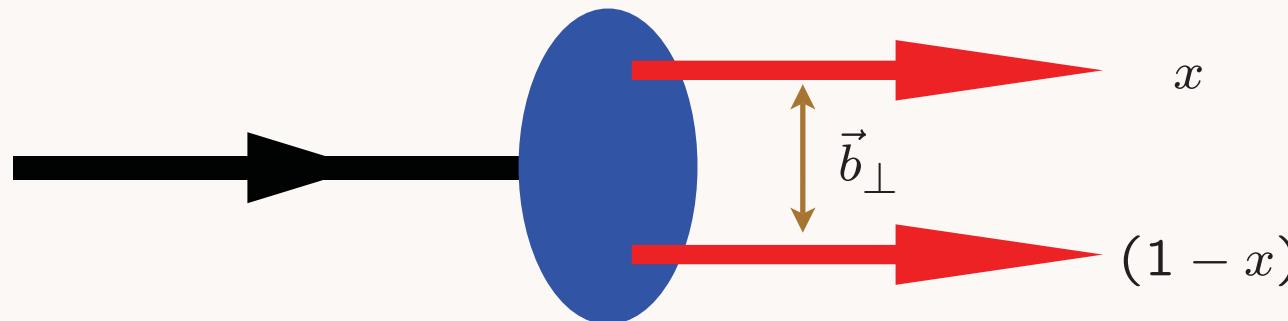
Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_\perp^2.$$

G. de Teramond, sjb



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2$$

confining potential:

Example: Pion LFWF

- Two parton LFWF bound state:

$$\tilde{\psi}_{\bar{q}q/\pi}^{HW}(x, \mathbf{b}_\perp) = \frac{\Lambda_{\text{QCD}} \sqrt{x(1-x)}}{\sqrt{\pi} J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)} |\mathbf{b}_\perp| \beta_{L,k} \Lambda_{\text{QCD}}\right) \theta\left(\mathbf{b}_\perp^2 \leq \frac{\Lambda_{\text{QCD}}^{-2}}{x(1-x)}\right),$$

$$\tilde{\psi}_{\bar{q}q/\pi}^{SW}(x, \mathbf{b}_\perp) = \kappa^{L+1} \sqrt{\frac{2n!}{(n+L)!}} [x(1-x)]^{\frac{1}{2}+L} |\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L(\kappa^2 x(1-x)\mathbf{b}_\perp^2).$$

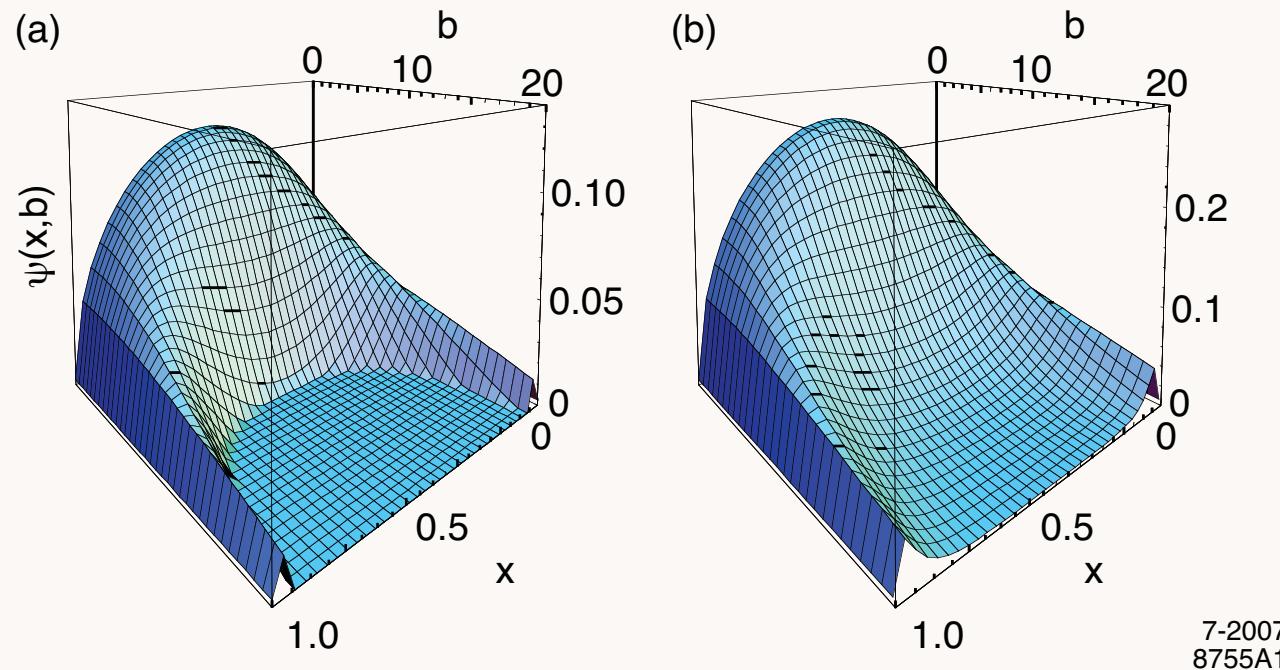
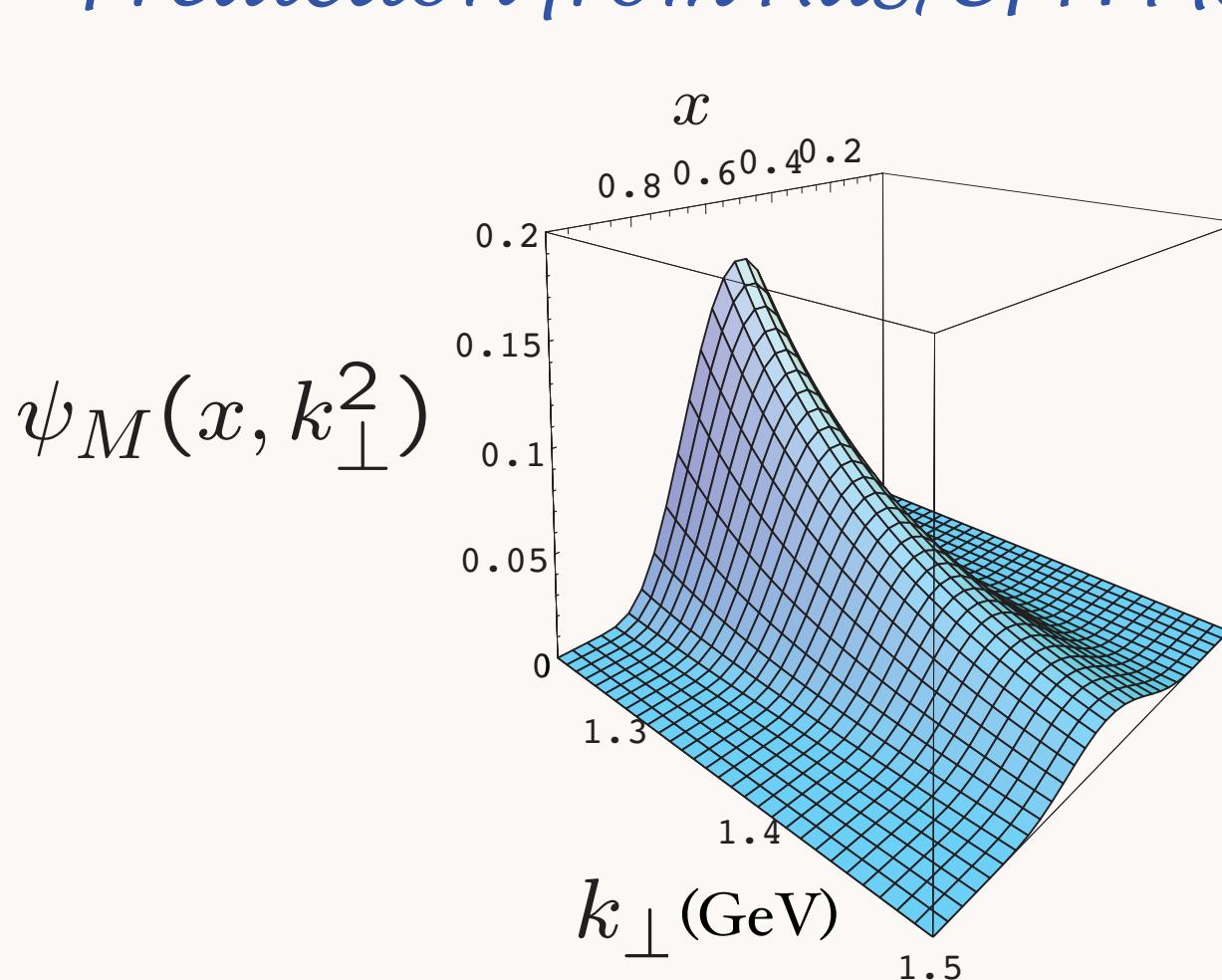


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$, (b) SW model $\kappa = 0.375 \text{ GeV}$.

Prediction from AdS/CFT: Meson LFWF



de Teramond, sjb

**“Soft Wall”
model**

$\kappa = 0.375$ GeV

massless quarks

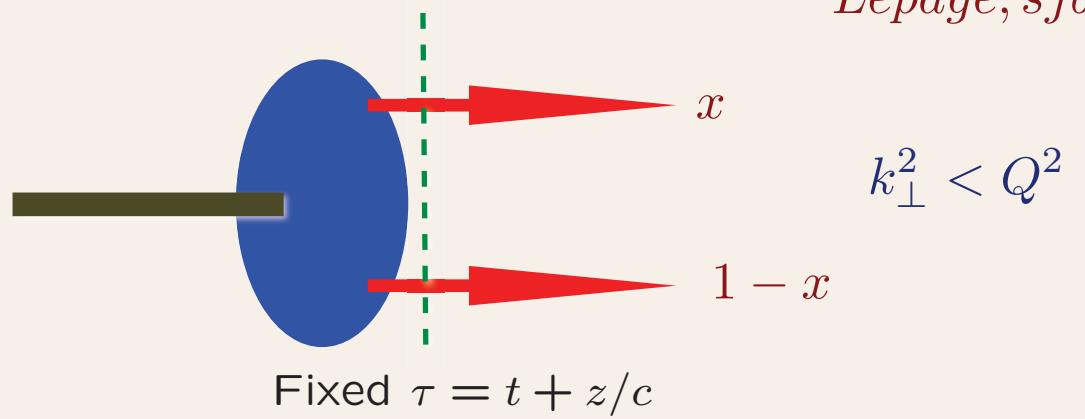
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \quad \phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Hadron Distribution Amplitudes

Lepage, sjb

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for mesons, baryons

Lepage, sjb

- Evolution Equations from PQCD,
OPE, Conformal Invariance

Frishman, Lepage, Sachrajda, sjb

Peskin Braun

Efremov, Radyushkin Chernyak et al

- Compute from valence light-front wavefunction in
light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_\pi = 1/5 = 0.20$$

$$\phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_\pi = 1/4 = 0.25$$

$$\phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $\langle \xi^2 \rangle_\pi = 0.28 \pm 0.03$

Donnellan et al.

Lattice (II) $\langle \xi^2 \rangle_\pi = 0.269 \pm 0.039$

Braun et al.